Channel Modeling for Advanced-Generation Wireless Systems

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Santa Rita do Sapucaí, October 17, 2016

1 Motivation

2 Quadrature μ Processes

3 The α-η-κ-μ Fading Model

4 Conclusions and Challenges

Motivation

- Accurate Description of Signal
- Current Models
  - Complex-Based
    Rayleigh, Hoyt, Rice, Beckmann
  - Envelope-Based Models
    Weibull, Nakagami-m, α-μ, η-μ, κ-μ, α-η-μ, α-κ-μ
- Evolution of Wireless Systems
  From personal-centered to device/thing centered
  Odd Scenarios may lead to nonunimodality

Motivation

- New Scenarios – Emerging Technologies
  - mmW spectrum – (20)30-300 GHz
  - Mechanisms – same as lower frequencies

Motivation

- New Scenarios – Emerging Technologies
  - Perception of Physical Phenomena
    - Signal attenuation
    - Penetration into solid
    - Atmospheric conditions

      Absorption by molecules of oxygen, water vapor, different gaseous components – peaks at 24 GHz, 60 GHz, 120 GHz, 200 GHz

      Raindrops – size of the wavelengths – potential scatterers
  - Direct paths, reflections, diffractions (negligible)

Motivation

- New Scenarios – Emerging Technologies
  - Reflections – specular, diffuse

    Surfaces with irregularities on the order of the magnitude of the wavelength are perceived as rougher, resulting in higher diffusion. Larger surfaces, nevertheless, contribute with specular components. The partial waves arising from the scattering process may present phase correlation due to spatially correlated surfaces. In addition, the variety of propagation scenarios as perceived at higher frequencies may render multipath clustering a more noticeable phenomenon. Moreover, it is expected that at higher frequencies, the nonlinear effect of the propagation medium is more pronounced.

Quadrature μ Processes

- Given an enveloped-based distribution what is the corresponding phase distribution?

$$f_{\mathbf{Z}}(\mathbf{z};\mathbf{A}) = f_{|\mathbf{Z}|}(\mathbf{z};\mathbf{A})\frac{f_{\mathbf{Z}}(\mathbf{z};\mathbf{A}{=}\mathbf{A}_0)}{f_{|\mathbf{Z}|}(\mathbf{z};\mathbf{A}{=}\mathbf{A}_0)}$$

Quadrature μ Process

- Quadrature μ Process Type I

$$f_Z(z) = \frac{\mu^{\mu/2} |z|^{\mu-1}}{\Omega^{\mu/2}\Gamma(\mu/2)} \exp\left(-\frac{\mu\, z^2}{\Omega}\right), \quad -\infty < z < \infty$$

Quadrature μ Process

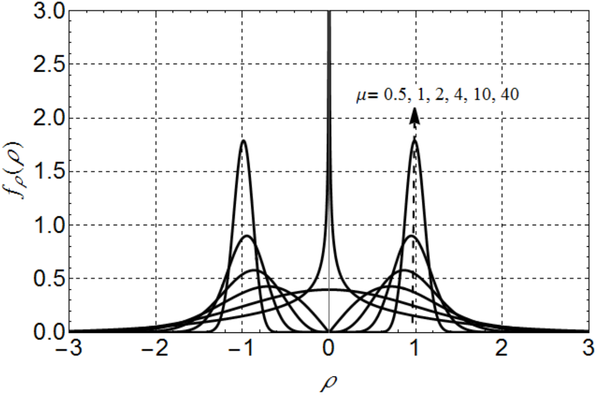
- Quadrature μ Process Type II

$$f_Z(z) = \frac{|z|^{\frac{\mu_z}{2}} \exp\left(-\frac{(z-\lambda_z)^2}{2\sigma_z^2}\right) I_{\frac{\mu_z}{2}-1}\left(\frac{|\lambda_z z|}{\sigma_z^2}\right)}{2\sigma_z^2 \left|\lambda_z\right|^{\frac{\mu_z}{2}-1} \cosh\left(\frac{z\lambda_z}{\sigma_z^2}\right)}$$

Quadrature μ Process

- Some Plots

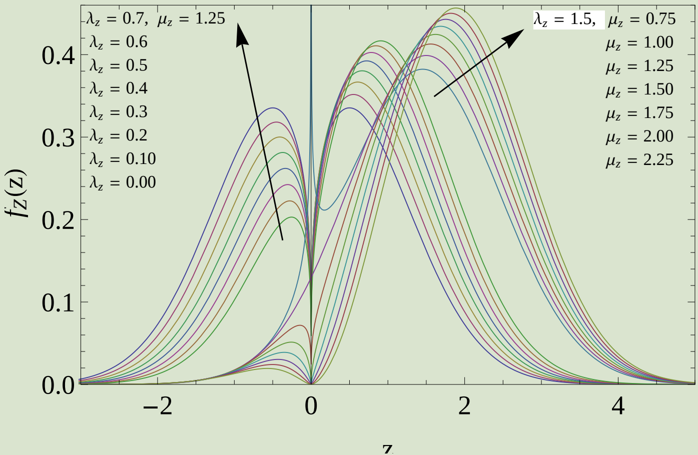
Quadrature μ Process Type I



Quadrature μ Process

- Some Plots

Quadrature μ Process Type II



The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model
  - $\alpha \rightarrow$ Nonlinear parameter

Complex Model

$$S = X + jY = R^{\frac{\alpha}{2}} \times exp(j\theta)$$

Joint PDF

$$f_{R,\Theta}(r,\theta) = \frac{\alpha}{2} r^{\alpha-1} f_X(r^{\frac{\alpha}{2}}\cos(\theta)) f_Y(r^{\frac{\alpha}{2}}\sin(\theta))$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model
  - Parameterization-0 (Raw Parameterization)
    - $\sigma_x^2, \sigma_y^2 \rightarrow$ Powers of the scattered waves of the individual multipath clusters of the in-phase and quadrature signals
    - $\lambda_x^2, \lambda_y^2 \rightarrow$ Powers of the dominant components of all cluster (location parameter) of the in-phase and quadrature signals, $-\infty < \lambda_x, < \infty, -\infty < \lambda_y, < \infty$
    - $\mu_x, \mu_y \rightarrow$ Number of multipath clusters (shape parameter) of the in-phase and quadrature signals

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model
  - Parameterization-1 (Local Parameterization)
    - κ_x, κ_y $\rightarrow$ Ratio of the total power of the dominant components and the total power of scattered waves of in-phase and quadrature signals
    - $\hat{r}_x^2$, $\hat{r}_y^2$ $\rightarrow$ The mean value $E(X^2)$ and $E(Y^2)$, given by the power of multipath cluster and dominant components
    - μ_x, μ_y $\rightarrow$ Number of multipath clusters (shape parameter) of the in-phase and quadrature signals

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model
  - Parameterization-2 (Global Parameterization)
    - $\hat{r}^{\alpha} \rightarrow$ Mean value $E(R^{\alpha}) = \mu_x\sigma_x^2 + \lambda_x^2 + \mu_y\sigma_y^2 + \lambda_y^2$
    - $\kappa \rightarrow$ Ratio of of the total power of the dominant components and the total power of the scattered waves, i.e. $\kappa = (\lambda_x^2 + \lambda_y^2)/(\mu_x\sigma_x^2 + \mu_y\sigma_y^2)$
    - $\eta \rightarrow$ Ratio of the total power of the in-phase and quadrature waves of the multipath cluster, i.e. $\eta = \mu_x\sigma_x/(\mu_y\sigma_y)$
    - $q \rightarrow$ Ratio of two ratios: The ratio of the power of dominant component to the power of the scattered wave of the in-phase signal and its counter part for the quadrature signal; $q = \lambda_x^2\mu_y\sigma_y^2/(\lambda_y^2\mu_x\sigma_x^2)$
    - $p \rightarrow$ Ratio of the number of multipath cluster; $p = \mu_x/\mu_y$
    - $\mu \rightarrow$ The total number of multipath clusters; $\mu = \mu_x + \mu_y$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model - Joint PDF

Parameterization-0 (Raw Parameterization)

$$f_{R,\Theta}(r,\theta) = \frac{\alpha r^{\alpha\left(1+\frac{\mu_x}{4}+\frac{\mu_y}{4}\right)-1} \left|\cos(\theta)\right|^{\frac{\mu_x}{2}} \left|\sin(\theta)\right|^{\frac{\mu_y}{2}}}{8\,\sigma_x^2\sigma_y^2 \; |\lambda_x|^{\frac{\mu_x}{2}-1} |\lambda_y|^{\frac{\mu_y}{2}-1} \exp\left(\frac{\lambda_x^2}{2\,\sigma_x^2}+\frac{\lambda_y^2}{2\,\sigma_y^2}\right)}$$
$$\times \exp\left(-\left(\frac{\cos^2(\theta)}{\sigma_x^2}+\frac{\sin^2(\theta)}{\sigma_y^2}\right)\frac{r^\alpha}{2}\right)$$
$$\times \exp\left(\left(\frac{\lambda_x\cos(\theta)}{\sigma_x^2}+\frac{\lambda_y\sin(\theta)}{\sigma_y^2}\right) r^{\frac{\alpha}{2}}\right)$$
$$\times \frac{I_{\frac{\mu_x}{2}-1}\left(\frac{|\lambda_x\cos(\theta)|\; r^{\frac{\alpha}{2}}}{\sigma_x^2}\right) I_{\frac{\mu_y}{2}-1}\left(\frac{\left|\lambda_y\sin(\theta)\right|\; r^{\frac{\alpha}{2}}}{\sigma_y^2}\right)}{\cosh\left(\frac{\lambda_x\cos(\theta) r^{\frac{\alpha}{2}}}{\sigma_x^2}\right)\cosh\left(\frac{\lambda_y\sin(\theta) r^{\frac{\alpha}{2}}}{\sigma_y^2}\right)}$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model - Joint PDF

Parameterization-1 (Local Parameterization)

$$f_{R,\Theta}(r,\theta)=\frac{\alpha\mu_x\mu_y(\kappa_x+1)^{\frac{\mu_x}{4}+\frac{1}{2}}(\kappa_y+1)^{\frac{\mu_y}{4}+\frac{1}{2}}|\cos(\theta)|^{\frac{\mu_x}{2}}|\sin(\theta)|^{\frac{\mu_y}{2}}r^{\alpha\left(\frac{\mu_x}{4}+\frac{\mu_y}{4}+1\right)-1}}{8\,\kappa_x^{\frac{\mu_x}{4}-\frac{1}{2}}\kappa_y^{\frac{\mu_y}{4}-\frac{1}{2}}\hat{r}_x^{\frac{\mu_x}{2}+1}\hat{r}_y^{\frac{\mu_y}{2}+1}\exp\left(\frac{\kappa_x\mu_x}{2}+\frac{\kappa_y\mu_y}{2}\right)}$$

$$\times\exp\left(-\left(\frac{(\kappa_x+1)\mu_x\cos^2(\theta)}{\hat{r}_x^2}+\frac{(\kappa_y+1)\mu_y\sin^2(\theta)}{\hat{r}_y^2}\right)\frac{r^\alpha}{2}\right)$$

$$\times\exp\left(\left(\frac{\sqrt{\kappa_x(\kappa_x+1)}\mu_x\cos(\theta)}{\hat{r}_x}+\frac{\sqrt{\kappa_y(\kappa_y+1)}\mu_y\sin(\theta)}{\hat{r}_y}\right)r^{\frac{\alpha}{2}}\right)$$

$$\times\frac{I_{\frac{\mu_x}{2}-1}\left(\frac{\sqrt{\kappa_x(\kappa_x+1)}\mu_x|\cos(\theta)|r^{\frac{\alpha}{2}}}{\hat{r}_x}\right)I_{\frac{\mu_y}{2}-1}\left(\frac{\sqrt{\kappa_y(\kappa_y+1)}\mu_y|\sin(\theta)|r^{\frac{\alpha}{2}}}{\hat{r}_y}\right)}{\cosh\left(\frac{\sqrt{\kappa_x(\kappa_x+1)}\mu_x\cos(\theta)r^{\frac{\alpha}{2}}}{\hat{r}_x}\right)\cosh\left(\frac{\sqrt{\kappa_y(\kappa_y+1)}\mu_y\sin(\theta)r^{\frac{\alpha}{2}}}{\hat{r}_y}\right)}$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model - Joint PDF

Parameterization-2 (Global Parameterization)

$$f_{R,\Theta}(r,\theta) = \frac{\alpha\mu^2 p(\eta+1)^2(\kappa+1)^{\frac{\mu}{2}+1} r^{\frac{\alpha}{2}(\mu+2)-1} |\sin(\theta)|^{\frac{\mu}{p+1}} |\cos(\theta)|^{\frac{\mu p}{p+1}}}{2\eta(p+1)^2 \left(\frac{\kappa}{\eta q+1}\right)^{\frac{\mu}{2}-1} (\eta q)^{\frac{\mu p}{2(p+1)}-\frac{1}{2}} \hat{r}^{\frac{\alpha}{2}(\mu+2)} \exp\left(\frac{\kappa\mu(\eta+1)(qp+1)}{(p+1)(\eta q+1)}\right)} \times \Phi(\theta,r) \times \Theta(\theta,r)$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model - Joint PDF

Parameterization-2 (Global Parameterization)

$$\Phi(\theta,r) = \exp\left(-\frac{\mu(\eta+1)(\kappa+1)\left(\eta\sin^2(\theta)+p\cos^2(\theta)\right)}{\eta(p+1)}\left(\frac{r}{\hat{r}}\right)^{\alpha}\right) \times \exp\left(\frac{2\mu(\eta+1)\cos\left(\theta-\phi\right)}{\eta(p+1)}\sqrt{\frac{\eta\kappa(\kappa+1)\left(\eta+qp^2\right)}{\eta q+1}}\left(\frac{r}{\hat{r}}\right)^{\alpha/2}\right)$$

- $\phi = \text{arg}(\text{sign}(\lambda_x) + j(1/p)((\eta/q))^{1/2}\text{sign}(\lambda_x))$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model - Joint PDF

Parameterization-2 (Global Parameterization)

$$\Theta(\theta, r) = \frac{I_{\frac{\mu}{p+1}-1}\left(\frac{2\mu(\eta+1)|\sin(\theta)|}{p+1}\sqrt{\frac{\kappa(\kappa+1)}{\eta q+1}}\left(\frac{r}{\hat{r}}\right)^{\alpha/2}\right)}{\cosh\left(\frac{2\mu(\eta+1)\sin(\theta)}{p+1}\sqrt{\frac{\kappa(\kappa+1)}{\eta q+1}}\left(\frac{r}{\hat{r}}\right)^{\alpha/2}\right)} \times \frac{I_{\frac{\mu p}{p+1}-1}\left(\frac{2\mu p(\eta+1)|\cos(\theta)|}{\eta(p+1)}\sqrt{\frac{\eta\kappa q(\kappa+1)}{\eta q+1}}\left(\frac{r}{\hat{r}}\right)^{\alpha/2}\right)}{\cosh\left(\frac{2\mu p(\eta+1)\cos(\theta)}{\eta(p+1)}\sqrt{\frac{\eta\kappa q(\kappa+1)}{\eta q+1}}\left(\frac{r}{\hat{r}}\right)^{\alpha/2}\right)}$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model

Envelope Based Model

$$R^{\alpha} = \sum_{i=1}^{\mu_x} (X_i + \lambda_{x_i})^2 + \sum_{i=1}^{\mu_y} (Y_i + \lambda_{y_i})^2$$

- $R^{\alpha} = U + V$

Envelope PDF

$$f_R(r) = \alpha r^{\alpha-1} \int_0^{r^{\alpha}} f_U(r^{\alpha} - \nu) f_V(\nu) d\nu$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Series Expansion

$$c_k = \frac{1}{k}\sum_{j=0}^{k-1} c_j d_{k-j}$$

$$m_k = \frac{1}{k}\sum_{j=0}^{k-1} m_j q_{k-j}$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Series Expansion
  - Parametrization 0 – Parametrization 1

PDF

$$f_R(r)=\frac{\alpha r^{\alpha\frac{\mu_x+\mu_y}{2}-1}\sum_{k=0}^{\infty}\frac{k!c_k L_k^{\frac{\mu_x+\mu_y}{2}-1}(2r^\alpha)}{\left(\frac{\mu_x+\mu_y}{2}\right)_k}}{2^{\frac{\mu_x+\mu_y}{2}}\Gamma\left(\frac{\mu_x+\mu_y}{2}\right)\exp\left(\frac{r^\alpha}{2}\right)}$$

CDF

$$F_R(r)=\frac{r^{\alpha\frac{\mu_x+\mu_y}{2}}\sum_{k=0}^{\infty}\frac{k!m_k L_k^{\frac{\mu_x+\mu_y}{2}}\left(\left(\frac{2}{\mu_x+\mu_y}+1\right)2r^\alpha\right)}{\left(\frac{\mu_x+\mu_y}{2}+1\right)_k}}{2^{\frac{\mu_x+\mu_y}{2}+1}\Gamma\left(\frac{\mu_x+\mu_y}{2}+1\right)\exp\left(\frac{r^\alpha}{2}\right)}$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Series Expansion
  - Parametrization 2

PDF

$$f_R\left(r\right) = \frac{\alpha r^{\alpha\mu-1}\sum_{k=0}^{\infty}\frac{k!c_k L_k^{\mu-1}(2r^{\alpha})}{(\mu)_k}}{2^{\mu}\Gamma(\mu)\exp\left(\frac{r^{\alpha}}{2}\right)}$$

CDF

$$F_R\left(r\right) = \frac{r^{\alpha\mu}\sum_{k=0}^{\infty}\frac{k!m_k L_k^{\mu}\left(\frac{2(\mu+1)r^{\alpha}}{\mu}\right)}{(\mu+1)_k}}{2^{\mu+1}\Gamma(\mu+1)\exp\left(\frac{r^{\alpha}}{2}\right)}$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Series Expansion
  - Parametrization 0

$$c_0 = \frac{2^{\mu_x+\mu_y}\left(1+3\sigma_x^2\right)^{-\frac{\mu_x}{2}}\left(1+3\sigma_y^2\right)^{-\frac{\mu_y}{2}}}{\exp\left(\frac{3\lambda_x^2}{6\sigma_x^2+2}+\frac{3\lambda_y^2}{6\sigma_y^2+2}\right)}$$

$$d_j = \frac{\mu_x}{2}\left(\frac{1-\sigma_x^2}{1+3\sigma_x^2}\right)^j + \frac{\mu_y}{2}\left(\frac{1-\sigma_y^2}{1+3\sigma_y^2}\right)^j - \frac{2j\lambda_x^2\left(1-\sigma_x^2\right)^{j-1}}{\left(1+3\sigma_x^2\right)^{j+1}} - \frac{2j\lambda_y^2\left(1-\sigma_y^2\right)^{j-1}}{\left(1+3\sigma_y^2\right)^{j+1}}$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Series Expansion
  - Parametrization 0

$$m_0 = \frac{2^{\mu_x+\mu_y+3}(\mu_x+\mu_y+2)^{\frac{\mu_x+\mu_y}{2}+1}}{(3(\mu_x+\mu_y)+8)} \times \frac{\left((\mu_x+\mu_y)\left(1+3\sigma_x^2\right)+8\sigma_x^2\right)^{-\frac{\mu_x}{2}}}{\left((\mu_x+\mu_y)\left(1+3\sigma_y^2\right)+8\sigma_y^2\right)^{\frac{\mu_y}{2}}} \times \exp\left(-\frac{{\lambda_x}^2(3(\mu_x+\mu_y)+8)}{2(\mu_x+\mu_y)\left(1+3\sigma_x^2\right)+16\sigma_x^2}\right) \times \exp\left(-\frac{{\lambda_y}^2(3(\mu_x+\mu_y)+8)}{2(\mu_x+\mu_y)\left(1+3\sigma_y^2\right)+16\sigma_y^2}\right)$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Series Expansion
    - Parametrization 0

$$
\begin{aligned}
q_j =& \frac{\mu_x}{2}\left(\frac{\left(1-\sigma_x^2\right)\left(\mu_x+\mu_y\right)}{\left(\mu_x+\mu_y\right)\left(1+3\sigma_x^2\right)+8\sigma_x^2}\right)^j \\
&+\frac{\mu_y}{2}\left(\frac{\left(1-\sigma_y^2\right)\left(\mu_x+\mu_y\right)}{\left(\mu_x+\mu_y\right)\left(1+3\sigma_y^2\right)+8\sigma_y^2}\right)^j+\left(-\frac{\mu_x+\mu_y}{3\left(\mu_x+\mu_y\right)+8}\right)^j \\
&-\frac{2j{\lambda_x}^2(\mu_x+\mu_y+2)(\mu_x+\mu_y)^j\left(1-\sigma_x^2\right)^{j-1}}{\left(\left(\mu_x+\mu_y\right)\left(1+3\sigma_x^2\right)+8\sigma_x^2\right)^{j+1}} \\
&-\frac{2j{\lambda_y}^2(\mu_x+\mu_y+2)(\mu_x+\mu_y)^j\left(1-\sigma_y^2\right)^{j-1}}{\left(\left(\mu_x+\mu_y\right)\left(1+3\sigma_y^2\right)+8\sigma_y^2\right)^{j+1}}
\end{aligned}
$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Series Expansion
  - Parametrization 1

$$c_0 = \frac{2^{\mu_x+\mu_y}\left(\frac{3\hat{r}_x^2}{(\kappa_x+1)\mu_x}+1\right)^{-\frac{\mu_x}{2}}\left(\frac{3\hat{r}_y^2}{(\kappa_y+1)\mu_y}+1\right)^{-\frac{\mu_y}{2}}}{\exp\left(\frac{3}{2}\left(\frac{\kappa_x\mu_x\hat{r}_x^2}{(\kappa_x+1)\mu_x+3\hat{r}_x^2}+\frac{\kappa_y\mu_y\hat{r}_y^2}{(\kappa_y+1)\mu_y+3\hat{r}_y^2}\right)\right)}$$

$$\begin{aligned} d_j = & \frac{\mu_x}{2}\left(1-\frac{4\hat{r}_x^2}{(\kappa_x+1)\,\mu_x+3\hat{r}_x^2}\right)^j + \frac{\mu_y}{2}\left(1-\frac{4\hat{r}_y^2}{(\kappa_y+1)\,\mu_y+3\hat{r}_y^2}\right)^j \\ & + \frac{2j\kappa_x\mu_x\hat{r}_x^2}{(\hat{r}_x^2-(\kappa_x+1)\mu_x)}\left(\frac{(\kappa_x+1)\mu_x}{(\kappa_x+1)\,\mu_x+3\hat{r}_x^2}\right)^{j+1}\left(1-\frac{\hat{r}_x^2}{(\kappa_x+1)\,\mu_x}\right)^j \\ & + \frac{2j\kappa_y\mu_y\hat{r}_y^2}{(\hat{r}_y^2-(\kappa_y+1)\mu_y)}\left(\frac{(\kappa_y+1)\mu_y}{(\kappa_y+1)\,\mu_y+3\hat{r}_y^2}\right)^{j+1}\left(1-\frac{\hat{r}_y^2}{(\kappa_y+1)\,\mu_y}\right)^j. \end{aligned}$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Series Expansion
  - Parametrization 1

$$m_0 = \frac{2\left(\frac{\mu_x+\mu_y}{2}+1\right)^{\frac{\mu_x+\mu_y}{2}+1}}{\left(\frac{3(\mu_x+\mu_y)}{8}+1\right)} \frac{\left(\frac{\mu_x+\mu_y}{8}+\frac{(3(\mu_x+\mu_y)+8)\hat{r}_y^2}{8(\kappa_y+1)\mu_y}\right)^{-\frac{\mu_y}{2}}}{\left(\frac{\mu_x+\mu_y}{8}+\frac{(3(\mu_x+\mu_y)+8)\hat{r}_x^2}{8(\kappa_x+1)\mu_x}\right)^{\frac{\mu_x}{2}}}$$
$$\times \exp\left(-\frac{(3(\mu_x+\mu_y)+8)\,\kappa_x\mu_x\hat{r}_x^2}{2\left((\kappa_x+1)\,(\mu_x+\mu_y)\,\mu_x+(3(\mu_x+\mu_y)+8)\,\hat{r}_x^2\right)}\right)$$
$$\times \exp\left(-\frac{(3(\mu_x+\mu_y)+8)\,\kappa_y\mu_y\hat{r}_y^2}{2\left((\kappa_y+1)\,(\mu_x+\mu_y)\,\mu_y+(3(\mu_x+\mu_y)+8)\,\hat{r}_y^2\right)}\right)$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Series Expansion
  - Parametrization 1

$$
\begin{aligned}
q_j = {} & \frac{\mu_x}{2}\left(\frac{(\mu_x+\mu_y)((\kappa_x+1)\mu_x-\hat{r}_x^2)}{(\kappa_x+1)(\mu_x+\mu_y)\mu_x+(3\,(\mu_x+\mu_y)+8)\hat{r}_x^2}\right)^j+\left(-\frac{\mu_x+\mu_y}{3\,(\mu_x+\mu_y)+8}\right)^j \\
& +\frac{\mu_y}{2}\left(\frac{(\mu_x+\mu_y)((\kappa_y+1)\,\mu_y-\hat{r}_y^2)}{(\kappa_y+1)(\mu_x+\mu_y)\mu_y+(3\,(\mu_x+\mu_y)+8)\hat{r}_y^2}\right)^j-\frac{2j\kappa_x(\mu_x+\mu_y+2)\hat{r}_x^2}{(\kappa_x+1)\,(\mu_x+\mu_y)} \\
& \times\left(1-\frac{\hat{r}_x^2}{(\kappa_x+1)\,\mu_x}\right)^{j-1}\left(\frac{(\kappa_x+1)(\mu_x+\mu_y)\mu_x}{(\kappa_x+1)(\mu_x+\mu_y)\mu_x+(3\,(\mu_x+\mu_y)+8)\hat{r}_x^2}\right)^{j+1} \\
& -\frac{2j\kappa_y(\mu_x+\mu_y+2)\hat{r}_y^2}{(\kappa_y+1)\,(\mu_x+\mu_y)}\left(1-\frac{\hat{r}_y^2}{(\kappa_y+1)\,\mu_y}\right)^{j-1} \\
& \times\left(\frac{(\kappa_y+1)(\mu_x+\mu_y)\mu_y}{(\kappa_y+1)(\mu_x+\mu_y)\mu_y+(3\,(\mu_x+\mu_y)+8)\hat{r}_y^2}\right)^{j+1}
\end{aligned}
$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Series Expansion
  - Parametrization 2

$$c_0 = \frac{8^{\mu}\left(\frac{3(p+1)\hat{r}^{\alpha}}{(\eta+1)(\kappa+1)\mu}+2\right)^{-\frac{\mu}{p+1}}\left(\frac{3\eta(p+1)\hat{r}^{\alpha}}{(\eta+1)(\kappa+1)\mu p}+2\right)^{-\frac{\mu p}{p+1}}}{\exp\left(\frac{3\kappa\mu\hat{r}^{\alpha}(\eta+1)(2\mu p(\eta+1)(\kappa+1)(\eta q+1)+3\eta\hat{r}^{\alpha}(p+1)(pq+1))}{(\eta q+1)(2\mu(\eta+1)(\kappa+1)+3\hat{r}^{\alpha}(p+1))(2\mu p(\eta+1)(\kappa+1)+3\eta\hat{r}^{\alpha}(p+1))}\right)}$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Series Expansion
  - Parametrization 2

$$
\begin{aligned}
d_j =\ & \frac{\mu}{p+1}\left(\frac{2\mu(\eta+1)(\kappa+1)-(p+1)\hat{r}^{\alpha}}{2\mu(\eta+1)(\kappa+1)+3(p+1)\hat{r}^{\alpha}}\right)^{j} \\
& + \frac{p\mu}{p+1}\left(\frac{2\mu p(\eta+1)(\kappa+1)-\eta\hat{r}^{\alpha}(p+1)}{2\mu p(\eta+1)(\kappa+1)+3\eta\hat{r}^{\alpha}(p+1)}\right)^{j} \\
& - \frac{8j\kappa\mu^2\eta p^2 q(\eta+1)^2(\kappa+1)}{\eta q+1}\,\frac{\left(2\mu p(\eta+1)(\kappa+1)-\eta\hat{r}^{\alpha}(p+1)\right)^{j-1}\hat{r}^{\alpha}}{\left(2\mu p(\eta+1)(\kappa+1)+3\eta\hat{r}^{\alpha}(p+1)\right)^{j+1}} \\
& - \frac{8j\kappa\mu^2(\eta+1)^2(\kappa+1)}{\eta q+1}\,\frac{\left(2\mu(\eta+1)(\kappa+1)-(p+1)\hat{r}^{\alpha}\right)^{j-1}\hat{r}^{\alpha}}{\left(2(\eta+1)(\kappa+1)\mu+3\hat{r}^{\alpha}(p+1)\right)^{j+1}}
\end{aligned}
$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Series Expansion
  - Parametrization 2

$$m_0 = \frac{8^{\mu+1}(\mu+1)^{\mu+1}\left(2\mu + \frac{(3\mu+4)(p+1)\hat{r}^{\alpha}}{(\eta+1)(\kappa+1)\mu}\right)^{-\frac{\mu}{p+1}}}{(3\mu+4)\left(2\mu + \frac{\eta\hat{r}^{\alpha}(3\mu+4)(p+1)}{\mu p(\eta+1)(\kappa+1)}\right)^{\frac{\mu p}{p+1}}}$$
$$\times \exp\left(-\frac{\kappa\mu\hat{r}^{\alpha}\left(2\mu^2 p(\eta+1)(\kappa+1) + \frac{\eta\hat{r}^{\alpha}(3\mu+4)(p+1)(pq+1)}{\eta q+1}\right)}{\left(\frac{2\mu^2(\eta+1)(\kappa+1)}{3\mu+4} + (p+1)\hat{r}^{\alpha}\right)\left(2p\mu^2(\kappa+1) + \frac{\eta\hat{r}^{\alpha}(3\mu+4)(p+1)}{\eta+1}\right)}\right)$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Series Expansion
  - Parametrization 2

$$
\begin{aligned}
q_j = {} & \frac{\mu}{p+1}\left(\frac{\mu(2\mu(\eta+1)(\kappa+1)-(p+1)\hat{r}^{\alpha})}{2\mu^2(\eta+1)(\kappa+1)+(3\mu+4)(p+1)\hat{r}^{\alpha}}\right)^j + \left(-\frac{\mu}{3\mu+4}\right)^j \\
& + \frac{\mu p}{p+1}\left(\frac{\mu(2\mu p(\eta+1)(\kappa+1)-\eta\hat{r}^{\alpha}(p+1))}{2p\mu^2(\eta+1)(\kappa+1)+\eta\hat{r}^{\alpha}(3\mu+4)(p+1)}\right)^j \\
& - \frac{8j\kappa\mu^{j+2}\eta p^2 q(\eta+1)^2(\kappa+1)(\mu+1)(2\mu p(\eta+1)(\kappa+1)-\eta\hat{r}^{\alpha}(p+1))^{j-1}}{(\eta q+1)\left(2p\mu^2(\eta+1)(\kappa+1)+\eta\hat{r}^{\alpha}(3\mu+4)(p+1)\right)^{j+1}\hat{r}^{-\alpha}} \\
& - \frac{8j\kappa\mu^{j+2}(\eta+1)^2(\kappa+1)(\mu+1)(2\mu(\eta+1)(\kappa+1)-(p+1)\hat{r}^{\alpha})^{j-1}\hat{r}^{\alpha}}{(\eta q+1)\left(2\mu^2(\eta+1)(\kappa+1)+(3\mu+4)(p+1)\hat{r}^{\alpha}\right)^{j+1}}
\end{aligned}
$$

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Special Cases
  - One parameter distribution from two parameter distribution
    - From α-μ, Nakagami-m is obtained with $\alpha_T = 2$ and $\mu_T = m$.
    - From α-μ, Weibull is obtained with $\alpha_T = \alpha$ and $\mu_T = 1$.
    - From η-μ, Nakagami-m is obtained with $\eta_T = 1$ and $2\mu_T = m$.
    - From η-μ, Nakagami-m is obtained with $\eta_T \to 0$ or $\eta_T \to \infty$ and $\mu_T = m$.
    - From η-μ, Hoyt can be obtained with $\eta_T = (1+b)\,/\,(1-b)$ and $\mu_T = 1/2$.

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Special Cases
  - One parameter distribution from two parameter distribution
    - From κ-μ, Nakagami-m is obtained with $\kappa_T \to 0$ and $\mu_T = m$.
    - From κ-μ, Rice is obtained with $\kappa_T = k$ and $\mu_T = 1$.
    - From η-κ(Beckmann), Hoyt is obtained with $\eta_T = (1 + b) \,/\, (1 - b)$ and $\kappa_T \to 0$.
    - From η-κ (Beckmann), Rice is obtained with $\eta_T = 1$ and $\kappa_T = k$.

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Special Cases
  - No parameter distribution from one parameter distribution
    - From Nakagami-m, Rayleigh is obtained with $m = 1$.
    - From Nakagami-m, semi-Gaussian is obtained with $m = 1/2$.
    - From Weibull, Rayleigh is obtained with $\alpha = 2$.
    - From Weibull, Negative Exponential is obtained with $\alpha = 1$.
    - From Hoyt, Rayleigh is obtained with $b = 0$.
    - From Hoyt, semi-Gaussian is obtained with $b \to \pm 1$.
    - From Rice, Rayleigh is obtained with $k \to 0$.

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Special Cases
  - Three-Fading-Parameters Models:
    - The α-κ-μ model is obtained from the α-η-κ-μ one with $\alpha_T = \alpha$, $\mu_T = \mu$, $\kappa_T = \kappa$, $\eta = p$, $q_T = q$, $\hat{r}_T = \hat{r}$.
    - The α-η-μ model is obtained from the α-η-κ-μ one with $\alpha_T = \alpha$, $2\mu_T = \mu$, $\kappa \to 0$, $\eta_T = \eta$, $p = 1$, $\hat{r}_T = \hat{r}$.

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Special Cases
  - Two-Fading-Parameter Models
    - The α-μ model is obtained from the α-η-κ-μ one with $\alpha_T = \alpha$, $\mu_T = \mu$, $\kappa \to 0$, $\eta = p$, $\hat{r}_T = \hat{r}$.
    - The η-μ model is obtained from the α-η-κ-μ one with $\alpha = 2$, $2\mu_T = \mu$, $\kappa \to 0$, $\eta_T = \eta$, $p = 1$, $\hat{r}_T = \hat{r}$.
    - The κ-μ model is obtained from the α-η-κ-μ one with $\alpha = 2$, $\mu_T = \mu$, $\kappa_T = \kappa$, $\eta = p$, $q_T = q$, $\hat{r}_T = \hat{r}$.
    - The η-κ (Beckmann)model is obtained from the α-η-κ-μ one with $\alpha = 2$, $\mu = 1$, $\kappa_T = \kappa$, $\eta_T = \eta$, $p = 1$, $q_T = q$, $\hat{r}_T = \hat{r}$.

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Some Plots

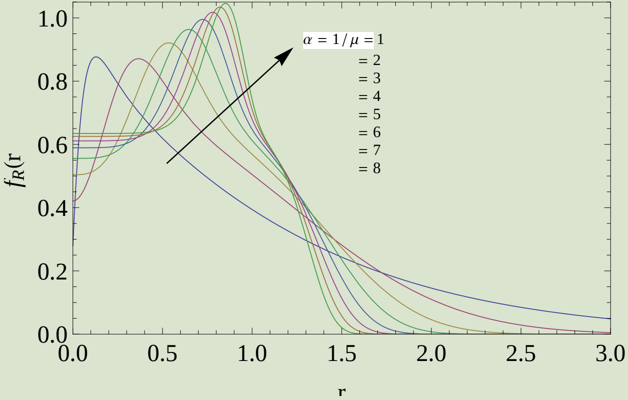


Figure: Envelope PDF for $\alpha\mu = 1$ ($\eta = 100$, $\kappa = 1$, $q = 1/10$, $p = 5$, $\hat{r} = 1$)

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Some Plots

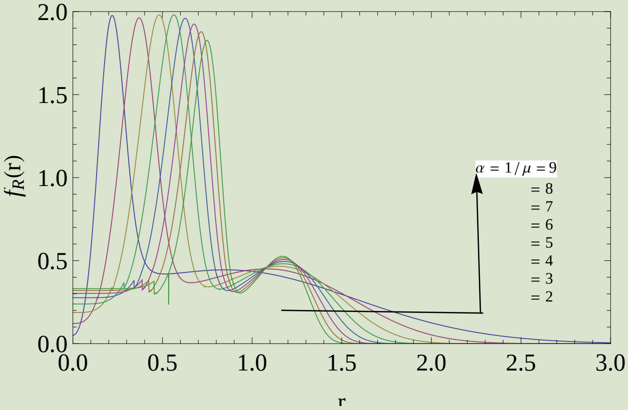


Figure: Envelope PDF for $\alpha\mu = 1$ ($\eta = 20$, $\kappa = 10$, $q = 1$, $p = 1/5$, $\hat{r} = 1$)

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Some Plots

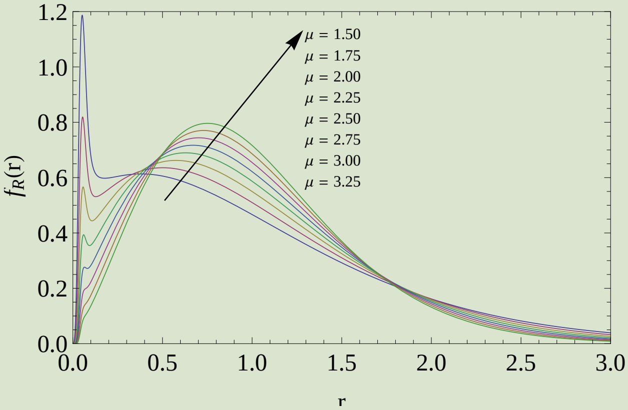


Figure: Envelope PDF for varying μ ($\alpha = 1$, $\eta = 20$, $\kappa = 10$, $q = 1$, $p = 1/5$, $\hat{r} = 1$)

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Some Plots

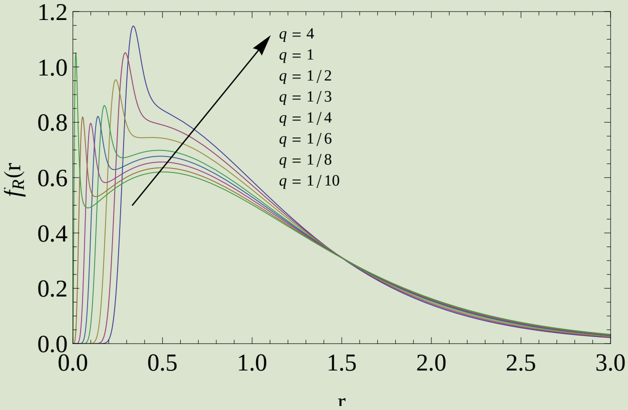


Figure: Envelope PDF for varying q ($\alpha = 1$, $\eta = 20$, $\kappa = 10$, $\mu = 1.75$, $p = 1/5$, $\hat{r} = 1$)

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Some Plots

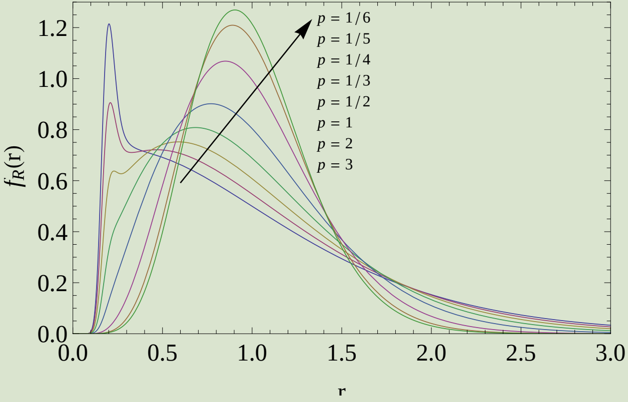


Figure: Envelope PDF for varying p ($\alpha = 1$, $\eta = 20$, $\kappa = 10$, $\mu = 1.75$, $q = 1/5$, $\hat{r} = 1$)

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Some Plots

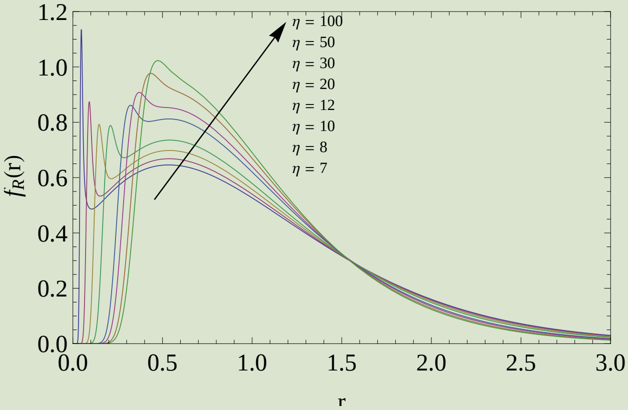


Figure: Envelope PDF for varying η ($\alpha = 1$, $\kappa = 10$, $\mu = 2.25$, $q = 1/5$, $p = 1/6$, $\hat{r} = 1$)

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Some Plots

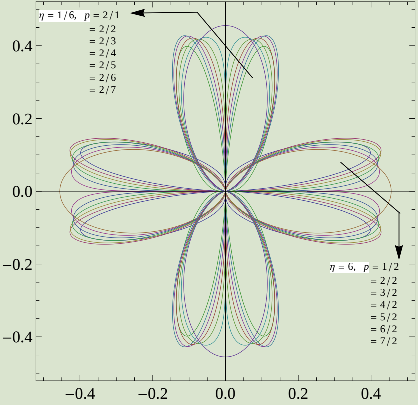


Figure: Phase PDF in polar coordinates, for varying p, no dominant components, and two symmetric values of η (α -irrelevant, $\kappa = 0$, $\mu = 2.25$, $q = 1$, $\hat{r}$-irrelevant)

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Some Plots

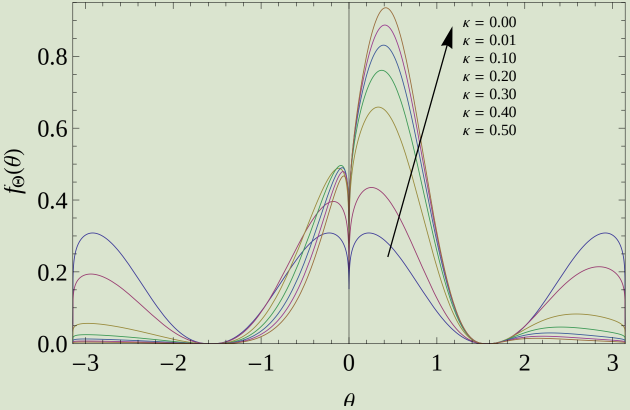


Figure: Phase PDF for varying κ (α -irrelevant, $\eta = 3$, $\mu = 2.25$, $q = 1$, $p = 3$, $\hat{r}$-irrelevant)

The α-η-κ-μ Fading Model

- The α-η-κ-μ Fading Model – Some Plots

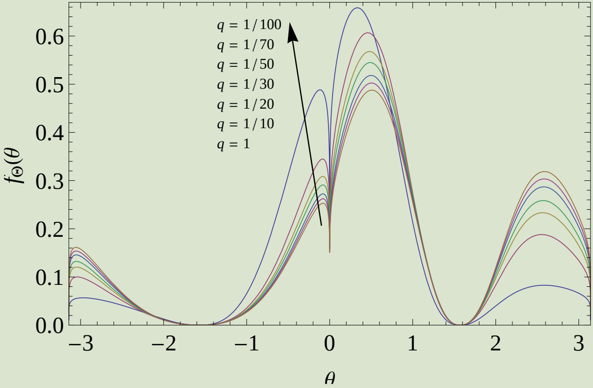


Figure: Phase PDF for varying q (α-irrelevant, $\eta = 3$, $\kappa = 0.1$, $\mu = 2.25$, $p = 3$, $\hat{r}$-irrelevant)

Conclusions and Challenges

Conclusions and Challenges

- To deepen the knowledge of the particular cases
- To obtain new particular cases
- To obtain new series expansions for envelope PDF and CDF
- To obtain other forms of parameterizations
- To obtain the general moments

Conclusions and Challenges

Conclusions and Challenges

- To find a way of estimating the parameters
- To propose efficient ways of simulation techniques
- To derive higher order statistics
- To propose correlation models
- To fit field data

Thank You!