Achievable Rate Region for the Gaussian MIMO-MAC with Cooperating Encoders

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Abstract

This paper considers a Gaussian two-user multiple-input multiple-output (MIMO) multiple-access channel (MAC) with cooperating encoders. After defining a form of cooperation strategy we derive achievable rate bounds by applying the results obtained by Willems for the MAC with generalized feedback. Then we characterize an achievable rate region by giving lower and upper bounds which are close together. The results obtained generally exceed the capacity region of a MIMO MAC without cooperation.

1 Introduction

Capacity regions of multiple-input multiple-output multiple-access (MIMO-MAC) channels are well described in [1], [2]. The characterization of these regions for the Gaussian MIMO-MAC is also treated in [3]. However, they were obtained without considering the possibility of transmitter cooperation. These regions give achievable rates for wireless systems that use multiple transmit and receive antennas.

A system emulating transmit antenna diversity (virtual MIMO) in a wireless environment was recently proposed by Sendonaris et al. for a Gaussian fading channel in [4], [5]. Diversity is achieved by allowing two users to cooperate before they transmit to the base station (BS). The benefits of user cooperation was demonstrated by comparing an achievable rate region with the two user single antenna MAC region. An achievable rate region for a discrete memoryless MAC with generalized feedback was described by Willems in [6]. The results of [4,7] can be obtained by applying the very general results of [6] to the scalar Gaussian MAC. User cooperation has also been investigated for the case of a three user MAC in [8].

In this paper we extend the results of [4, 7] by obtaining an achievable rate region for the Gaussian MIMO-MAC with cooperating encoders. First, we define a form of cooperation strategy by using auxiliary variables (usually called "cloud centers"). These variables introduce correlation between the signals transmitted by the users. Then, we derive achievable rate bounds by applying the results obtained in [6]. And finally, we characterize an achievable rate region by giving lower and upper bounds which are close together.

In Section 2 we describe the communication system model and make some definitions. In Section 3 we characterize an achievable rate region for the model of Section 2. Then in Section 4 we give some examples of cooperating systems and obtain their achievable regions. Finally, in Section 5 we make some final comments and conclusions.

2 System Model

We consider a two user Gaussian vector memoryless MAC where users can cooperate when transmitting their messages. This is illustrated in Fig. 1. The BS receiver has n_{r0} antennas and user i, $i = 1, 2$, transmits and receives with n_{ti} and n_{ri} antennas, respectively.

Users 1 and 2 generate their messages W_1 and W_2 independently and uniformly distributed over the sets $\{1, 2, \dots, 2^{NR_1}\}$ and $\{1, 2, \dots, 2^{NR_2}\}$. Let $\mathbf{y}_i \in \mathbb{R}^{n_{ri}\times 1}$, $i = 0, 1, 2$, be the received signals at the BS, user 1 and user 2, respectively. Encoder E_i, $i = 1, 2$ maps the message W_i and a sequence of previous received signals $(\mathbf{y}_i^1, \mathbf{y}_i^2, \dots, \mathbf{y}_i^{k-1})$ into the next channel input $\mathbf{x}_i^k \in \mathbb{R}^{n_{ti}}$, $k = 1, 2, \dots, N$. Upon receiving the sequence $(\mathbf{y}_0^1, \mathbf{y}_0^2, \dots, \mathbf{y}_0^N)$ the BS receiver's decoder obtain message estimates $\hat{W}_1$ and $\hat{W}_2$. An error occurs whenever $(\hat{W}_1, \hat{W}_2) \neq (W_1, W_2)$. A $(2^{NR_1}, 2^{NR_2}, N)$ code consists of two sets of N encoding functions and a BS receiver's decoding function.

The discrete time-invariant channel model is given by (where time index k is dropped)

$$\mathbf{y}_0 = \mathbf{H}_{10}\mathbf{x}_1 + \mathbf{H}_{20}\mathbf{x}_2 + \mathbf{z}_0 \quad (1)$$

$$\mathbf{y}_1 = \mathbf{H}_{21}\mathbf{x}_2 + \mathbf{z}_1 \quad (2)$$

$$\mathbf{y}_2 = \mathbf{H}_{12}\mathbf{x}_1 + \mathbf{z}_2 \quad (3)$$

where $\mathbf{z}_i \sim \mathcal{N}(0, N_i\mathbf{I}_{n_{ri}})$, $i = 0, 1, 2$, are independent additive white Gaussian noises at the BS, user 1 and user 2, respectively. The matrix $\mathbf{H}_{i0} \in \mathbb{R}^{n_{r0}\times n_{ti}}$ models the channel between user i , $i = 1, 2$, and the BS whereas matrices $\mathbf{H}_{12} \in \mathbb{R}^{n_{r2}\times n_{t1}}$ and $\mathbf{H}_{21} \in \mathbb{R}^{n_{r1}\times n_{t2}}$ models the inter-user channels. Each channel matrix is assumed to be known to their corresponding transmitter and

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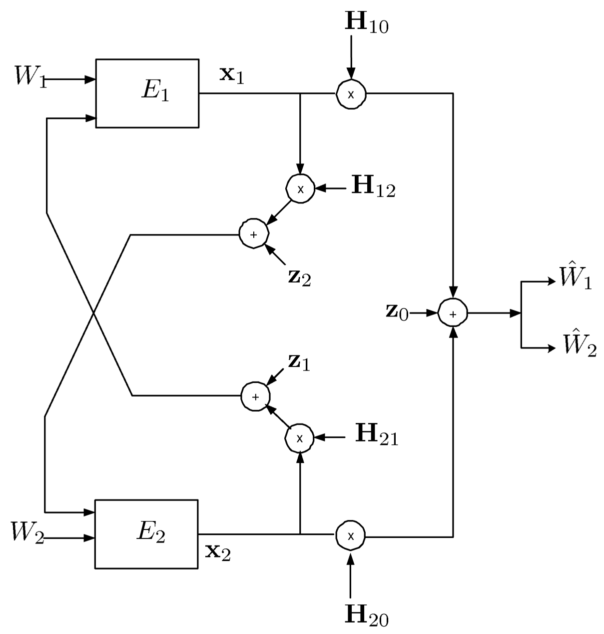

Figure 1: MIMO MAC Cooperation channel model

receiver. Additionally, we assume the power constraints $\operatorname{tr}(E\left[\mathbf{X}_i\mathbf{X}_i^T\right]) \leq P_i, i = 1, 2$, where $E\left[\cdot\right]$ stands for the average operator, $(\cdot)^T$ stands for transpose and $\operatorname{tr}(\cdot)$ is the trace operator.

The rate pair (R_1, R_2) is achievable for the Gaussian vector MAC with cooperation, if for any $\varepsilon > 0$ and for sufficiently large N there exists a sequence of $(2^{NR_1}, 2^{NR_2}, N)$ codes such that the error probability is less than ε. The set of all achievable rate pairs is the capacity region for the multiple access system described in this section. In the next section we will characterize an achievable rate region for this system.

3 Achievable Rate Region

In [6] an achievable rate region for the discrete memoryless MAC with generalized feedback was described. As the result of [6] is very general, we argue in the Appendix A that the result of [6] can also be applied to the Gaussian vector memoryless MAC considered in the last section.

User 1 divides its message W_1 into two parts, $W_{10} \in \{1, 2, \ldots, 2^{NR_{10}}\}$ and $W_{12} \in \{1, 2, \ldots, 2^{NR_{12}}\}$, and then uses signals $\mathbf{x}_{10} \in \mathbb{R}^{n_{t1}\times 1}$ to sent the first part directly to the BS and signals $\mathbf{x}_{12} \in \mathbb{R}^{n_{r1}\times 1}$ to sent the second part via user 2. User 2 also divides its message in a similar form: $W_{20} \in \{1, 2, \ldots, 2^{NR_{20}}\}$ and $W_{21} \in \{1, 2, \ldots, 2^{NR_{21}}\}$. Note that $R_1 = R_{10} + R_{12}$ and $R_2 = R_{20} + R_{21}$. User 2 has estimated the second part of user's 1 message and vice-versa. Based on these previous estimated messages they cooperate by simultaneously defining two vectors, $\mathbf{u}_1 \in \mathbb{R}^{n_{t1}\times 1}$ and $\mathbf{u}_2 \in \mathbb{R}^{n_{t2}\times 1}$. We call the definition of these vectors as a cooperation strategy. The users's signals can thus be decomposed as

$$\mathbf{x}_1 = \mathbf{x}_{10} + \mathbf{x}_{12} + \mathbf{u}_1 \quad (4)$$
$$\mathbf{x}_2 = \mathbf{x}_{20} + \mathbf{x}_{21} + \mathbf{u}_2 \quad (5)$$

Now define the covariance matrices: $\mathbf{Q}_i = E\left[\mathbf{X}_i\mathbf{X}_i^T\right]$, $\mathbf{Q}_{i0} = E\left[\mathbf{X}_{i0}\mathbf{X}_{i0}^T\right]$, $\mathbf{Q}_{Ui} = E\left[\mathbf{U}_i\mathbf{U}_i^T\right]$, $i = 1, 2$, $\mathbf{Q}_{12} = E\left[\mathbf{X}_{12}\mathbf{X}_{12}^T\right]$ and $\mathbf{Q}_{21} = E\left[\mathbf{X}_{21}\mathbf{X}_{21}^T\right]$. We assume that all vectors $\mathbf{x}_{i0}, \mathbf{u}_i, i = 1, 2$, $\mathbf{x}_{12}$ and $\mathbf{x}_{21}$ have a Gaussian distribution. We show in the Appendix A that the rate pair (R_1, R_2) is achievable if the inequalities (6)–(11) are simultaneously satisfied.

In these inequalities, $\mathbf{H} = [\mathbf{H}_{10}\ \mathbf{H}_{20}]$ and $\mathbf{Q} = \begin{pmatrix} \mathbf{Q}_1 & \mathbf{Q}_3 \\ \mathbf{Q}_3^T & \mathbf{Q}_2 \end{pmatrix}$, with $\mathbf{Q}_3 = E\left[\mathbf{U}_1\mathbf{U}_2^T\right]$. Note that matrices $\mathbf{Q}_1$ and $\mathbf{Q}_2$ can be expressed as

$$\mathbf{Q}_1 = \mathbf{Q}_{10} + \mathbf{Q}_{12} + \mathbf{Q}_{U1} \quad (12)$$
$$\mathbf{Q}_2 = \mathbf{Q}_{20} + \mathbf{Q}_{21} + \mathbf{Q}_{U2} \quad (13)$$

By using Lemma 7.1 of [6] it is possible to show that an equivalent set of inequalities defining achievable rate pairs is:

$$R_1 \leq F_{10} + F_{12} \quad (14)$$
$$R_2 \leq F_{20} + F_{21} \quad (15)$$
$$R_1 + R_2 \leq \min\{(F_0 + F_{12} + F_{21}), F\} \quad (16)$$

where F_{10}, F_{12}, F_{20}, F_{21},F_0 and F are the right-hand sides of inequalities (6), (8), (7), (9), (10) and (11), respectively.

Let $I_1 = F_{10} + F_{12}$, $I_2 = F_{20} + F_{21}$ and $I_3 = \min\{(F_0 + F_{12} + F_{21}), F\}$. We show in Appendix B that $I_3 \leq I_1 + I_2$ holds true. This implies that for a fixed input $\mathbf{Q_{i0}}, i = 1, 2$, $\mathbf{Q_{12}}$, $\mathbf{Q_{21}}$, and cooperation strategy $\mathbf{Q}$, the achievable rate pairs define in general a pentagon. The convex hull of the union of all possible pentagons is an achievable region. In order to obtain this region, a brute-force approach is to generate all pentagons, store their union and then apply a convex hull algorithm. This may be computationally hard. In case the union itself is convex, an easier way to get the boundary of the region is to maximize a weighted sum of the rates R_1, R_2 [1], [3], i.e., to maximize $a_1R_1 + a_2R_2$, with $a_1 + a_2 = 1$. If the union is not convex, the convex boundary obtained by maximizing a weighted sum of the rates R_1, R_2 is an upper bound for the achievable region. Unfortunately, we cannot say that the union of the regions defined by inequalities (14), (15) and (16) is convex.

Following the same reasoning of [1], [3] the upper bound can be characterized by solving the following optimization problem:

$$\max[(a_2 - a_1)(F_{20} + F_{21}) + a_1 \min((F_0 + F_{12} + F_{21}), F)] \quad (17)$$

$$R_{10} \leq \frac{1}{2}\log_2 \det\left(\mathbf{I}_{n_{r0}} + \frac{\mathbf{H}_{10}\mathbf{Q}_{10}\mathbf{H}_{10}^T}{N_0}\right) \quad (6)$$

$$R_{20} \leq \frac{1}{2}\log_2 \det\left(\mathbf{I}_{n_{r0}} + \frac{\mathbf{H}_{20}\mathbf{Q}_{20}\mathbf{H}_{20}^T}{N_0}\right) \quad (7)$$

$$R_{12} \leq \frac{1}{2}\log_2 \det\left(\mathbf{I}_{n_{r2}} + \left(N_2\mathbf{I}_{n_{r2}} + \mathbf{H}_{12}\mathbf{Q}_{10}\mathbf{H}_{12}^T\right)^{-1}\mathbf{H}_{12}\mathbf{Q}_{12}\mathbf{H}_{12}^T\right) \quad (8)$$

$$R_{21} \leq \frac{1}{2}\log_2 \det\left(\mathbf{I}_{n_{r1}} + \left(N_1\mathbf{I}_{n_{r1}} + \mathbf{H}_{21}\mathbf{Q}_{20}\mathbf{H}_{21}^T\right)^{-1}\mathbf{H}_{21}\mathbf{Q}_{21}\mathbf{H}_{21}^T\right) \quad (9)$$

$$R_{10} + R_{20} \leq \frac{1}{2}\log_2 \det\left(\mathbf{I}_{n_{r0}} + \frac{\mathbf{H}_{10}\mathbf{Q}_{10}\mathbf{H}_{10}^T + \mathbf{H}_{20}\mathbf{Q}_{20}\mathbf{H}_{20}^T}{N_0}\right) \quad (10)$$

$$R_{10} + R_{12} + R_{20} + R_{21} \leq \frac{1}{2}\log_2 \det\left(\mathbf{I}_{n_{r0}} + \frac{\mathbf{H}\mathbf{Q}\mathbf{H}^T}{N_0}\right) \quad (11)$$

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subject to

$$\mathbf{Q}_{10}, \mathbf{Q}_{20}, \mathbf{Q}_{12}, \mathbf{Q}_{21}, \mathbf{Q}_{U1}, \mathbf{Q}_{U2} \geq 0 \quad (18)$$

$$\mathrm{tr}\left(\mathbf{Q}_1\right) \leq P_1 \quad (19)$$

$$\mathrm{tr}\left(\mathbf{Q}_2\right) \leq P_2 \quad (20)$$

with $a_1 \leq a_2$, $a_1 + a_2 = 1$ and $\mathbf{Q}_3$ is a cross-correlation matrix. $\mathbf{A} \geq 0$ denotes that $\mathbf{A}$ is a positive semi-definite matrix.

The functions F_{12}, F_{21} and F are not concave in the space of matrices given in (18) and $\mathbf{Q}_3$ is not a positive semi-definite matrix. Hence, the above optimization problem is not in the class of convex programming problems. However, it was possible to use very efficient global optimization routines to solve the above problem.

Now suppose that we find the optimal $\mathbf{Q}_{10}^*$ and $\mathbf{Q}_{12}^*$ that maximizes $F_{10}+F_{12}$. Then using matrices $\mathbf{Q}_{10}^*$ and $\mathbf{Q}_{12}^*$, we find the other optimal strategies which maximizes (16). This define an achievable pentagon. The same procedure can be done to find a second achievable pentagon by first obtaining $\mathbf{Q}_{20}^*$ and $\mathbf{Q}_{21}^*$ that maximizes $F_{20} + F_{21}$. And finally, a third pentagon is obtained by maximizing (16) and using these optimal strategies to find R_1^* and R_2^*. The union of these three pentagons is achievable. We can enlarge even further this region if we allow time sharing combination of the corner points [3, Fig. 5]. This last region is a lower bound for the achievable region of our MIMO MAC with cooperation. In the next section we will show results for both lower and upper bounds.

4 Results

We have obtained results for two different systems. In the following all signals x_{ij}, u_i are zero mean and unit variance Gaussian variables.

System 1: Consider the case where the transmitters have one antenna each and the BS has two antennas, i.e., $n_{ti} = n_{ri} = 1$ $i = 1, 2$ and $n_{r0} = 2$. This system is denoted as system $1 \times 1 \times 2$. Set $\mathbf{x}_{i0} = (\sqrt{P_{i0}}x_{i0})$,$i = 1, 2$, $\mathbf{u}_1 = (\sqrt{P_1}u_1)$, and, $\mathbf{u}_2 = (\sqrt{P_2}u_2)$. Set also $\mathbf{x}_{12} = (\sqrt{P_{12}}x_{12})$ and $\mathbf{x}_{21} = (\sqrt{P_{21}}x_{21})$. Then $\mathbf{x}_i = (x_i)$,$i = 1, 2$, can be expressed as:

$$x_1 = \sqrt{P_{10}}\,x_{10} + \sqrt{P_{12}}\,x_{12} + \sqrt{P_1}\,u_1$$
$$x_2 = \sqrt{P_{20}}\,x_{20} + \sqrt{P_{21}}\,x_{21} + \sqrt{P_2}\,u_2$$

In the superposition block Markov encoding process, cooperation is based on previously estimated messages. This implies that components x_{ij} are independent of $u_i, i = 1, 2$. Then, $\mathbf{Q}_1 = [P_{10} + P_{12} + P_1]$, $\mathbf{Q}_2 = [P_{20} + P_{21} + P_2]$ and $\mathbf{Q}_3 = [\sqrt{P_1\,P_2}\rho]$, with $|\rho| \leq 1$.

System 2: Consider the case where both transmitters have two antennas and the BS has four antennas, i.e., $n_{ti} = n_{ri} = 2$ $i = 1, 2$ and $n_{r0} = 4$. This system is denoted as system $2 \times 2 \times 4$. Assume the following:

$$\mathbf{x}_{i0}^T = (\sqrt{P_{i0}^{(1)}}x_{i0}^{(1)}, \sqrt{P_{i0}^{(2)}}x_{i0}^{(2)}), i = 1, 2$$
$$\mathbf{u}_1^T = (\sqrt{P_1^{(1)}}u_1, \sqrt{P_1^{(2)}}u_2)$$
$$\mathbf{u}_2^T = (\sqrt{P_2^{(1)}}u_3, \sqrt{P_2^{(2)}}u_4)$$
$$\mathbf{x}_{12}^T = (\sqrt{P_{12}^{(1)}}x_{12}^{(1)}, \sqrt{P_{12}^{(2)}}x_{12}^{(2)})$$
$$\mathbf{x}_{21}^T = (\sqrt{P_{21}^{(1)}}x_{21}^{(1)}, \sqrt{P_{21}^{(2)}}x_{21}^{(2)})$$

Then $\mathbf{x}_i^T = (x_{i1}, x_{i2})$,$i = 1, 2$, can be expressed as:

$$x_{11} = \sqrt{P_{10}^{(1)}}x_{10}^{(1)} + \sqrt{P_{12}^{(1)}}x_{12}^{(1)} + \sqrt{P_1^{(1)}}u_1$$
$$x_{12} = \sqrt{P_{10}^{(2)}}x_{10}^{(2)} + \sqrt{P_{12}^{(2)}}x_{12}^{(2)} + \sqrt{P_1^{(2)}}u_2$$
$$x_{21} = \sqrt{P_{20}^{(1)}}x_{20}^{(1)} + \sqrt{P_{21}^{(1)}}x_{21}^{(1)} + \sqrt{P_2^{(1)}}u_3$$
$$x_{22} = \sqrt{P_{20}^{(2)}}x_{20}^{(2)} + \sqrt{P_{21}^{(2)}}x_{21}^{(2)} + \sqrt{P_2^{(2)}}u_4$$

Again, the components u_j, $j = 1, 2, 3, 4$ are independent of components $x_{ij}^{(k)}$. Then, $\mathbf{Q}_i$, $i = 1, 2, 3$

are given by

$$\mathbf{Q}_1 = \begin{pmatrix} P_{10}^{(1)} & \rho_{10}\sqrt{P_{10}^{(1)}P_{10}^{(2)}} \\ \rho_{10}\sqrt{P_{10}^{(1)}P_{10}^{(2)}} & P_{10}^{(2)} \end{pmatrix} + \\ + \begin{pmatrix} P_{12}^{(1)} & \rho_{12}\sqrt{P_{12}^{(1)}P_{12}^{(2)}} \\ \rho_{12}\sqrt{P_{12}^{(1)}P_{12}^{(2)}} & P_{12}^{(2)} \end{pmatrix} + \\ + \begin{pmatrix} P_{1}^{(1)} & 0 \\ 0 & P_{1}^{(2)} \end{pmatrix}$$

$$\mathbf{Q_2} = \begin{pmatrix} P_{20}^{(1)} & \rho_{20}\sqrt{P_{20}^{(1)}P_{20}^{(2)}} \\ \rho_{20}\sqrt{P_{20}^{(1)}P_{20}^{(2)}} & P_{20}^{(2)} \end{pmatrix} + \\ + \begin{pmatrix} P_{21}^{(1)} & \rho_{21}\sqrt{P_{21}^{(1)}P_{21}^{(2)}} \\ \rho_{21}\sqrt{P_{21}^{(1)}P_{21}^{(2)}} & P_{21}^{(2)} \end{pmatrix} + \\ + \begin{pmatrix} P_{2}^{(1)} & 0 \\ 0 & P_{2}^{(2)} \end{pmatrix}$$

$$\mathbf{Q_3} = \begin{pmatrix} \sqrt{P_1^{(1)}P_2^{(1)}}\sigma_{11} & \sqrt{P_1^{(1)}P_2^{(2)}}\sigma_{12} \\ \sqrt{P_1^{(2)}P_2^{(1)}}\sigma_{21} & \sqrt{P_1^{(2)}P_2^{(2)}}\sigma_{22} \end{pmatrix}$$

where $|\rho_{ij}| \leq 1$ and $|\sigma_{ij}| \leq 1$.

The achievable rate region using cooperation has been compared to a non-cooperation case in which the capacity region is given in [3].

Besides the non-cooperation case, an outer region was generated by maximizing the three inequalities (14)–(16) independently.

The total cooperation line is obtained by evaluating a point-to-point MIMO with n_{r0} receiving and $(n_{t1} + n_{t2})$ transmitting antennas [9].

Fig. 2 and 3 show results for systems $1 \times\ 1 \times 2$ and $2 \times\ 2 \times 4$, respectively. For the system of Fig. 2, the cooperation upper and lower bounds are not shown since they are almost indistinguishable from one another and coincide with the outer region. As can be observed in both figures, cooperation enlarges significantly the rate region. It is worthwhile noting that this enlargement is only possible when the inter–user channel matrices $\mathbf{H}_{12}$ and $\mathbf{H}_{21}$ represent better channels than the direct links channels $\mathbf{H}_{10}$ and $\mathbf{H}_{20}$. In other words, the users are close to each other.

The point where user 2 acts as a relay to user 1, $(R_1^*, 0)$, where R_1^* is the maximum of the right-hand side of (14), is an achievable rate for a MIMO Relay channel [10]. The fact that $R_2 = 0$ does not mean that $P_2 = 0$, since user 2 is cooperating with user 1. A similar conclusion holds for the case where user 1 acts as a relay to user 2.

5 Conclusions

An achievable rate region for the two-user Gaussian MIMO MAC with cooperating encoders was

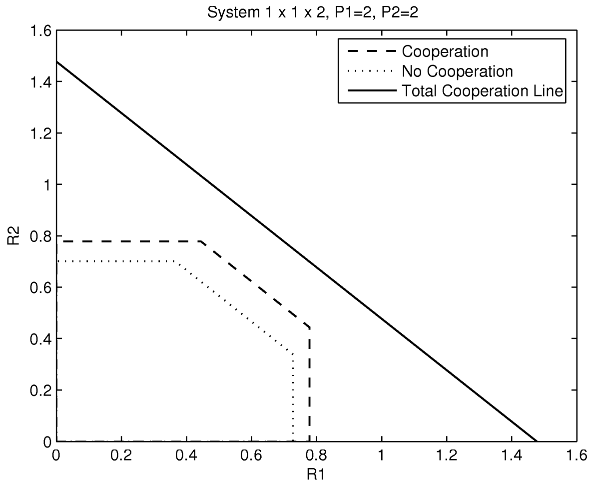

Figure 2: Achievable Rate Region for a 1x1x2 system with $(\mathbf{H}_{10} = [0.65\ 0.67], \mathbf{H}_{20} = [0.67\ 0.61], H_{12} = H_{21} = 0.985)$

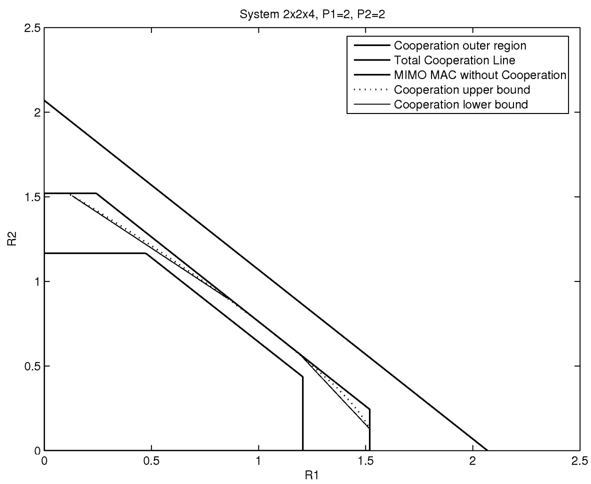

Figure 3: Achievable Rate Region for a 2x2x4 system with $\mathbf{H}_{10} = [0.5\ 0.45; 0.55\ 0.5; 0.4\ 0.6; 0.6\ 0.55], \mathbf{H}_{20} = [0.3\ 0.6; 0.45\ 0.7; 0.6\ 0.24; 0.34\ 0.7], \mathbf{H}_{12} = \mathbf{H}_{21} = [0.95\ 0.93; 0.93\ 0.99]$.

presented. The region generally enlarges the capacity region of a MIMO MAC without cooperation. We have also evaluated upper and lower bounds for this region. Numerical results were obtained and they showed that the bounds are close together.

Appendix A

Willems gives in [6, Theorem 7.1, Lemma 7.1] an achievable rate region for a discrete memoryless MAC with generalized feedback. The achievability proof is based on superposition block Markov encoding and backward decoding. The set of inequalities defining achievable rate pairs can be expressed as

$$R_1 \le I\left(V_1;Y_2|X_2,U\right)+I\left(X_1;Y|X_2,V_1,U\right) \quad (21)$$

$$R_2 \le I\left(V_2;Y_1|X_1,U\right)+I\left(X_2;Y|X_1,V_2,U\right) \quad (22)$$

$$\begin{aligned}R_1+R_2 \le \min\{&I\left(V_1;Y_2|X_2,U\right)\\&+I\left(V_2;Y_1|X_1,U\right)+I\left(X_1,X_2;Y|V_1,V_2,U\right),\\&I\left(X_1,X_2;Y\right)\}\end{aligned} \quad (23)$$

over the joint probability

$$\begin{aligned}&P(u,v_1,v_2,x_1,x_2,y_1,y_2,y)=\\&=P(u)P(v_1|u)P(v_2|u)P(x_1|v_1,u)\times\\&\times P(x_2|v_2,u)P(y,y_1,y_2|x_1,x_2).\end{aligned}$$

By considering an appropriate definition of the joint AEP property [11] for jointly typical decoding for continuous random variables and making the substitutions: $U \equiv (\mathbf{U}_1,\mathbf{U}_2) \equiv \mathbf{U}$, $V_1 \equiv \mathbf{X}_{12}$, $V_2 \equiv \mathbf{X}_{21}$,$X_1 \equiv \mathbf{X}_1$,$X_2 \equiv \mathbf{X}_2$, $Y_1 \equiv \mathbf{Y}_1$,$Y_2 \equiv \mathbf{Y}_2$ and $Y \equiv \mathbf{Y}_0$, inequalities (14), (15) and (16) can be derived. Note that in our case $P(y,y_1,y_2|x_1,x_2) = P(y|x_1,x_2)P(y_1|x_1,x_2)P(y_2|x_1,x_2)$, because of the noise independency.

For example, the first mutual information in (14), i.e. F_{10}, can be evaluated using the chain rule as

$$\begin{aligned}&I\left(\mathbf{X}_1;\mathbf{Y}|\mathbf{X}_2,\mathbf{X}_{12},\mathbf{U}\right)=\\&h\left(\mathbf{Y}|\mathbf{X}_2,\mathbf{X}_{12},\mathbf{U}\right)-h\left(\mathbf{Y}|\mathbf{X}_1,\mathbf{X}_2,\mathbf{X}_{12},\mathbf{U}\right)\\&=\frac{1}{2}\log_2\det\left(\pi e\left(N_0\mathbf{I}_{n_{r0}}+\mathbf{H}_{10}\mathbf{Q}_{10}\mathbf{H}_{10}^T\right)\right)-\\&-\frac{1}{2}\log_2\det\left(\pi e\left(N_0\mathbf{I}_{n_{r0}}\right)\right)\\&=\frac{1}{2}\log_2\det\left(\mathbf{I}_{n_{r0}}+\frac{\mathbf{H}_{10}\mathbf{Q}_{10}\mathbf{H}_{10}^T}{N_0}\right)\end{aligned}$$

All the other evaluations of mutual informations follow the same rationale and will not be shown here.

Appendix B

Consider inequalities (21), (22) and (23). Let

$$\begin{aligned}I_1 &= I\left(V_1;Y_2|X_2,U\right)+I\left(X_1;Y|X_2,V_1,U\right)\\I_2 &= I\left(V_2;Y_1|X_1,U\right)+I\left(X_2;Y|X_1,V_2,U\right)\\I_3 &= \min\{I_{3A},I_{3B}\}\\&= \min\{I\left(V_1;Y_2|X_2,U\right)+I\left(V_2;Y_1|X_1,U\right)\\&\quad +I\left(X_1,X_2;Y|V_1,V_2,U\right),I\left(X_1,X_2;Y\right)\}.\end{aligned}$$

To show that $I_3 \le I_1 + I_2$ is equivalent to show that $I_{3A} \le I_1 + I_2$, which in turn is equivalent to show that $I\left(X_1,X_2;Y|V_1,V_2,U\right) \le I\left(X_1;Y|X_2,V_1,U\right)+I\left(X_2;Y|X_1,V_2,U\right)$.

Due to independence of (X_1,Y) with respect to V_2 and of (X_2,Y) with respect to V_1, we can write that $I\left(X_1;Y|X_2,V_1,U\right) = I\left(X_1;Y|X_2,V_1,V_2,U\right)$ and $I\left(X_2;Y|X_1,V_2,U\right) = I\left(X_2;Y|X_1,V_1,V_2,U\right)$. However, conditioned on V_1,V_2,U, X_2 and X_1 are independent. Therefore, we can apply the same reasoning used in [11] for a two-user MAC without cooperation, to conclude that $I_3 \le I_1 + I_2$.

6 Acknowledgement

The authors would like to thank Paulo A. V. Ferreira for helpful discussions.

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