Power Allocation for Coded OFDM via Linear Programming

Alireza Kenarsari-Anhari and Lutz Lampe

***Abstract*—The combination of bit-interleaved coded modulation and orthogonal frequency-division multiplexing (BIC-OFDM) forms a powerful coded modulation scheme for transmission over wideband channels. Recently, Moon and Cox [1] presented a new power allocation method to minimize the bit-error rate (BER) of BIC-OFDM. It requires the solution of a convex optimization problem and is limited to (complex) binary transmission. Motivated by their work, in this letter we present an alternative power allocation method, which has the advantages of being a *linear program* and applicable to *arbitrary signal constellations*. Our approach relies on a BER approximation which becomes tight for asymptotically large signal-to-noise ratios. Simulative evidence shows that the proposed power allocation method achieves a performance very close to that from [1] for the case of quadrature phase-shift keying.**

***Index Terms*—Power loading, bit-interleaved coded modulation (BICM), orthogonal frequency division multiplexing (OFDM), linear program.**

I. Introduction

BIT-INTERLEAVED coded modulation (BICM) [2] has gained immense popularity for coded multilevel transmission. In combination with orthogonal frequency-division multiplexing (OFDM), i.e., BIC-OFDM, it is a powerful technique for transmission over frequency selective channels [3], which has been adopted in a number of recent standards.

OFDM enables transmitter side adaptation according to the present channel conditions, assuming that the channel remains unchanged over a sufficiently long interval. In particular, numerous algorithms for bit-loading and power allocation per OFDM sub-carrier have been developed, cf. e.g. [4]–[6]. Recently, [1] has studied the problem of power allocation for BIC-OFDM aiming at the minimization of bit-error rate (BER) under a power budget constraint, i.e.,

$$\begin{aligned} \min_{\underline{p}} \quad & P_{\mathrm{BER}} \\ \text{s.t.} \quad & \sum_{i=1}^{L} p_i \le P_{\mathrm{T}} \,, \\ & p_i \ge 0 \quad \forall i \in \{1, \ldots, L\} \,, \end{aligned} \tag{1}$$

where $\underline{p} \triangleq [p_1, \ldots, p_L]$ denotes the vector of powers allocated to each OFDM sub-carrier, P_{BER} denotes the BER, P_{T} is the maximal transmit power, and L is the number of OFDM sub-carriers. Using the union bound approach to approximate P_{BER}, it was shown [1] that (1) is a convex optimization problem. However, the solution presented in [1] is limited to (complex) binary transmission, i.e., binary and quadrature phase-shift keying (BPSK and QPSK), since linearity of coding and modulation was required.

Motivated by the approach in [1], in this letter we revisit the problem of power allocation for BIC-OFDM. We stipulate an approximative binary channel model for BIC-OFDM and make use of a newly derived expression for BICM error event probabilities from [7] to arrive at a simplified objective function.[1] This allows us to translate the optimization problem into a linear program (LP). Solving this LP using standard numerical methods is much faster than solving the general convex optimization problem obtained in [1] (cf. [9]). Numerical results show that the LP-based power allocation achieves a performance very close to that from convex programming of [1] for QPSK. Furthermore, the proposed method is applicable to arbitrary signal constellations and thus overcomes the restriction of BPSK and QPSK signaling needed in [1].

The rest of this letter is organized as follows. Section II introduces the system model for BIC-OFDM transmission. In Section III, a method for performance evaluation of BIC-OFDM is presented, based on which the power allocation optimization problem is formulated as an LP. Selected simulation results are shown in Section IV to illustrate the performance of the proposed method. Section V concludes the letter.

II. System Model

We consider a BIC-OFDM system with L sub-carriers. At the transmitter, the codeword $\underline{c} = [c_1, c_2, \cdots, c_N]$ generated from a linear binary encoder is bit-wise interleaved into $\underline{c}^\pi = [c_1^\pi, c_2^\pi, \cdots, c_N^\pi]$. The interleaved codeword is partitioned into blocks of r binary symbols, which are input to a subsequent mapper $\mu\,\{0,1\}^r \to \mathcal{X}$ such that $x_i = \mu\left(c^\pi_{(i-1)r+1}, \cdots, c^\pi_{ir}\right)$ is the signal point transmitted over the ith sub-carrier. The signal constellation $\mathcal{X}$ can be arbitrary, but most commonly PSK or quadrature amplitude modulation (QAM) constellations are considered. Furthermore, binary-reflected Gray mapping is applied.

Assuming a sufficiently long cyclic prefix and coherent reception, the equivalent baseband channel model is given by

$$y_i = \sqrt{p_i}\, h_i\, x_i + z_i \,, \quad i = 1, \cdots, L \,, \tag{2}$$

where y_i, h_i, p_i, and z_i are the received symbol, the frequency-domain channel gain, the allocated power, and the additive white Gaussian noise (AWGN) sample for the ith sub-carrier with variance $\mathbb{E}\{|z_i|^2\} = \sigma_z^2$, respectively. For (2) we assumed that $L = N/r$ is an integer.

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[1] In course of the review of this letter, it was brought to our attention that a similar criterion has been derived in unpublished material of the doctoral thesis [8].

At the receiver, the demapper outputs bit-wise reliability metrics

$$\lambda_{i,j} = -\min_{a\in\mathcal{X}_{j,1}} \|y_i - \sqrt{p_i}h_i a\|^2 + \min_{a\in\mathcal{X}_{j,0}} \|y_i - \sqrt{p_i}h_i a\|^2 , \tag{3}$$

$j = 1, \ldots, r$, for the r coded bits transmitted over the ith sub-carrier. $\mathcal{X}_{j,b}$ denotes the set of symbols in $\mathcal{X}$ with the jth bit in the binary label fixed to b. Finally, the metrics are deinterleaved and input to the maximum-likelihood sequence decoder of the binary code in order to retrieve the information bits.

III. Power Allocation Method

In this section, we present the new power allocation method. To this end, we first derive an expression for the probability of decoding errors, which relies on a simplified BIC-OFDM channel model and the result from [7]. We then show that this expression allows us to formulate the power allocation problem for BIC-OFDM with arbitrary constellations as an LP.

A. Error Event Probability

For a given vector of frequency-domain channel gains $\underline{h} \triangleq [h_1, \ldots, h_L]$ we model the effective channel between encoder output at the transmitter and decoder input at the receiver as a memoryless binary-input output symmetric (MBIOS) channel. This model is only an approximation for BIC-OFDM, as it neglects the dependencies between binary symbols c_k mapped to the same transmitted symbol x_i. However, their effect on the overall error probability of BIC-OFDM is negligible as long as interleaving distributes these c_k across dominant error events.

Let us identify an error event by the tuple $(d_{\rm H}, j)$, where $d_{\rm H}$ denotes its Hamming weight and j its index within the group of events with distance $d_{\rm H}$. Under the MBIOS channel model, the probability for this error event can be written as

$$P_{\rm e}(d_{\rm H}, j, \underline{h}) = \Pr(\Delta \le 0) , \tag{4}$$

where

$$\Delta \triangleq \sum_{k=1}^{d_{\rm H}} \lambda_{s_k, b_k} , \tag{5}$$

is the accumulated metric difference, and s_k and b_k denote the sub-carrier index and label position of the kth non-zero bit for the event, i.e., s_k and b_k are functions of $(d_{\rm H}, j)$. The bit metrics in (5) are mutually independent for the MBIOS model, and thus we can apply the PDF approximation for $\lambda_{i,j}$ developed in [7] to arrive at

$$P_{\rm e}(d_{\rm H}, j, \underline{h}) = \sum_{l_1=1}^{n} \cdots \sum_{l_{d_{\rm H}}=1}^{n} \left[\prod_{k=1}^{d_{\rm H}} \beta_{b_k, l_k}\right] \times Q\left(\sqrt{\left[\sum_{k=1}^{d_{\rm H}} p_{s_k} h_{s_k}^2 \, (l_k d_{\rm m})^2\right] \frac{1}{2\sigma_z^2}}\right) , \tag{6}$$

where $d_{\rm m}$ is the minimum Euclidean distance between signal points of $\mathcal{X}$, and n and $\beta_{j,l}$ are parameters solely defined by $\mathcal{X}$ (cf. [7, Section III.B] for details).

B. Linear Program Power Allocation

The error event probability can be used as a lower bound for the BIC-OFDM BER:

$$P_{\rm BER} \ge \max_{d_{\rm H}, j} \left[c(d_{\rm H}, j) P_{\rm e}(d_{\rm H}, j, \underline{h})\right] , \tag{7}$$

where the factor $c(d_{\rm H}, j)$ accounts for the number of errors caused by an error event. This bound becomes tight for asymptotically large signal-to-noise ratio (SNR). Considering the expression (6), the lower bound (7) will be asymptotically dominated by the component with the minimum effective squared Euclidean distance

$$d_{\rm E}^2(d_{\rm H}, j) \triangleq \sum_{k=1}^{d_{\rm H}} p_{s_k} h_{s_k}^2 d_{\rm m}^2 . \tag{8}$$

Thus, we suggest to apply power allocation such that the minimum of $d_{\rm E}^2(d_{\rm H}, j)$ is maximized. That is, the power allocation problem (1) can be reformulated as

$$\begin{aligned} \max_{\underline{p}} \quad & \min_{d_{\rm H}, j} d_{\rm E}^2(d_{\rm H}, j) \\ \text{s.t.} \quad & \sum_{i=1}^{L} p_i \le P_{\rm T} , \\ & p_i \ge 0 \quad \forall i \in \{1, \ldots, L\} . \end{aligned} \tag{9}$$

Considering (8), this problem can be re-written as (recall that the index s_k is a function of $(d_{\rm H}, j)$)

$$\begin{aligned} \max_{\underline{p}} \quad & t \\ \text{s.t.} \quad & t \le \sum_{k=1}^{d_{\rm H}} p_{s_k} h_{s_k}^2 \quad \forall (d_{\rm H}, j) \\ & \sum_{i=1}^{L} p_i \le P_{\rm T} , \\ & p_i \ge 0 \quad \forall i \in \{1, \ldots, L\} , \end{aligned} \tag{10}$$

which is an LP. The number of inequality constraints in (10) needs to be limited by considering only significant error events with $d_{\rm H} \le d_{\rm H,max}$, as has been done in [1]. Different from the convex program in [1], the LP is independent of the SNR. Using CVX, a package for specifying and solving convex programs [10], we have observed that the LP is solved ten times faster than the convex program from [1] (for a given SNR).

IV. Numerical Results

In this section, we present selected simulation results for the proposed power allocation method. We have used the WLAN IEEE 802.11a OFDM system with 48 active sub-carriers and the quasi-standard memory-6 convolutional code with generator polynomials $[133, 171]_8$ and rate $R_{\rm c} = 1/2$. All error events with Hamming weight $10 \le d_{\rm H} \le 14$ have been considered in (10). The channel realization $\underline{h}$ is randomly generated according to an exponentially decaying power delay profile and normalized to $\sum_{i=1}^{L} |h_i|^2 = L$.

Figure 1 compares the BER performances for (i) uniform power allocation (UPA), (ii) minimum BER allocation for uncoded transmission according to [6], (iii) power allocation (PA) for BIC-OFDM according to [1], and (iv) the proposed PA from (10) as function of the bit-wise SNR measure

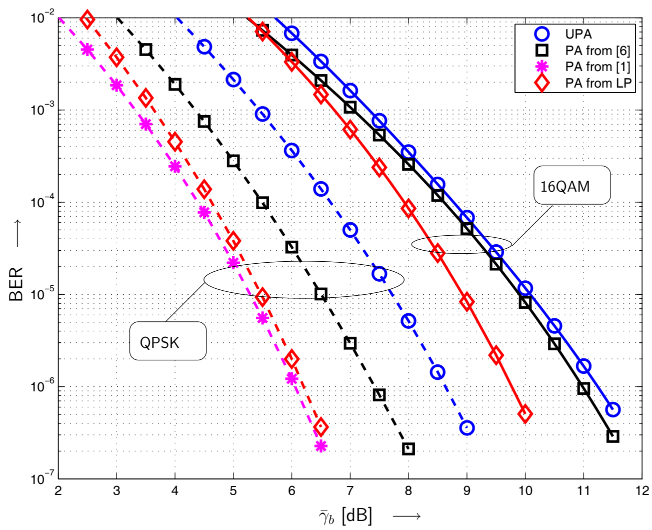

Fig. 1. BER of BIC-OFDM transmission systems with QPSK and 16QAM. Uniform power allocation (UPA), power allocation (PA) for uncoded transmission according to [6], PA according to [1], and PA using the proposed LP (10) are compared.

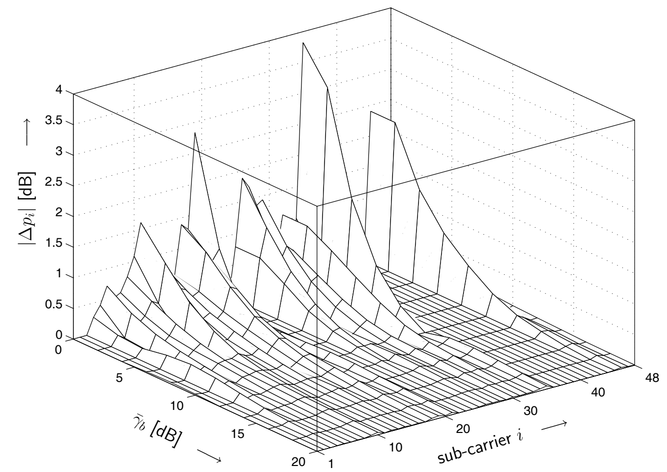

Fig. 2. Absolute difference $|\Delta p_i|$ between the power allocation solutions obtained from [1] and the LP (10) (for QPSK).

$\bar{\gamma}_b = P_{\mathrm{T}}/(\sigma_z^2 r R_{\mathrm{c}})$. QPSK and 16QAM are considered for all methods but the method from [1], which is only applicable to QPSK. We observe that the proposed method clearly outperforms UPA and PA designed for uncoded transmission. More importantly, its performance closely approaches that achieved with the considerably more complex method from [1] for the case of QPSK.

The difference between the PA solutions obtained from [1] and from the LP (10) is plotted in Figure 2. It can be seen that the LP solution converges to the PA from convex programming as SNR grows. This is due to the increasing dominance of the minimum distance error event for the overall error rate with increasing SNR.

Finally, Figure 3 illustrates the effect of LP-based PA on the distance profile of BIC-OFDM. For this purpose, the empirical cumulative density function (CDF) of d_{E}^2 defined in (8) is shown for UPA and the proposed PA. We observe that by maximizing the minimum distance, the LP effectively shifts the profile towards larger distances. This in turn reduces the overall error rate as has been seen in Figure 1.

V. Conclusions

We have developed a new power allocation policy for BIC-OFDM transmission. It is based on maximizing the minimum effective Euclidean distance of error events and requires the solution of an LP. Extensions of this approach to include bit loading are currently investigated.

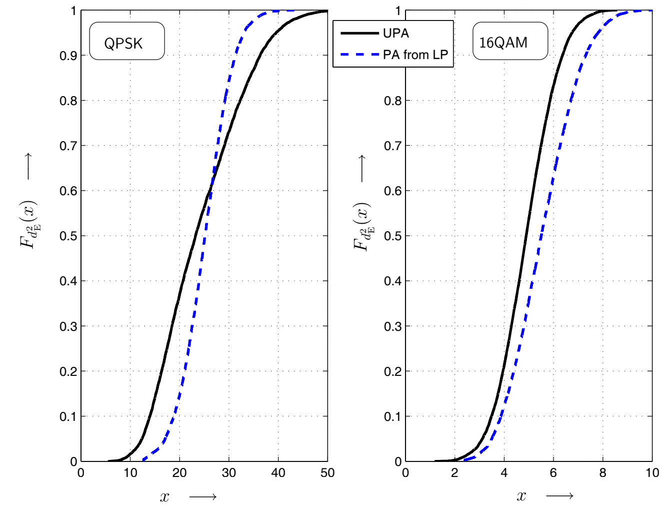

Fig. 3. CDF of d_{E}^2 from (8) for uniform power allocation (UPA) and PA with the proposed LP (10).

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