Capacidade do Canal MIMO em Arranjos ULA e URA

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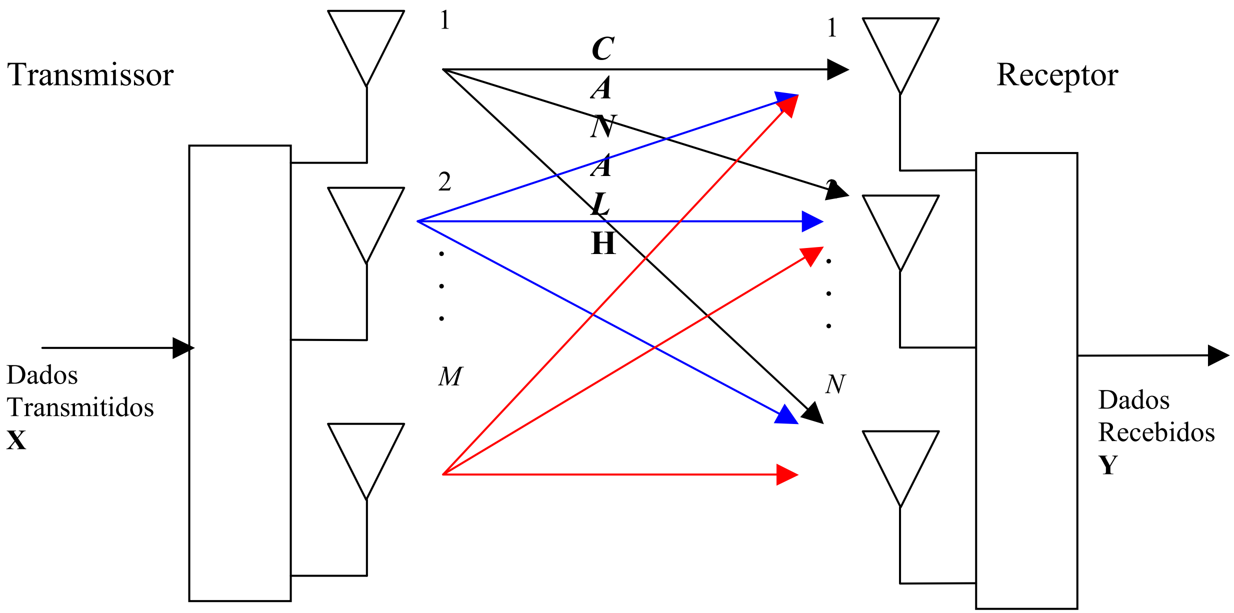
Canal MIMO



$$\begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} h_{11} & \cdots & h_{1n} \\ \vdots & \ddots & \vdots \\ h_{m1} & \cdots & h_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_m \end{bmatrix},$$

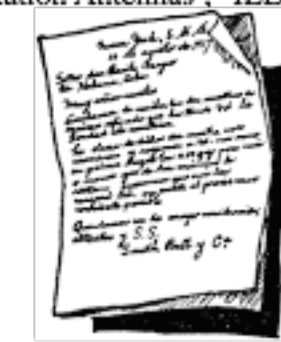
$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

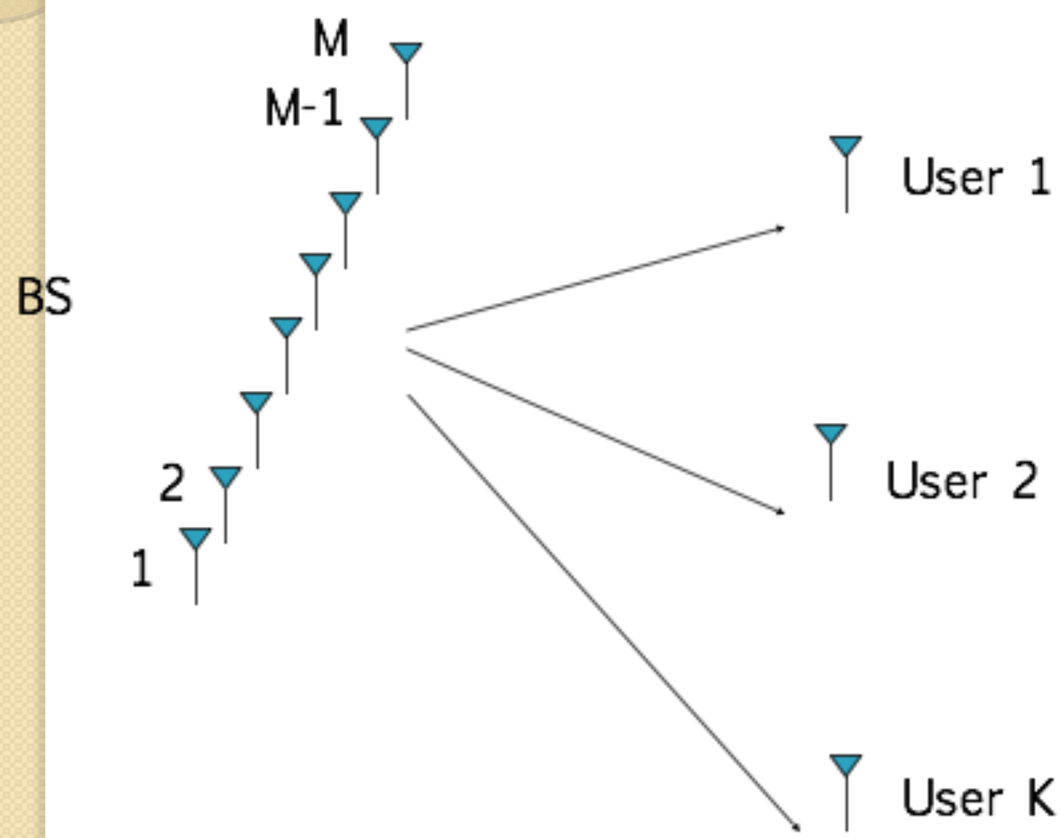
$$n \sim N(0, \sigma^2 \mathbf{I})$$

Massive MIMO

T. L. Marzetta, "The case for MANY (greater than 16) antennas as the base station," in *Proc. ITA*, San Diego, CA, USA, Jan. 2007.

Thomas L. Marzetta , "Noncooperative Cellular Wireless with Unlimited Numbers of Base Station Antennas ," IEEE Trans. Commun. 2010.





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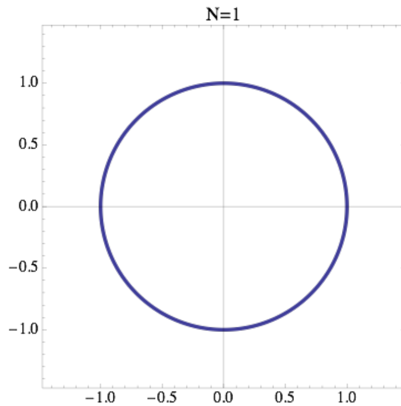
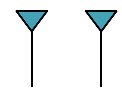
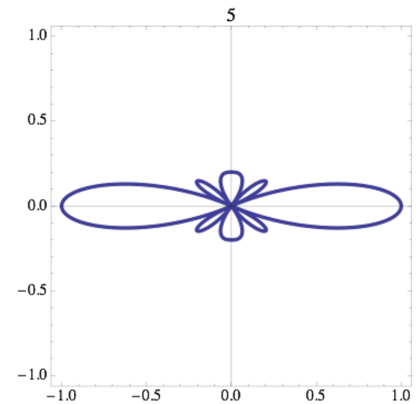
Antenna Array Gain

1 Element



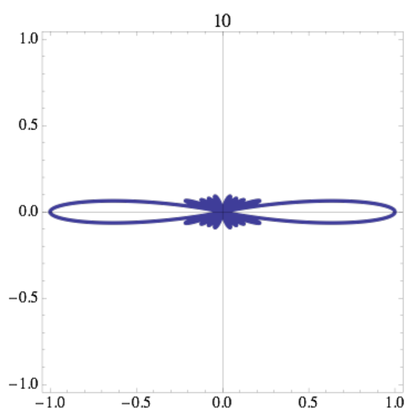
2 Elements

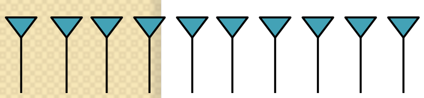




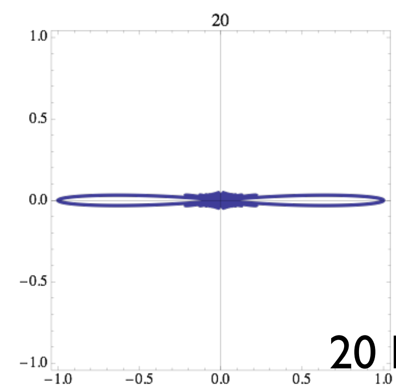
Antenna Aperture λ / D

10 Elements

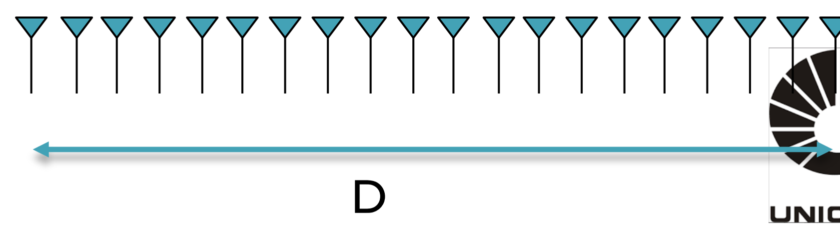




20 Elements



20 Elements



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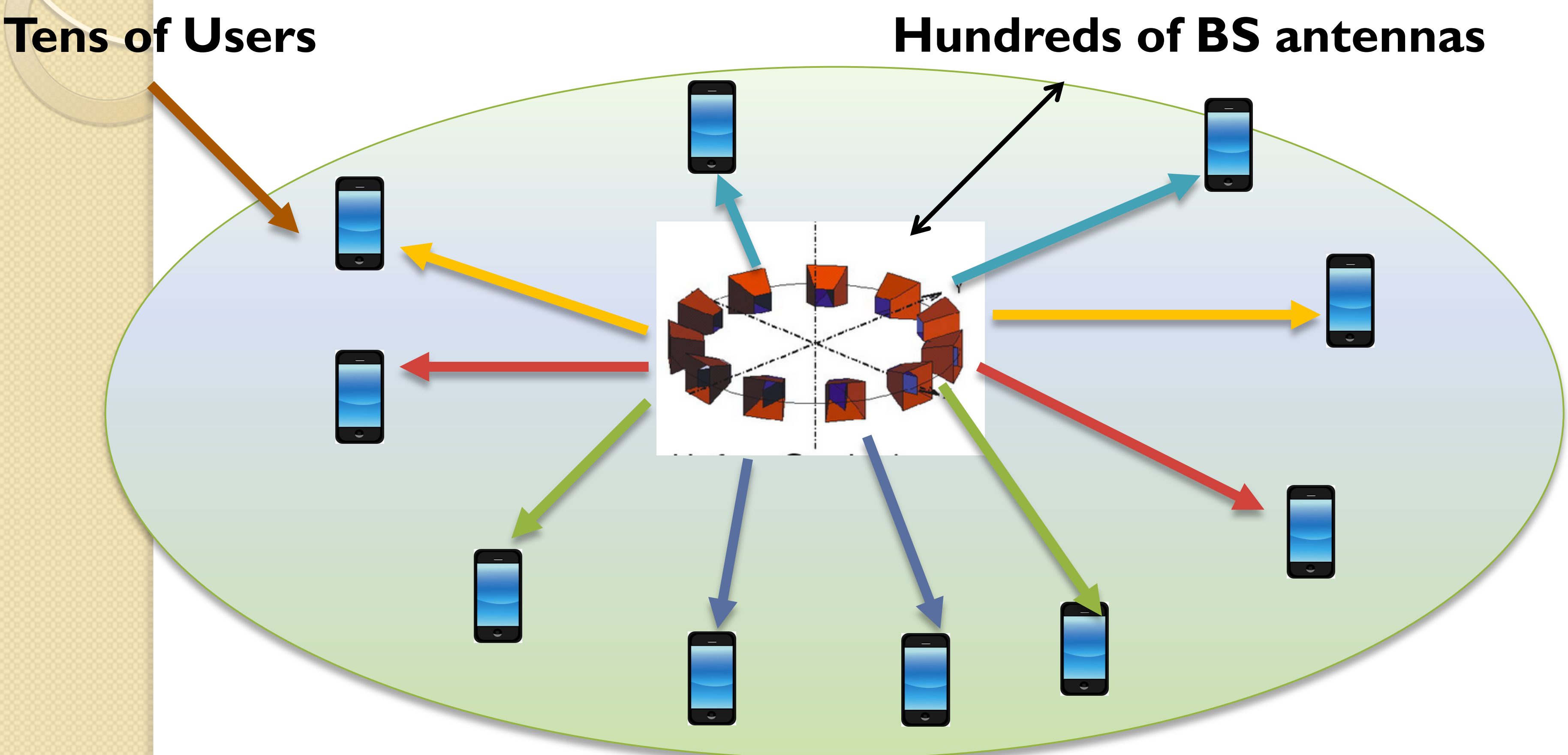
What is Massive MIMO

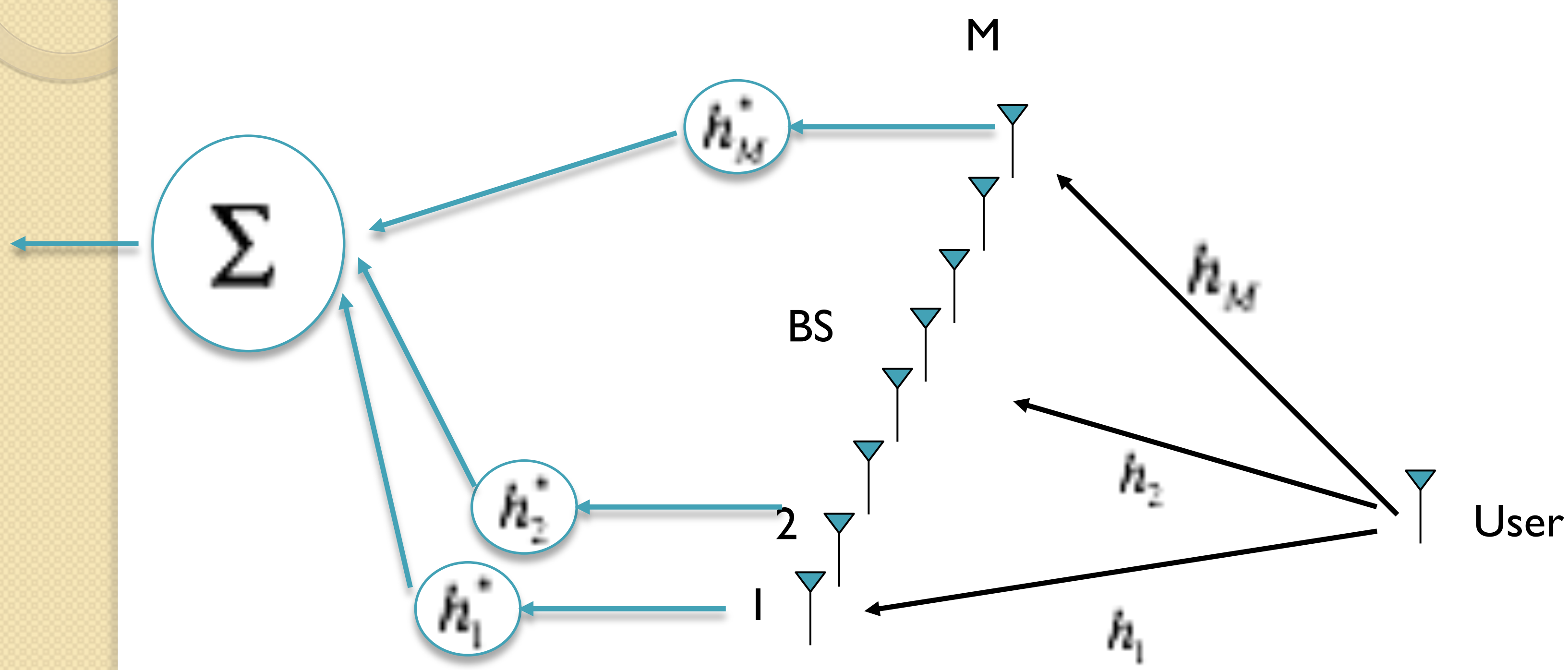
Essentially multiuser MIMO with lots of base station antennas



A very large antenna array at each base station
A large number of users are served simultaneously
An excess of base station (BS) antennas

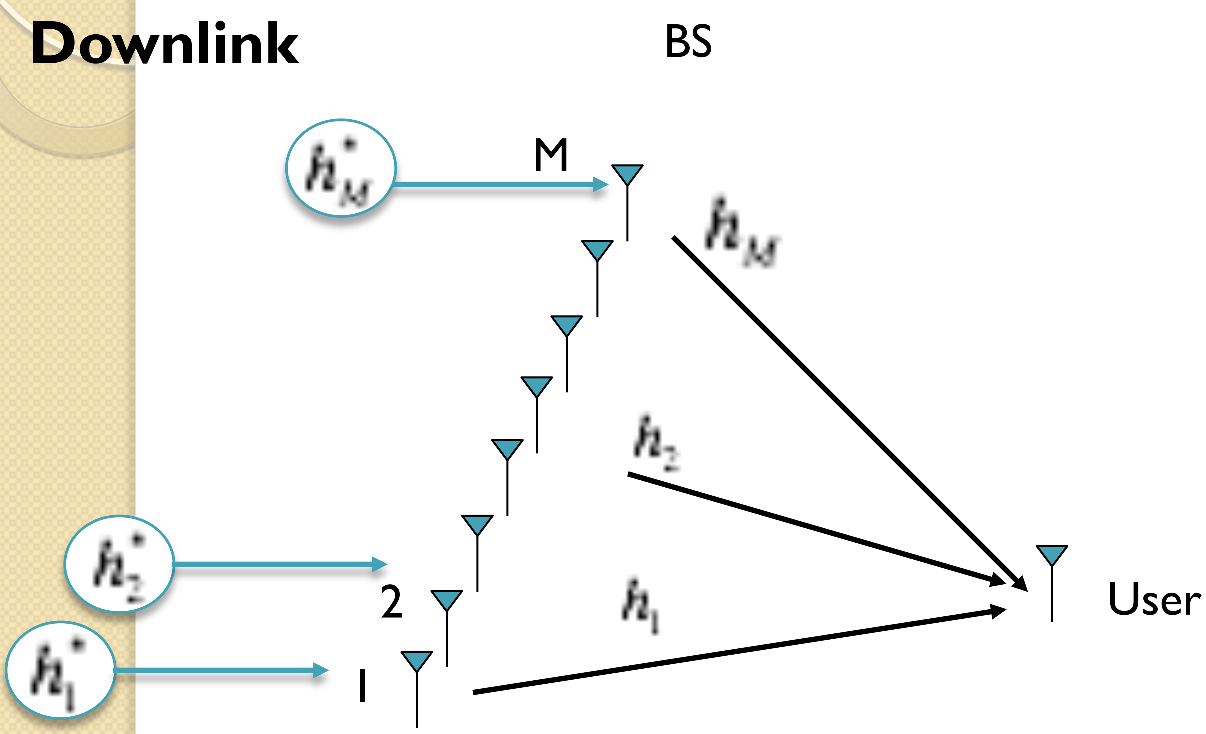
Maximal Ratio Combining

Uplink



Maximal Ratio Transmission

Downlink



Knowledge of the Channel at the transmitter side.
Reciprocity!

Canal de Múltiplo Acesso (MAC)

- Sinal recebido

$$\mathbf{y}_{ul} = \sqrt{p_u} \sum_{k=1}^{K} \mathbf{h}_k s_k + \mathbf{n} = \sqrt{p_u} \mathbf{H} \mathbf{s} + \mathbf{n},$$

$$C_{sum,ul} = \log_2 \det \left(\mathbf{I}_K + p_u \mathbf{H}^H \mathbf{H} \right).$$

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Favorable Propagation

$$\mathbf{h}_i^H \mathbf{h}_j = \begin{cases} 0, & i \neq j \quad i,j = 1, ..., K \\ \|\mathbf{h_k}\|^2 \neq 0, & k = 1, ..., k \end{cases}.$$

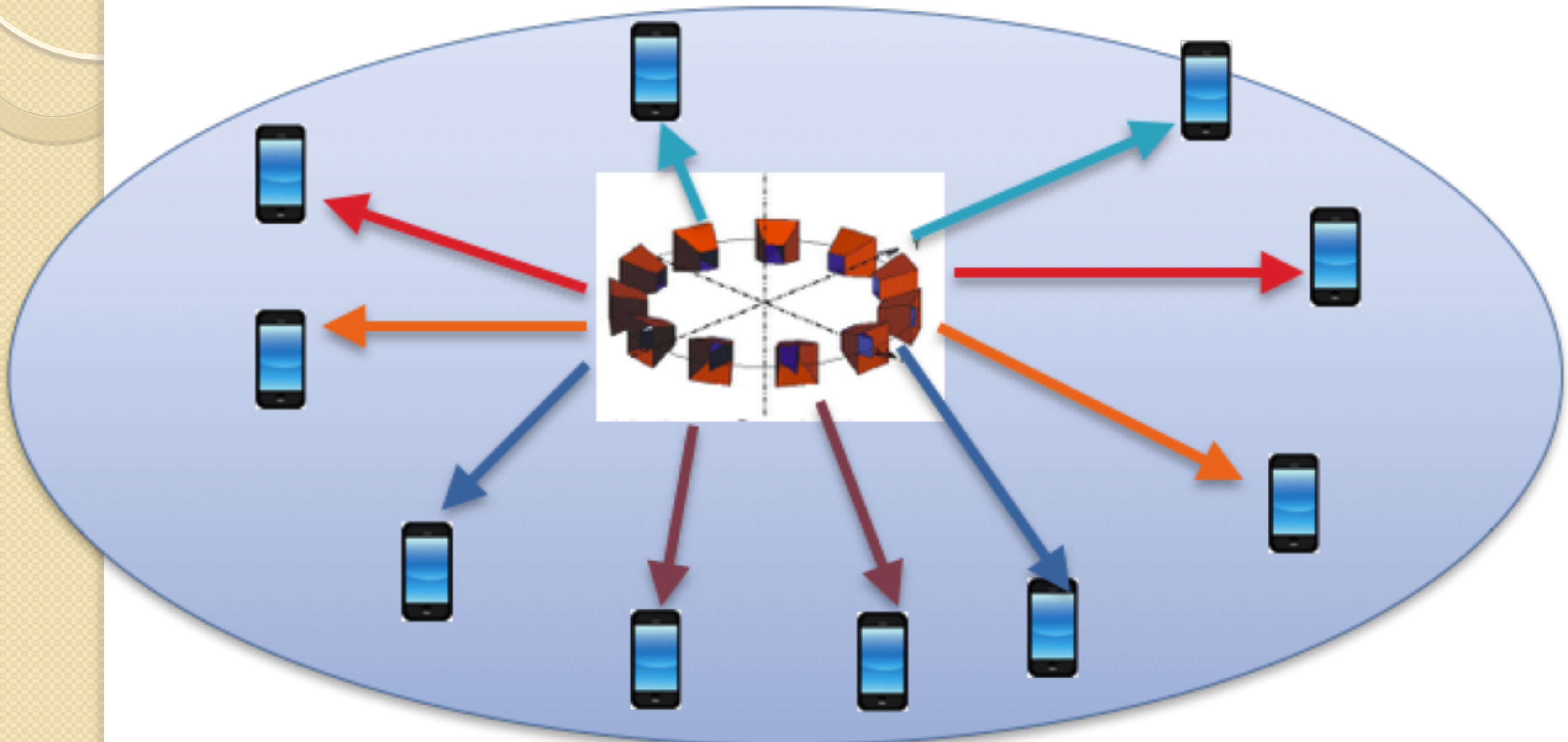
$$\frac{1}{M}\mathbf{h}_i^H \mathbf{h}_j \to 0, \qquad M \to \infty.$$

$$\mathbf{H}^H \mathbf{H} = \mathrm{Diag}\{\|\mathbf{h}_1\|^2, ..., \|\mathbf{h}_K\|^2\}$$

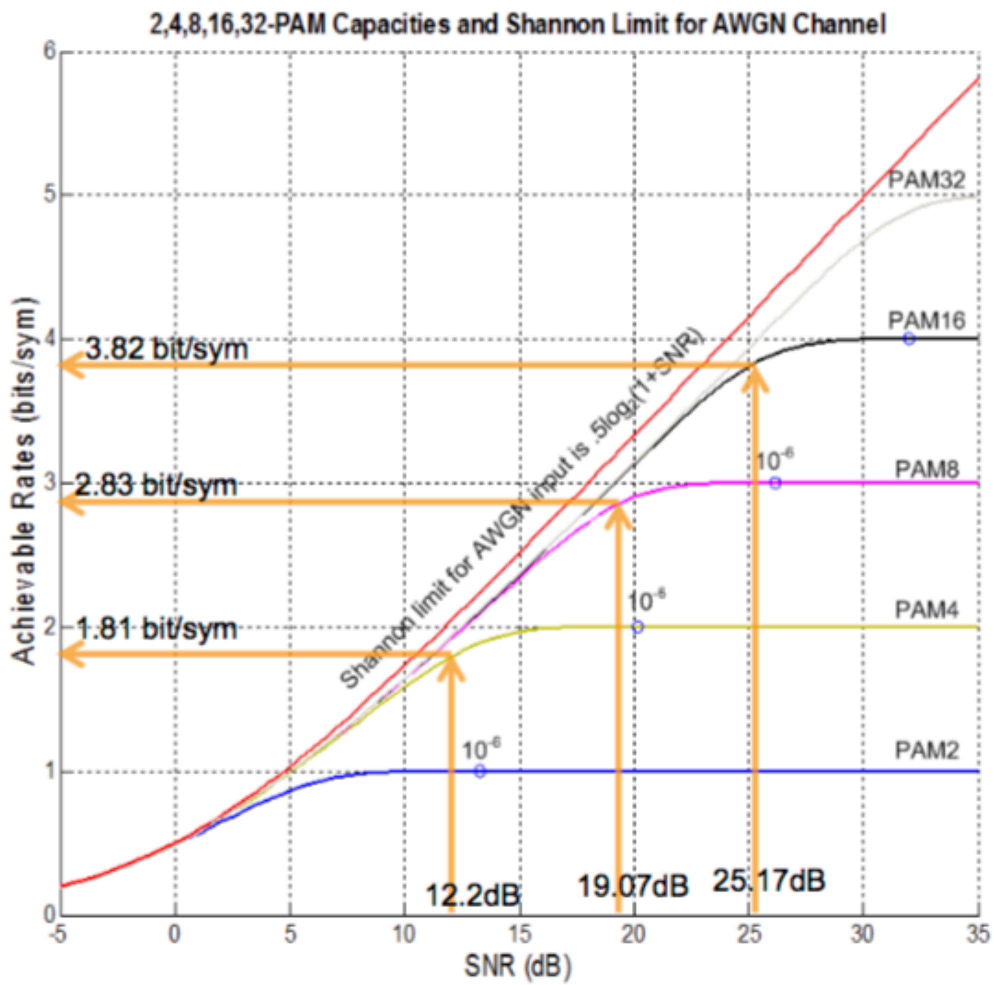
Condition Number:

$$\kappa(\mathbf{H}^H \mathbf{H}) = \frac{\sigma_{max}}{\sigma_{min}} = 1$$

Favorable Propagation



Capacidade com Restrições Espaciais



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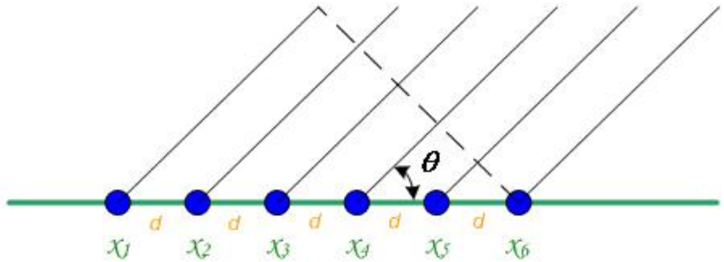
Uniform Linear Array Model (ULA)



$$\mathbf{h}_k = \mathrm{e}^{i\xi_k} \begin{bmatrix} 1 & \mathrm{e}^{-i\pi \sin(\phi_k)} & \dots & \mathrm{e}^{-i\pi(M-1)\sin(\phi_k)} \end{bmatrix}^T$$

$$\mathbf{H}^H\mathbf{H} = \begin{pmatrix} \mathbf{h}_i^H\mathbf{h}_i & \mathbf{h}_i^H\mathbf{h}_j \\ \mathbf{h}_j^H\mathbf{h}_i & \mathbf{h}_i^H\mathbf{h}_i \end{pmatrix} \text{ for } i \neq j$$

Diagonal

$$\mathbf{h}_i^H\mathbf{h}_i = \mathrm{e}^{-i0\pi(\sin(\phi_i)-\sin(\phi_i))} + \mathrm{e}^{-i1\pi(\sin(\phi_i)-\sin(\phi_i))} + \dots + \mathrm{e}^{-iM\pi(\sin(\phi_i)-\sin(\phi_i))} = M$$

Out Diagonal

$$\mathbf{h}_i^H\mathbf{h}_j = \sum_{m=0}^{M} \mathrm{e}^{-imx} = \frac{-1+\mathrm{e}^{iMx}}{-1+\mathrm{e}^{ix}}$$

Uniform Linear Array Model

$$\mathbf{h}_i^H \mathbf{h}_j = \sum_{m=0}^{M} \mathrm{e}^{-imx} = \frac{-1 + \mathrm{e}^{iMx}}{-1 + \mathrm{e}^{ix}}$$

$$\mathbf{H}^H \mathbf{H} = \begin{pmatrix} M & \frac{-1+\mathrm{e}^{iMx}}{-1+\mathrm{e}^{ix}} \\ \frac{-1+\mathrm{e}^{-iMx}}{-1+\mathrm{e}^{-ix}} & M \end{pmatrix}$$

$$x = \pi\left(\sin\left(\phi_i\right) - \sin\left(\phi_j\right)\right)$$

Eigenvalues calculated for two users:

$$\lambda_1 = M - \frac{\sqrt{(-1 + \cos(x))(-1 + \cos(Mx))}}{1 - \cos(x)},$$

$$\lambda_2 = M + \frac{\sqrt{(-1 + \cos(x))(-1 + \cos(Mx))}}{1 - \cos(x)}.$$

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Capacidade Média Espacial

$$C(\phi) = \log_2 \det\left(I + \rho \mathbf{H}(\phi)^{\mathbf{H}} \mathbf{H}(\phi) \right) = \log_2 \prod \left(1 + \rho \lambda_i(\phi)\right)$$

$$C(\phi) = \sum_{i=1}^{M} \log_2\left(1 + \rho \lambda_i(\phi)\right)$$

$$C = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} C(\phi)\, d\phi$$

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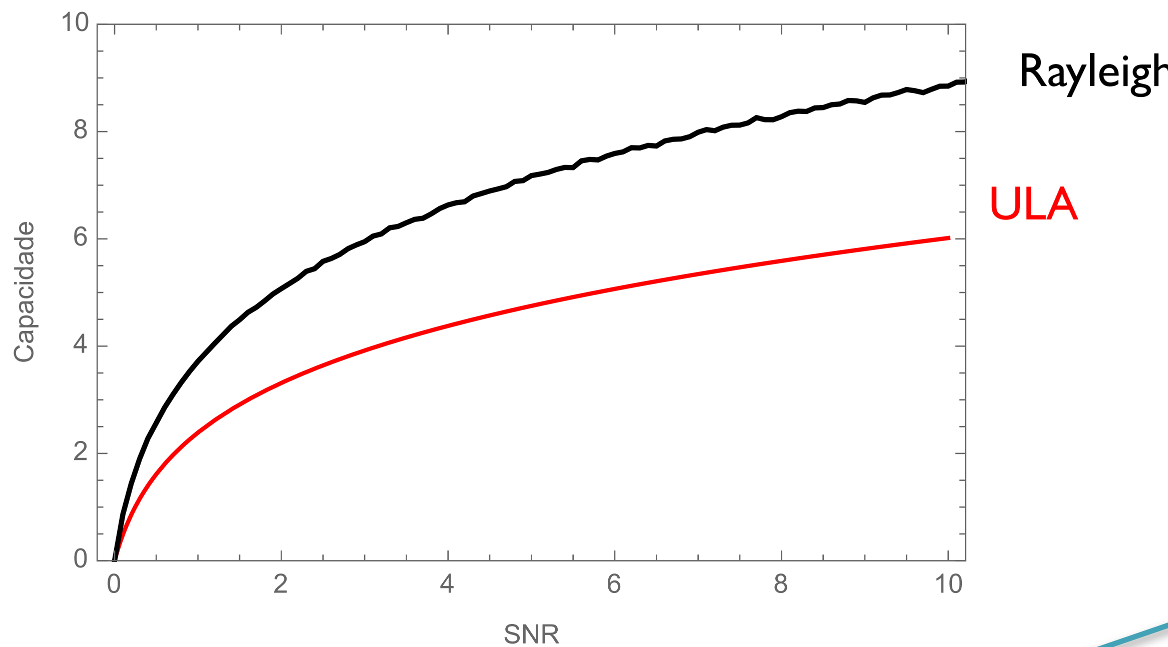
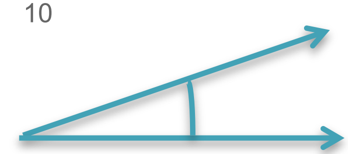
Resultados ULA

Dois usuários e duas antenas.



M=2

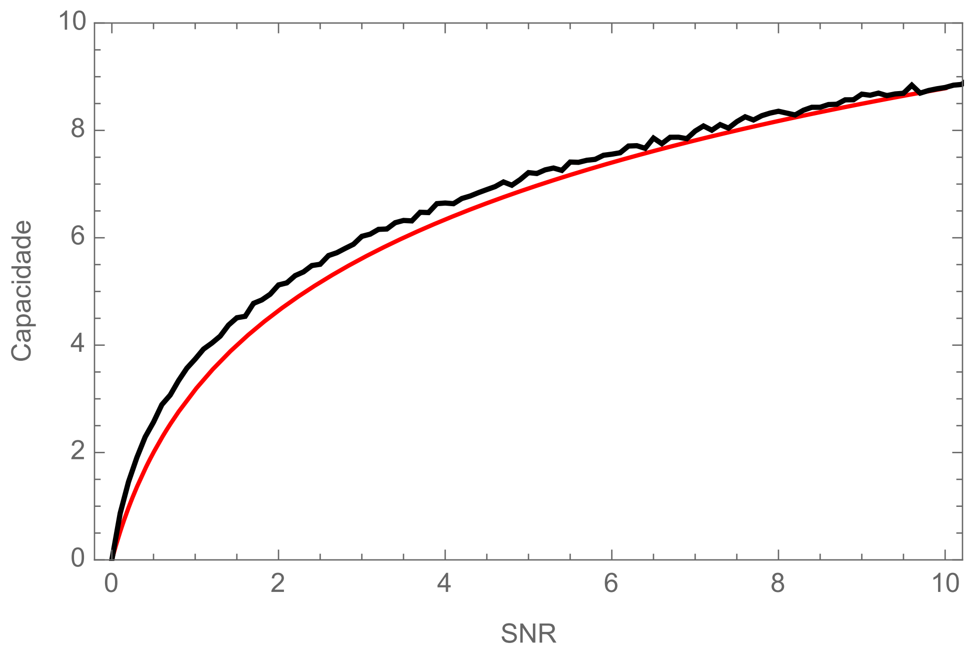


$$\Delta\phi = \pi / 20$$

$$\Delta\phi = \phi_1 - \phi_2 = \pi / 20$$

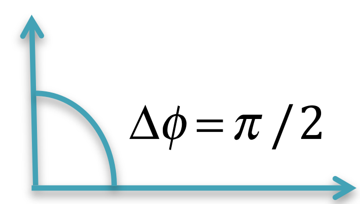
Resultados ULA

Dois usuários e duas antenas.



Rayleigh

ULA



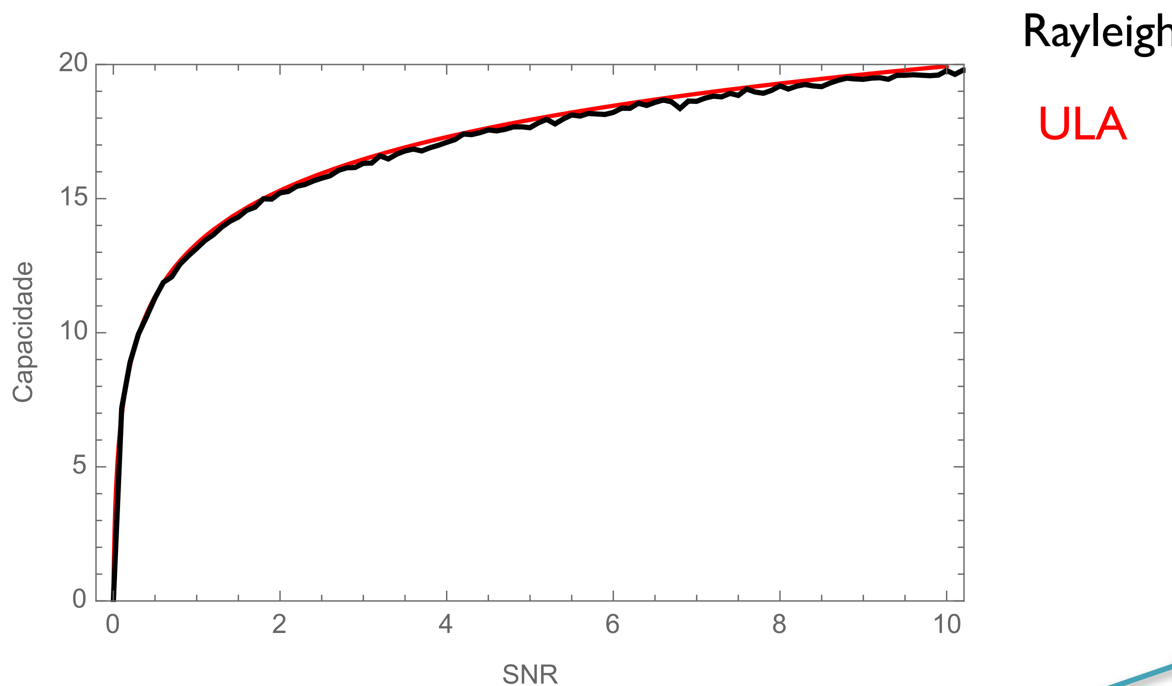
$\Delta\phi = \pi / 2$

K=2,

M=2

$$\Delta\phi = \phi_1 - \phi_2 = \pi / 2$$

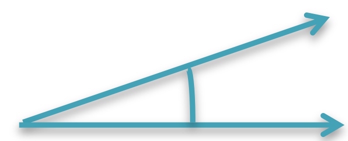
Resultados ULA



K=2

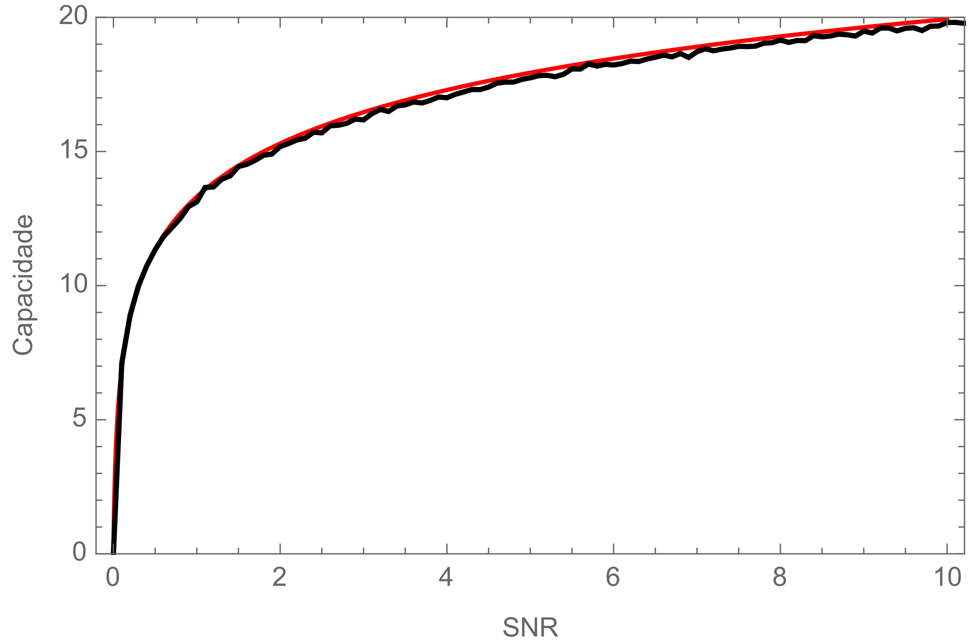
M=100

$$\Delta\phi = \phi_1 - \phi_2 = \pi / 20$$



$$\Delta\phi = \pi / 20$$

Resultados ULA

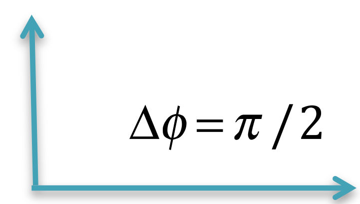


Rayleigh

ULA

K=2
M=100

$$\Delta\phi = \phi_1 - \phi_2 = \pi / 2$$



$$\Delta\phi = \pi / 2$$

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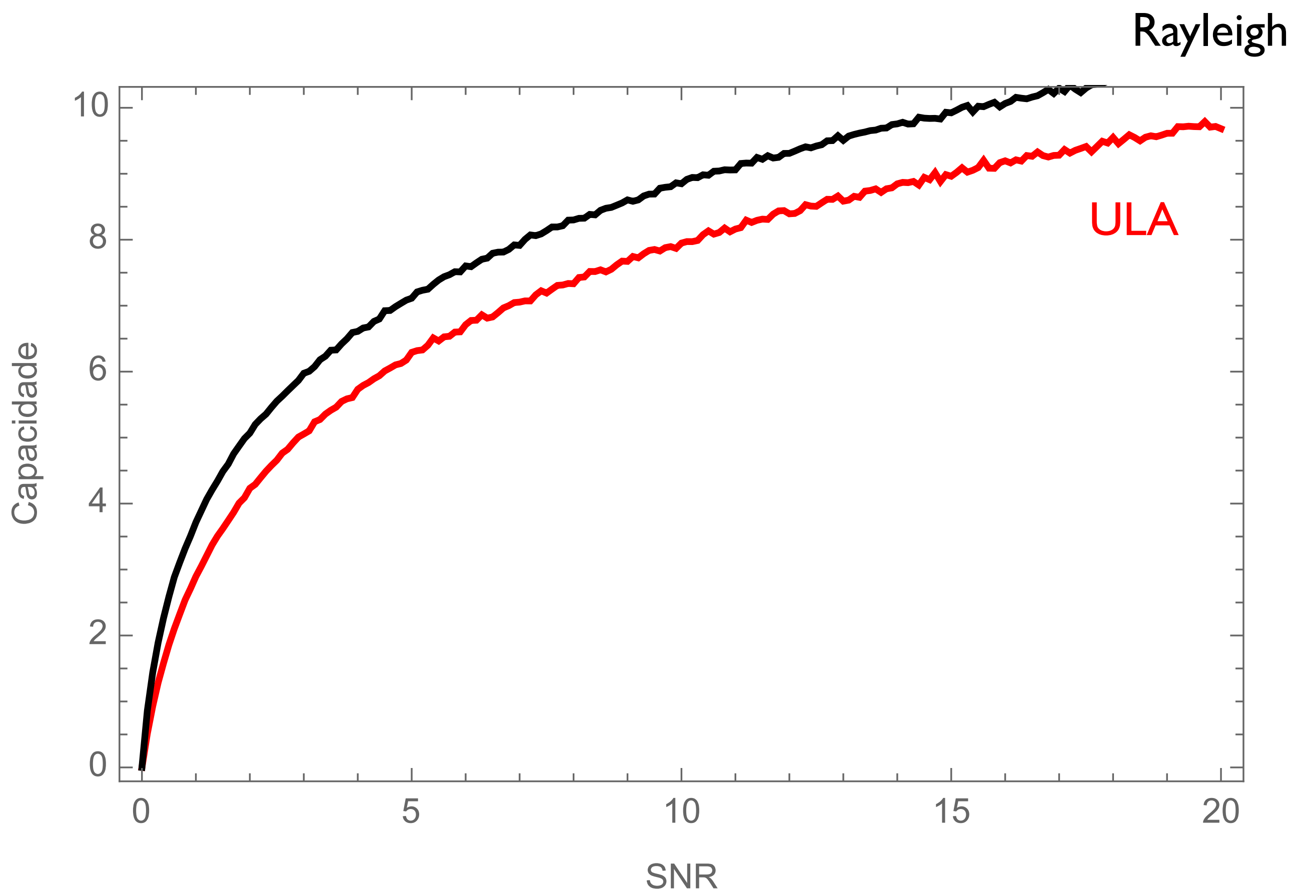
Resultados ULA

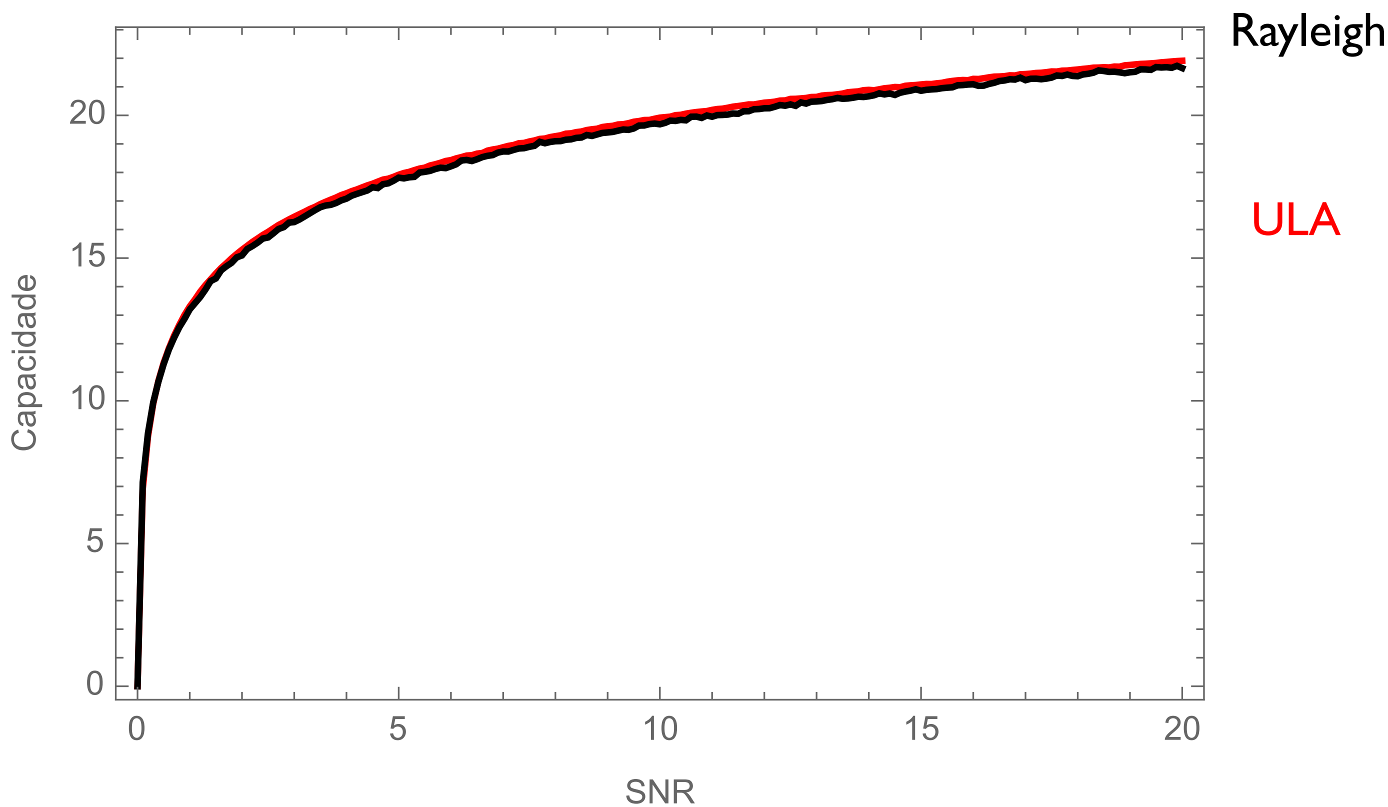


K=2

M=2

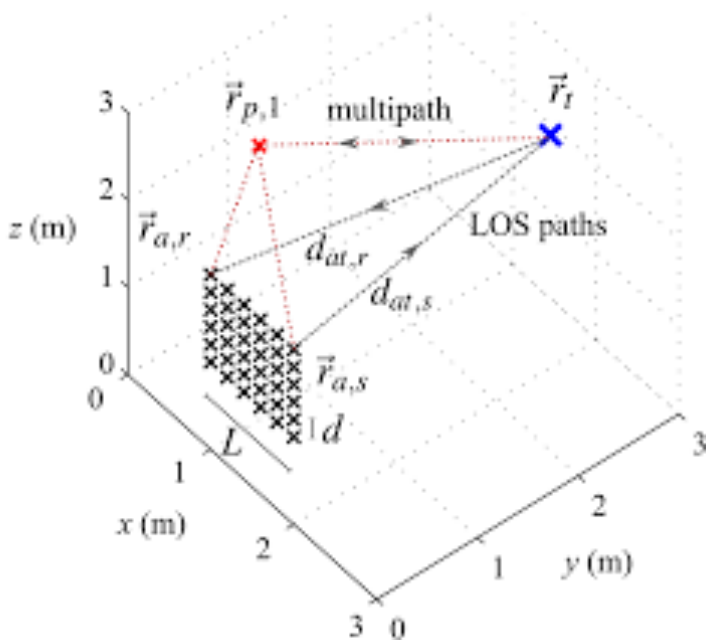
Ângulos Aleatórios entre $-\pi/2$ e $\pi/2$

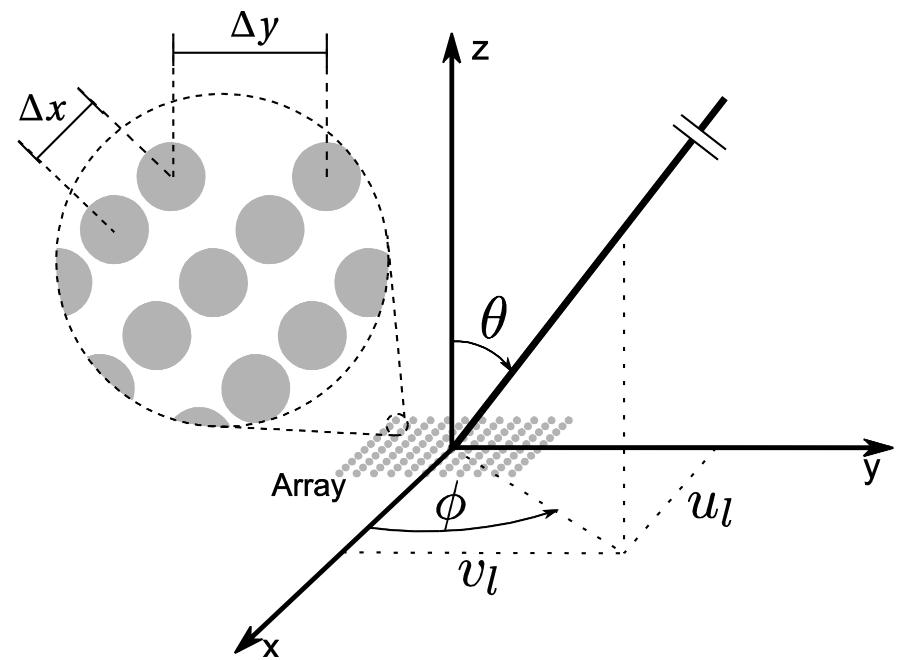
Resultados ULA



K=2
M=100
Ângulos Aleatórios entre $-\pi/2$ e $\pi/2$

Uniform Rectangular Array Model (URA)





Uniform Rectangular Array Model (URA)

$$h_{k,m_x,m_y} = \mathrm{e}^{-i\pi(m_x-1)\mu_k}\mathrm{e}^{-i\pi(m_y-1)\nu_k}$$

$$\mathbf{h}_{URA,k} = \mathrm{e}^{i\xi_k}\mathrm{vec}\{\mathbf{\Psi}_k\}$$

$$\mathbf{\Psi}_k = \left[1 \; \mathrm{e}^{-i\pi\nu_k} \;\; ... \;\; \mathrm{e}^{-i\pi(M_y-1)\nu_k}\right]^T \left[1 \; \mathrm{e}^{-i\pi\mu_k} \;\; ... \;\; \mathrm{e}^{-i\pi(M_x-1)\mu_k}\right]$$

$$\begin{aligned}
\mathbf{h}_i^H\mathbf{h}_i &= \mathrm{e}^{i0\pi\mu_i}\mathrm{e}^{i0\pi\nu_i}\mathrm{e}^{-i0\pi\mu_i}\mathrm{e}^{-i0\pi\nu_i} \\
&+ \mathrm{e}^{i1\pi\mu_i}\mathrm{e}^{i0\pi\nu_i}\mathrm{e}^{-i1\pi\mu_i}\mathrm{e}^{-i0\pi\nu_i} \\
&+ ... \\
&+ \mathrm{e}^{iM_x\pi\mu_i}\mathrm{e}^{i0\pi\nu_i}\mathrm{e}^{-iM_x\pi\mu_i}\mathrm{e}^{-i0\pi\nu_i} \\
&+ \mathrm{e}^{i0\pi\mu_i}\mathrm{e}^{i1\pi\nu_i}\mathrm{e}^{-i0\pi\mu_i}\mathrm{e}^{-i1\pi\nu_i} \\
&+ ... \\
&+ \mathrm{e}^{iM_x\pi\mu_i}\mathrm{e}^{iM_y\pi\nu_i}\mathrm{e}^{-iM_x\pi\mu_i}\mathrm{e}^{-iM_y\pi\nu_i} \\
&= M_xM_y
\end{aligned}$$

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Uniform Rectangular Array Model (URA)

$$\begin{aligned}
\mathbf{h}_i^H \mathbf{h}_j &= \mathrm{e}^{i0\pi\mu_i} \mathrm{e}^{i0\pi\nu_i} \mathrm{e}^{-i0\pi\mu_j} \mathrm{e}^{-i0\pi\nu_j} \\
&+ \mathrm{e}^{i1\pi\mu_i} \mathrm{e}^{i0\pi\nu_i} \mathrm{e}^{-i1\pi\mu_j} \mathrm{e}^{-i0\pi\nu_j} \\
&+ \ldots \\
&+ \mathrm{e}^{iM_x\pi\mu_i} \mathrm{e}^{i0\pi\nu_i} \mathrm{e}^{-iM_x\pi\mu_j} \mathrm{e}^{-i0\pi\nu_j} \\
&+ \mathrm{e}^{i0\pi\mu_i} \mathrm{e}^{i1\pi\nu_i} \mathrm{e}^{-i0\pi\mu_j} \mathrm{e}^{-i1\pi\nu_j} \\
&+ \ldots \\
&+ \mathrm{e}^{iM_x\pi\mu_i} \mathrm{e}^{iM_y\pi\nu_i} \mathrm{e}^{-iM_x\pi\mu_j} \mathrm{e}^{-iM_y\pi\nu_j} \\
&= \frac{-1 + \mathrm{e}^{iM_x x}}{-1 + \mathrm{e}^{ix}} \frac{-1 + \mathrm{e}^{iM_y y}}{-1 + \mathrm{e}^{iy}}
\end{aligned}$$

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Capacidade do Canal URA

$$\mathbf{H}^{\mathbf{H}}\mathbf{H}(\theta,\phi)=\begin{pmatrix} M_x M_y & \left(\frac{e^{iM_x x}-1}{e^{ix}-1}\right)\left(\frac{e^{iM_y y}-1}{e^{iy}-1}\right) \\ \left(\frac{e^{-iM_x x}-1}{e^{-ix}-1}\right)\left(\frac{e^{-iM_y y}-1}{e^{-iy}-1}\right) & M_x M_y \end{pmatrix}$$

$$C(\theta,\phi)=\log_2\det\left(I+\rho\mathbf{H}(\theta,\phi)^{\mathbf{H}}\mathbf{H}(\theta,\phi)\right)=\log_2\prod\left(1+\rho\lambda_i(\theta,\phi)\right)$$

$$C(\theta,\phi)=\sum_{i=1}^{M}\log_2\left(1+\rho\lambda_i(\theta,\phi)\right)$$

$$C=\frac{2}{\pi^2}\int_0^{\pi/2}\int_{-\pi/2}^{\pi/2}C(\theta,\phi)\,d\phi\,d\theta$$

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Uniform Rectangular Array Model

- Eigenvalues can be calculated in a similar way of ULA.

$$\lambda_1 = M_x M_y - \frac{\sqrt{(-1+\cos(x))(-1+\cos(M_x x))}}{1-\cos(x)} \frac{\sqrt{(-1+\cos(y))(-1+\cos(M_y y))}}{1-\cos(y)},$$

$$\lambda_2 = M_x M_y + \frac{\sqrt{(-1+\cos(x))(-1+\cos(M_x x))}}{1-\cos(x)} \frac{\sqrt{(-1+\cos(y))(-1+\cos(M_y y))}}{1-\cos(y)}.$$

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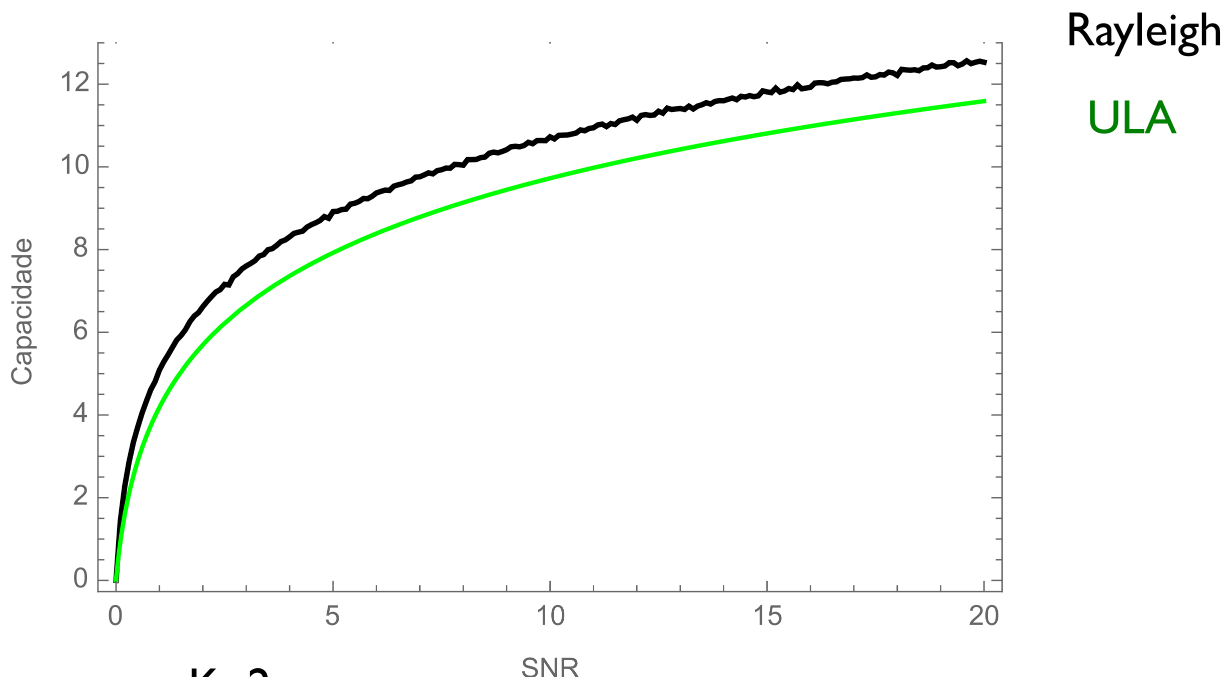
Resultado caso URA



K=2

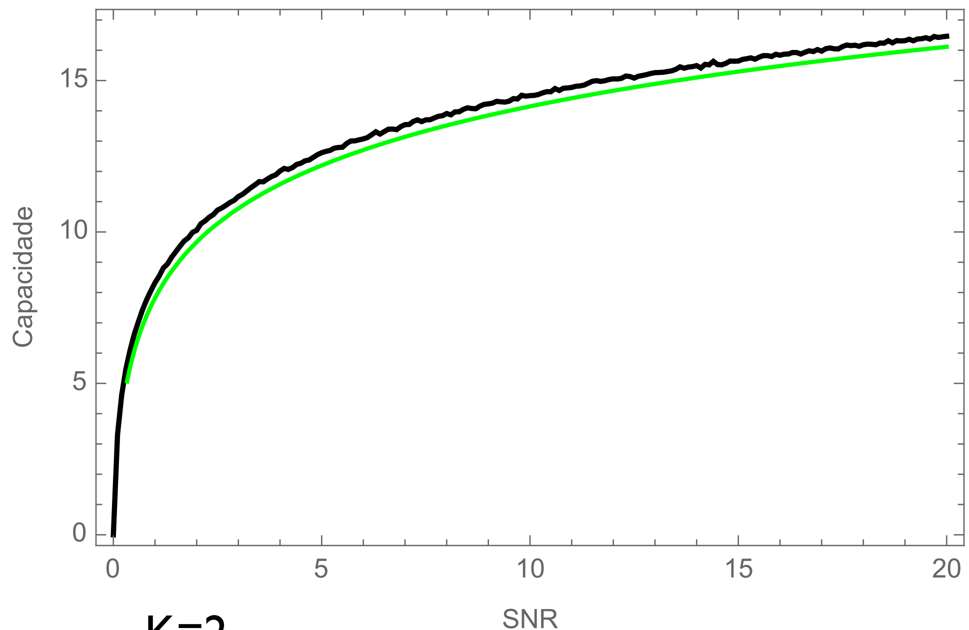
M=2

Ângulos Aleatórios:

Azimute entre $-\pi/2$ e $\pi/2$

Elevação entre 0 e $\pi/2$

Resultado caso URA



Rayleigh

ULA

K=2

M=4

Ângulos Aleatórios:

Azimute entre $-\pi/2$ e $\pi/2$

Elevação entre 0 e $\pi/2$

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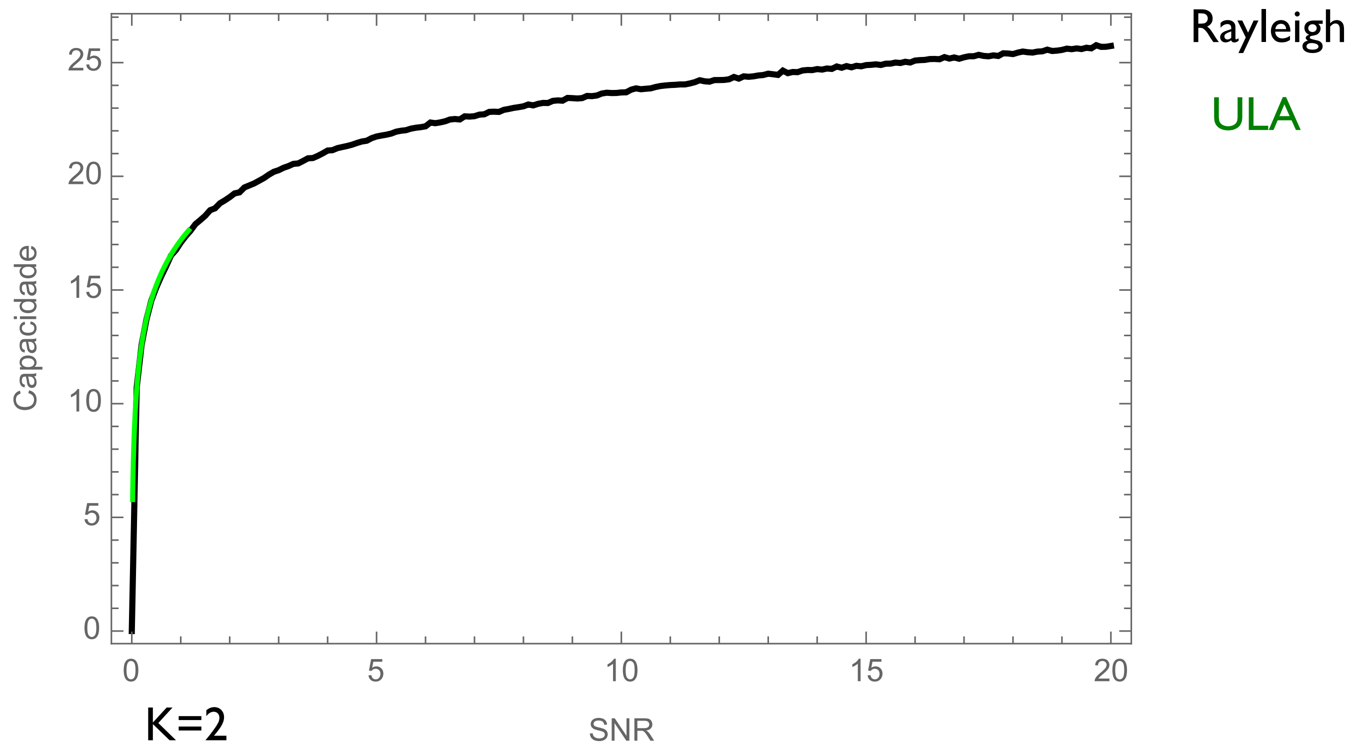
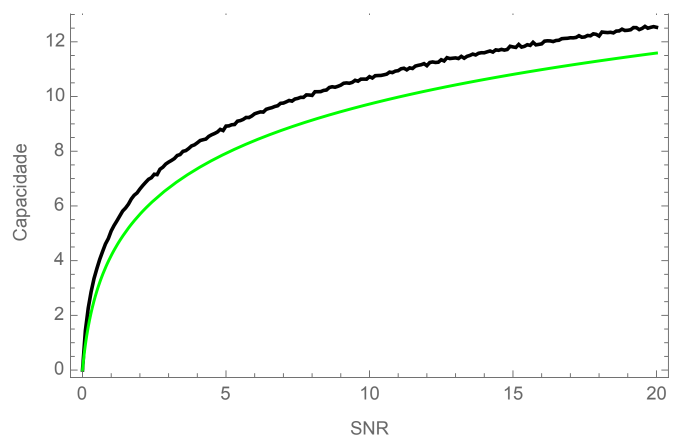
Resultado caso URA



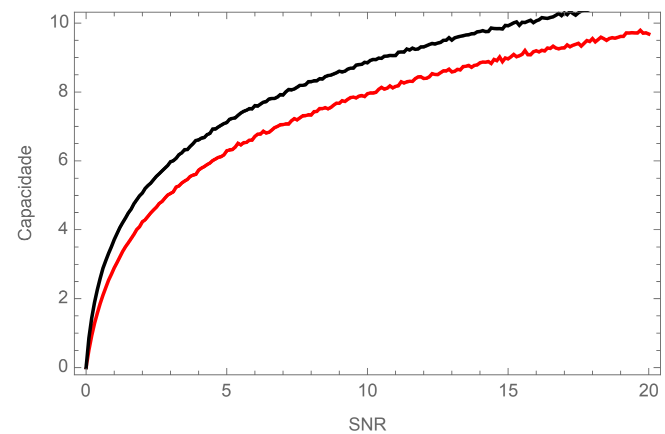
K=2
M=20
Ângulos Aleatórios:
Azimute entre $-\pi/2$ e $\pi/2$
Elevação entre 0 e $\pi/2$

URA x ULA

URA



ULA

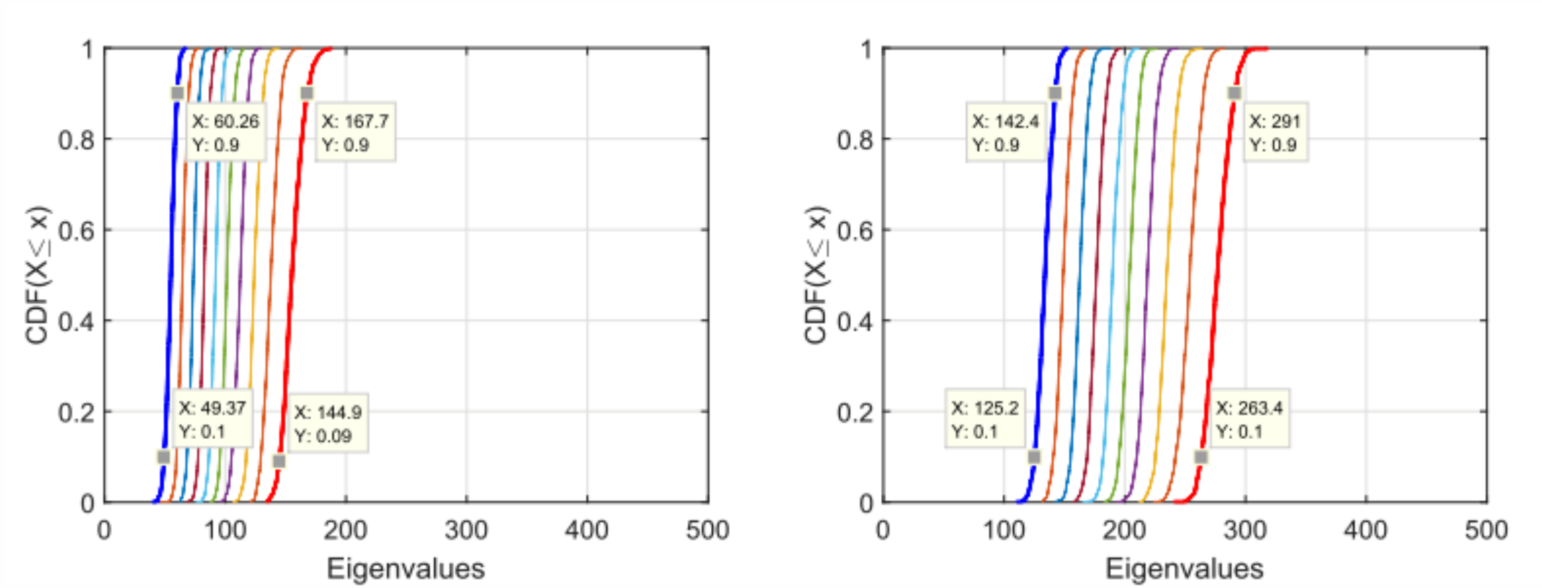


K=2
M=2

Caso Geral com K Usuários e M Antenas

Simulações

Caso Rayleigh



(a) K=10, M=100 (b) K=10, M=200

Figure 17 – CDF of the eigenvalues of the Gramian matrix $\mathbf{H}^H\mathbf{H}$ with the first and last eigenvalues highlighted.

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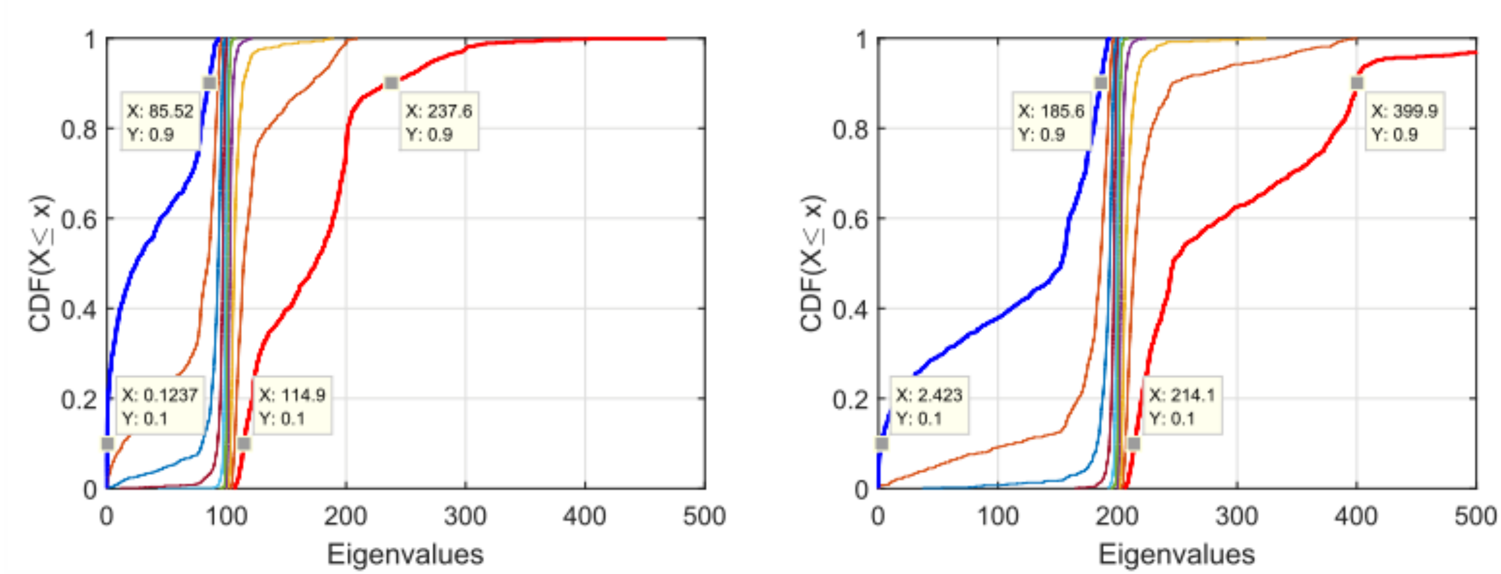
Resultados Numéricos ULA

Ângulos Uniformemente Distribuídos entre –π/2 e π/2



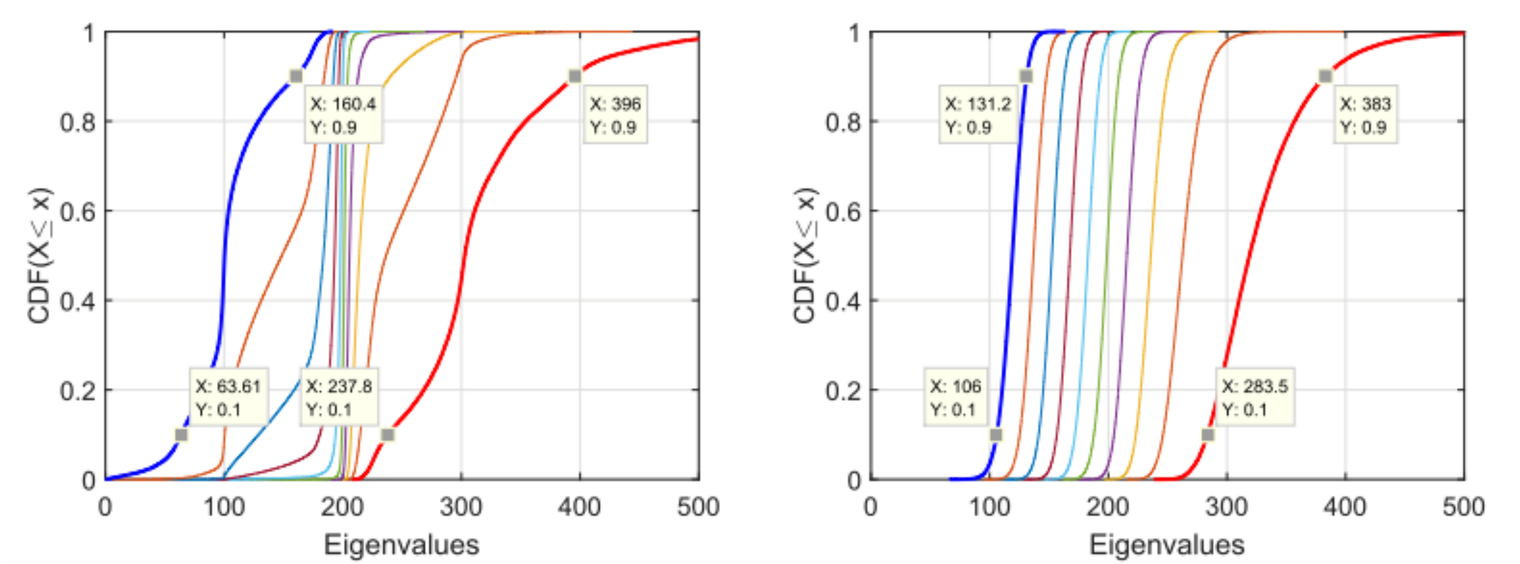
(a) K=10, M=100

(b) K=10, M=200

Figure 18 – CDF of the eigenvalues of the Gramian matrix $\mathbf{H}^H\mathbf{H}$ with the first and last eigenvalues highlighted.

Resultados Numéricos ULA

Ângulos Uniformemente Distribuídos entre -π/2 e π/2 e Múltiplos percursos.

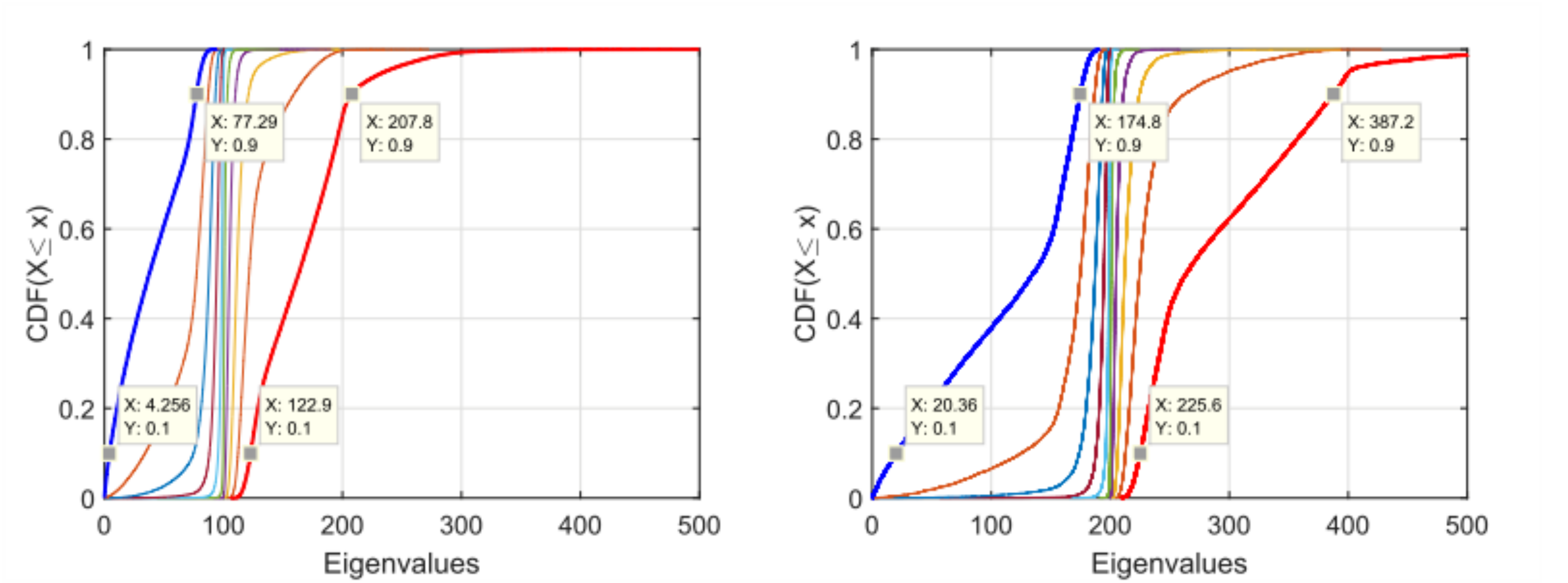


(a) K=10, M=200, L=2 (b) K=10, M=200, L=10

Figure 19 – CDF of the eigenvalues of the Gramian matrix $\mathbf{H}^H\mathbf{H}$ with the first and last eigenvalues highlighted when there are multiple paths.

Resultados Numéricos URA

Ângulos Uniformemente Distribuídos entre –π/2 e π/2



(a) K=10, M_x=10, M_y=10

(b) K=10, M_x=20, M_y=10

Figure 21 – CDF of the eigenvalues of the Gramian matrix $\mathbf{H}^H\mathbf{H}$ with the first and last eigenvalues highlighted.

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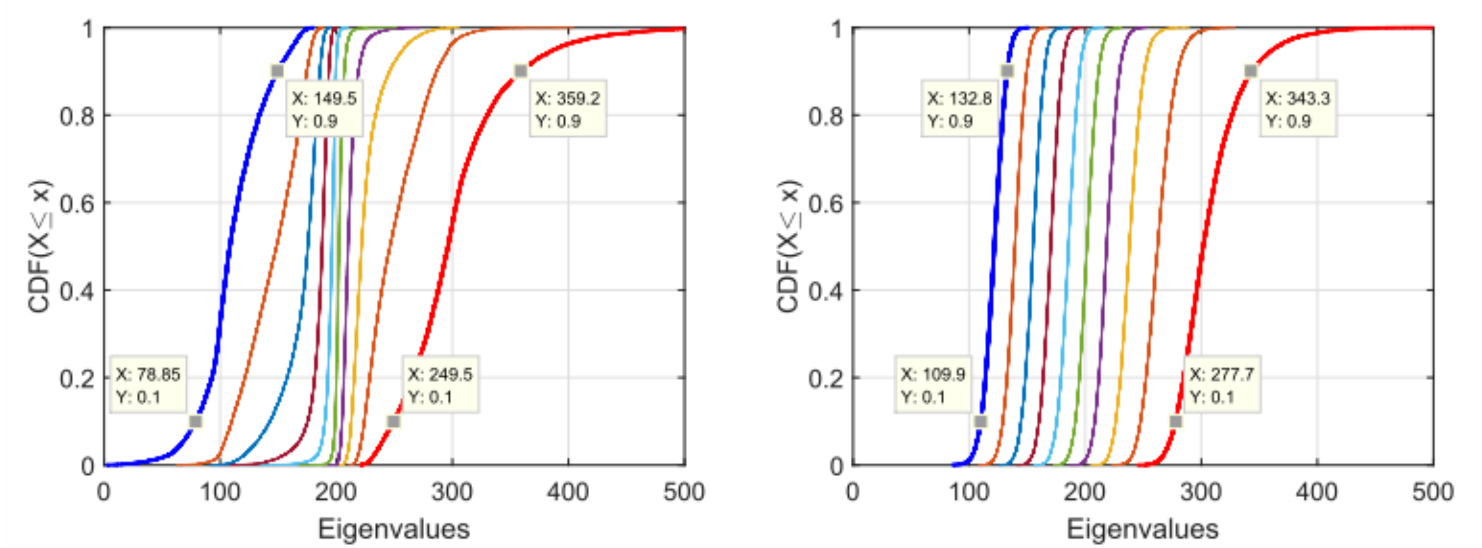
Resultados Numéricos URA

Ângulos Uniformemente Distribuídos entre –π/2 e π/2 e Múltiplos percursos.



(a) K=10, M_x=20, M_y=10, L=2 (b) K=10, M_x=20, M_y=10, L=10

Figure 22 – CDF of the eigenvalues of the Gramian matrix $\mathbf{H}^H\mathbf{H}$ with the first and last eigenvalues highlighted in nLoS case for URA.

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Conclusões

- O arranjo espacial das antenas limita a capacidade do sistema.
- Sistemas com arranjos URA são espacialmente mais eficientes, quando comparados os sistemas ULA, e apresentam praticamente a mesma capacidade.

Trabalhos Futuros

- Obter expressões analíticas para o caso geral com K usuários e M antenas.

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Thank you