Wideband Waveform Optimization for Energy Detector Receiver with Practical Considerations

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***Abstract*—This paper deals with waveform optimization problems raised from advanced radio system prototyping conducted recently. Motivated by increasing demands for wireless sensor networks, simple receivers paired with sophisticated transmitters are considered. Among a few receiver candidates, energy detector (or square law) receiver is adopted in this paper. The receiver includes an energy detector followed by an integrator to generate a decision statistic. To compensate performance loss suffered due to the use of the simple receiver, an optimal waveform with respect to a known channel impulse response is transmitted, given all constraints posed by practical limitations. The optimization goal is to maximize signal energy at the output of the integrator at the receiver. Thanks to the recent technology advanced in semiconductors, waveform level (at multiple of Nyquist sampling rate) arbitrary precoding is possible. However, reduction in quantization and transmitted peak power is still very desired for conservation in implementation complexity and power consumption. These motivate us to consider various practical constraints in waveform optimization. A number of solutions are provided in this paper in conjunction with numerical verification using measured data.**

***Index Terms*—Wideband waveform optimization, energy detector, QCQP, SDP**

I. INTRODUCTION

Motivated by increasing demand for cheap wireless sensor networks, a wideband radio system combining waveform precoding and simple receivers is considered in this paper. Among receiver options is energy detector receiver that is attractive because of its compromising between performance and simplicity. The central idea is to pair the simple receiver with a sophisticated transmitter which is able to transmit an optimal waveform to achieve performance gain. Such type of transmitter featured by an arbitrary waveform generator has been tested in our laboratorial UWB test-bed [1], thanks to the recent technology advanced in semiconductors. Although multiple of Nyquist sampling rate has been feasible for Giga-Hertz bandwidth, reduction in quantization and transmitted peak power is very desired from implementation perspective.

Four practical cases are considered in this paper, which are all critical issues when we built our laboratorial UWB test-bed [1]. There should be a tradeoff between energies within and outside of integration window if inter-symbol-interference (ISI) has to be concerned. Because of the hardware limitation or implementation simplicity, a low-resolution quantization has to be employed sometimes. Thus the design for binary waveform and ternary waveform are studied. Meanwhile, binary waveform or ternary waveform can be treated as one kind of robust waveform, if channel estimation is not accurate. For waveform design in the digital domain, one of the challenging issues is Peak-to-Average Power Ratio (PAPR) because of the nonlinear devices used in the system. The transmitted peak power should be reduced such that the nonlinear devices can work in the proper region. All the practical cases are formulated as the corresponding optimization problems. By using the advanced optimization tool, the optimal solution or suboptimal solution to these optimization problems can be achieved. The work in this paper can not only make the contribution to the theoretical research in waveform optimization and synthesis [2] but also give the guidance for the real system design and implementation.

The rest of the paper is organized as follows. The system is described in Section II. Practical considerations for wideband waveform design are presented in Section III. Numerical results are provided in Section IV, followed by some remarks given in Section V.

II. SYSTEM DESCRIPTION AND OPTIMAL WAVEFORM

The system architecture is shown in Figure 1. We limit our discussion to a single-user scenario, and consider the transmitted signal with On-off keying (OOK) modulation given by

$$s(t) = \sum_{j=-\infty}^{\infty} d_j p(t - jT_b) \tag{1}$$

where T_b is the symbol duration, $p(t)$ is the transmitted symbol waveform defined over $[0, T_p]$ and $d_j \in \{0, 1\}$ is j-th transmitted bit. Without loss of generality, assume the minimal propagation delay is equal to zero. The energy of $p(t)$ is E_p,

$$\int_0^{T_p} p^2(t)\, dt = E_p \tag{2}$$

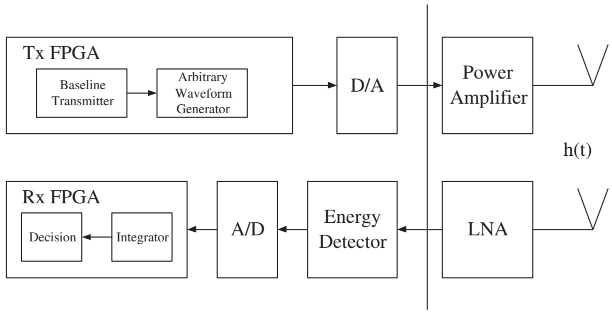

Fig. 1. System architecture.

The received noise-polluted signal at the output of the receiver front-end filter is

$$r(t) = h(t) \otimes s(t) + n(t) = \sum_{j=-\infty}^{\infty} d_j x(t - jT_b) + n(t), \tag{3}$$

where $h(t), t \in [0, T_h]$ is the multipath impulse response that takes into account the effect of channel impulse response, the RF front-ends in the transceivers including antennas. $h(t)$ is available at the transmitter [3] [4]. "$\otimes$" denotes convolution operation. $n(t)$ is a low-pass additive zero-mean Gaussian noise with one-sided bandwidth W and one-sided power spectral density N_0. $x(t)$ is the received noiseless symbol-"1" waveform defined as

$$x(t) = h(t) \otimes p(t) \tag{4}$$

We further assume that $T_b \geq T_h + T_p \stackrel{\text{def}}{=} T_x$, i.e. no existence of ISI.

An energy detector performs square operation to $r(t)$ without any explicit analog filter at the receiver. Then the integrator does the integration over a given integration window T_I. Corresponding to the time index k, the k-th decision statistic at the output of the integrator is given by

$$z_k = \int_{kT_b+T_{I0}}^{kT_b+T_{I0}+T_I} r^2(t)dt \tag{5}$$

$$= \int_{kT_b+T_{I0}}^{kT_b+T_{I0}+T_I} (d_k x(t - kT_b) + n(t))^2 dt \tag{6}$$

where T_{I0} is the starting time of integration for each symbol and $0 \leq T_{I0} < T_{I0} + T_I \leq T_x \leq T_b$.

An approximately equivalent SNR for the energy detector receiver, which provides the same detection performance when applied to a coherent receiver, is given as [5]

$$\mathrm{SNR_{eq}} = \frac{2\left(\int_{T_{I0}}^{T_{I0}+T_I} x^2(t)\,dt\right)^2}{2.3T_I W N_0^2 + N_0 \int_{T_{I0}}^{T_{I0}+T_I} x^2(t)\,dt} \tag{7}$$

For best performance, the equivalent SNR $\mathrm{SNR_{eq}}$ should be maximized. Define,

$$E_I = \int_{T_{I0}}^{T_{I0}+T_I} x^2(t)\,dt \tag{8}$$

For given T_I, N_0 and W, $\mathrm{SNR_{eq}}$ is the increasing function of E_I. So the maximization of $\mathrm{SNR_{eq}}$ in Equation (7) is equvalent to the maximization of E_I in Equation (8).

So the optimization problem to get the optimal $\mathbf{p}$ is shown below,

$$\begin{array}{l} \max \int_{T_{I0}}^{T_{I0}+T_I} x^2(t)\,dt \\ s.t. \int_0^{T_p} p^2(t)\,dt = E_p \end{array} \tag{9}$$

In order to solve the optimization problem (9), numerical approach is employed in this paper. In other words, $p(t)$, $h(t)$ and $x(t)$ are uniformly sampled (assumed at Nyquist rate), and the optimization problem (9) will be converted to its corresponding discrete-time form. Assume the sampling period is T_s. $T_p/T_s = N_p$, $T_h/T_s = N_h$ and $T_x/T_s = N_x$. So $N_x = N_p + N_h$.

$p(t)$, $h(t)$ and $x(t)$ are represented by $p_i, i = 0, 1, \ldots, N_p$, $h_i, i = 0, 1, \ldots, N_h$ and $x_i, i = 0, 1, \ldots, N_x$ respectively [5].

Define,

$$\mathbf{p} = [p_0\ p_1\ \cdots\ p_{N_p}]^T \tag{10}$$

and

$$\mathbf{x} = [x_0\ x_1\ \cdots\ x_{N_x}]^T \tag{11}$$

Construct channel matrix $\mathbf{H}_{(N_x+1)\times(N_p+1)}$,

$$(\mathbf{H})_{i,j} = \begin{cases} h_{i-j}, 0 \leq i - j \leq N_h \\ 0, \quad else \end{cases} \tag{12}$$

where $(\bullet)_{i,j}$ denotes the entry in the i-th row and j-th column of the matrix or vector. Meanwhile, for vector, taking $\mathbf{p}$ as an example, $(\mathbf{p})_{i,1}$ is equivalent to p_{i-1}.

The matrix expression of Equation (4) is,

$$\mathbf{x} = \mathbf{H}\mathbf{p} \tag{13}$$

and the constraint in the optimization problem (9) can be expressed as,

$$\|\mathbf{p}\|_2^2 T_s = E_p \tag{14}$$

where "$\|\bullet\|_2$" denotes the norm-2 of the vector. In order to make the whole paper consistent, we further assume,

$$\|\mathbf{p}\|_2^2 = 1 \tag{15}$$

Let $T_I/T_s = N_I$ and $T_{I0}/T_s = N_{I0}$. The entries in $\mathbf{x}$ within integration window constitute $\mathbf{x}_I$ as,

$$\mathbf{x}_I = [x_{N_{I0}}\ x_{N_{I0}+1}\ \cdots\ x_{N_{I0}+N_I}]^T \tag{16}$$

and E_I in Equation (8) can be equivalently shown as,

$$E_I = \|\mathbf{x}_I\|_2^2 T_s \tag{17}$$

Simply dropping T_s in E_I will not affect the optimization objective, so E_I is redefined as,

$$E_I = \|\mathbf{x}_I\|_2^2 \tag{18}$$

Similar to Equation (13), $\mathbf{x}_I$ can be obtained by,

$$\mathbf{x}_I = \mathbf{H}_I \mathbf{p} \tag{19}$$

where $(\mathbf{H}_I)_{i,j} = (\mathbf{H})_{N_{I0}+i,j}$ and $i = 1, 2, \ldots, N_I + 1$ as well as $j = 1, 2, \ldots, N_p + 1$.

The optimization problem (9) can be represented by its discrete-time form as,

$$\begin{aligned}&\max E_I\\&s.t.\ \|\mathbf{p}\|_2^2=1\end{aligned}\tag{20}$$

The optimal solution $\mathbf{p}^*$ for the optimization problem (20) is the dominant eigen-vector in the following eigen-function [5],

$$\mathbf{H}_I^T\mathbf{H}_I\mathbf{p}=\lambda\mathbf{p}\tag{21}$$

Furthermore, E_I^* will be obtained by Equation (18) and Equation (19).

III. Waveform Design with Practical Considerations

A. Tradeoff between Energies Within And Outside Of Integration Window

The energy outside of the integration window needs to be concerned sometimes, say, when ISI has to be considered. In order to reduce ISI, the energies within and outside of integration window should be balanced, which means the energy within integration window should be maximized and the energy outside of integration window should be minimized.

The entries in $\mathbf{x}$ outside of integration window constitute $\mathbf{x}_{\bar{I}}$ as,

$$\mathbf{x}_{\bar{I}}=[x_0\ \cdots\ x_{N_{I0-1}}\ x_{N_{I0}+N_I+1}\ \cdots\ x_{N_x}]^T\tag{22}$$

and the energy outside of integration window $E_{\bar{I}}$ can be expressed as,

$$E_{\bar{I}}=\|\mathbf{x}_{\bar{I}}\|_2^2\tag{23}$$

Similar to Equation (19), $\mathbf{x}_{\bar{I}}$ can be obtained by,

$$\mathbf{x}_{\bar{I}}=\mathbf{H}_{\bar{I}}\mathbf{p}\tag{24}$$

where $(\mathbf{H}_{\bar{I}})_{i,j}=(\mathbf{H})_{i,j}$ when $i=1,\ldots,N_{I0}$ and $(\mathbf{H}_{\bar{I}})_{i-(N_I+1),j}=(\mathbf{H})_{i,j}$ when $i=N_{I0}+N_I+2,\ldots,N_x+1$ as well as $j=1,2,\ldots,N_p+1$.

In order to balance energies within and outside of integration window, the tradeoff factor α is introduced. The range of α is from 0 to 1. Given α, the optimization problems is formulated as follows,

$$\begin{aligned}&\max \alpha E_I-(1-\alpha)E_{\bar{I}}\\&s.t.\ \|\mathbf{p}\|_2^2=1\end{aligned}\tag{25}$$

The optimal solution $\mathbf{p}^*$ for the optimization problem (25) is the dominant eigen-vector in the following eigen-function,

$$\left[\alpha\mathbf{H}_I^T\mathbf{H}_I-(1-\alpha)\mathbf{H}_{\bar{I}}^T\mathbf{H}_{\bar{I}}\right]\mathbf{p}=\lambda\mathbf{p}\tag{26}$$

B. Binary Waveform

If the transmitted waveform is constrained to the binary waveform because of the hardware limitation or implementation simplicity, which means $p_i, i=0,1,\ldots,N_p$ is equal to $-\frac{1}{\sqrt{1+N_p}}$ or $\frac{1}{\sqrt{1+N_p}}$, then the optimization problem is expressed as,

$$\begin{aligned}&\max E_I\\&s.t. [(\mathbf{p})_{i,1}]^2=\tfrac{1}{1+N_p}, i=0,1,\ldots,N_p\end{aligned}\tag{27}$$

One suboptimal solution $\mathbf{p}_{b1}^*$ to the optimization problem (27) is derived from the optimal solution $\mathbf{p}^*$ of the optimization problem (20). When $\mathbf{p}^*$ is obtained, then

$$(\mathbf{p}_{b1}^*)_{i,1}=\begin{cases}\frac{1}{\sqrt{1+N_p}}, (\mathbf{p}^*)_{i,1}\geq 0\\ -\frac{1}{\sqrt{1+N_p}}, (\mathbf{p}^*)_{i,1}<0\end{cases}\tag{28}$$

This simple method can lead to the optimal solution to the optimization problem (27) when $T_I\rightarrow 0$, which can be proofed by CauchySchwarz inequality, but if T_I is greater than zero, there is still a improvement potential to this suboptimal solution obtained from Equation (28).

It is well known that the optimization problem (27) is Quadratically Constrained Quadratic Program (QCQP) and general QCQP is NP-hard, so a semidefinite relaxation method is proposed to give the suboptimal solution to this optimization problem.

Define,

$$\mathbf{P}=\mathbf{p}\mathbf{p}^T\tag{29}$$

$\mathbf{P}$ should be a symmetric positive semidefinite matrix, i.e. $\mathbf{P}\succ= 0$ and rank of $\mathbf{P}$ should be equal to 1. Reformulate E_I as,

$$\begin{aligned}E_I&=\mathbf{p}^T\mathbf{H}_I^T\mathbf{H}_I\mathbf{p}&(30)\\&=\text{trace}\left(\mathbf{H}_I^T\mathbf{H}_I\mathbf{p}\mathbf{p}^T\right)&(31)\\&=\text{trace}\left(\mathbf{H}_I^T\mathbf{H}_I\mathbf{P}\right)&(32)\end{aligned}$$

Rank constraint is nonconvex constraint, so after dropping it, QCQP is relaxed to the Semidefinite Program (SDP),

$$\begin{aligned}&\max\text{trace}\left(\mathbf{H}_I^T\mathbf{H}_I\mathbf{P}\right)\\&s.t. (\mathbf{P})_{i,i}=\tfrac{1}{1+N_p}, i=0,1,\ldots,N_p\\&\quad\mathbf{P}\succ= 0\end{aligned}\tag{33}$$

The optimal solution $\mathbf{P}^*$ of the optimization problem (33) can be obtained by using CVX tool [6] and the value of the objective function in the optimizatiion problem (33) gives the upper bound of the optimal value in the optimization problem (27). Project the dominant eigen-vector of $\mathbf{P}^*$ on $-\frac{1}{\sqrt{1+N_p}}$ and $\frac{1}{\sqrt{1+N_p}}$ based on Equation (28), the suboptimal solution $\mathbf{p}_{b2}^*$ is achieved [7].

Finally, the designed binary waveform is,

$$\mathbf{p}_b^*=\arg\max_{\mathbf{p}\in\left\{\mathbf{p}_{b1}^*,\mathbf{p}_{b2}^*\right\}}\mathbf{p}^T\mathbf{H}_I^T\mathbf{H}_I\mathbf{p}\tag{34}$$

C. Ternary Waveform

If the transmitted waveform is constrained to the ternary waveform, which means $p_i, i=0,1,\ldots,N_p$ is equal to three levels, i.e. $-c$, 0 or c, then the optimization problem is expressed as,

$$\begin{aligned}&\max E_I\\&s.t.[(\mathbf{p})_{i,1}]^2=c^2\,\text{or}\,0, i=0,1,\ldots,N_p\\&\quad\|\mathbf{p}\|_2^2=1\end{aligned}\tag{35}$$

where the value of c will be determined later.

The optimization problem (35) is still NP-hard and can be approximately reformulated as,

$$\begin{array}{l} \max E_I \\ s.t. \text{Cardinality}(\mathbf{p}) \leq k \\ \quad 1 \leq k \leq N_p + 1 \qquad \|\mathbf{p}\|_2^2 = 1 \end{array} \tag{36}$$

where $\text{Cardinality}(\mathbf{p})$ denotes the number of non-zero entries of $\mathbf{p}$ and cardinality constraint is also a nonconvex constraint.

Because k is the integer number between 1 and $N_p + 1$, the optimization problem (36) can be decomposed into $N_p + 1$ independent and parallel sub-problems and each sub-problem is shown as,

$$\begin{array}{l} \max E_I \\ s.t. \text{Cardinality}(\mathbf{p}) \leq k \\ \quad \|\mathbf{p}\|_2^2 = 1 \end{array} \tag{37}$$

where k is equal to 1, 2, $\cdots$, or $N_p + 1$;

The sub-problems (37) can be solved in parallel and then the solutions are combined to get the solution of the original optimization problem (35). Reuse the definition in Equation (29) and the sub-problem (37) can converted to the following SDP by semidefinite relaxation combined with l1 heuristic [7],

$$\begin{array}{l} \max \text{trace}\left(\mathbf{H}_I^T \mathbf{H}_I \mathbf{P}\right) \\ s.t. \text{trace}(\mathbf{P}) = 1 \\ \quad \mathbf{a}^T |\mathbf{P}| \mathbf{a} \leq k \\ \quad \mathbf{P} \succ= 0 \end{array} \tag{38}$$

where $\mathbf{a}$ is all-1 column vector and,

$$\|\mathbf{p}\|_2^2 = \mathbf{p}^T \mathbf{p} \tag{39}$$

$$= \text{trace}\left(\mathbf{p}\mathbf{p}^T\right) \tag{40}$$

$$= \text{trace}(\mathbf{P}) \tag{41}$$

The CVX tool [6] is also operated to get the optimal solution $\mathbf{P}_k^*$ of SDP (38). From the dominant eigen-vector $\mathbf{p}_k^*$ of $\mathbf{P}_k^*$ and the threshold p_{thk}, the solution for the sub-problem (37) can be achieved as,

$$(\mathbf{p}_{\text{tk}}^*)_{i,1} = \begin{cases} c_k, (\mathbf{p}_k^*)_{i,1} > p_{\text{thk}} \\ 0, \left|(\mathbf{p}_k^*)_{i,1}\right| \leq p_{\text{thk}} \\ -c_k, (\mathbf{p}_k^*)_{i,1} < p_{\text{thk}} \end{cases} \tag{42}$$

where

$$\begin{array}{c} p_{\text{thk}} = \arg \max\limits_{\{p_{\text{th}}\}} (\mathbf{p}_{\text{tk}}^*)^T \mathbf{H}_I^T \mathbf{H}_I \mathbf{p}_{\text{tk}}^* \\ s.t. \text{Cardinality}(\mathbf{p}_{\text{tk}}^*) \leq k \end{array} \tag{43}$$

and

$$c_k = \frac{1}{\sqrt{\text{Cardinality}(\mathbf{p}_{\text{tk}}^*)}} \tag{44}$$

Finally, the designed ternary waveform is,

$$\mathbf{p}_t^* = \arg \max_{\mathbf{p} \in \left\{\mathbf{p}_{\text{tk}}^*, k=1,2,\ldots,N_p+1\right\}} \mathbf{p}^T \mathbf{H}_I^T \mathbf{H}_I \mathbf{p} \tag{45}$$

D. Peak-to-Average Power Ratio

PAPR is one of major concerns in waveform design. Because of nonlinearity caused by nonlinear devices such as Digital-to-Analog Converter (DAC) and Power Amplifier (PA), maximal transmitted power has to be backed up, resulting in inefficient utilization. PAPR in OFDM has been well studied. In this paper, PAPR is handled under a unified optimization framework. It is defined as,

$$\text{PAPR} = \frac{\|\mathbf{p}\|_\infty^2}{\|\mathbf{p}\|_2^2 \big/ (N_p + 1)} \tag{46}$$

where

$$\|\mathbf{p}\|_\infty = \max\left(|p_0|, |p_1|, \cdots, |p_{N_p}|\right) \tag{47}$$

If $\|\mathbf{p}\|_2^2$ is given, reducing PAPR is equivalent to setting the upper bound for $\|\mathbf{p}\|_\infty$. So the optimization problem can be expressed as,

$$\begin{array}{l} \max E_I \\ s.t. \|\mathbf{p}\|_2^2 = 1 \\ \quad \|\mathbf{p}\|_\infty \leq \text{ub} \end{array} \tag{48}$$

The bound constraint $\|\mathbf{p}\|_\infty \leq \text{ub}$ can also be written as,

$$-\text{ub} \leq p_i \leq \text{ub}, i = 0, 1, \ldots, N_p \tag{49}$$

which can be further simplified as,

$$p_i^2 \leq (\text{ub})^2, i = 0, 1, \ldots, N_p \tag{50}$$

Reuse the definition in Equation (29), the optimization problem (46) can be relaxed to SDP,

$$\begin{array}{l} \max \text{trace}\left(\mathbf{H}_I^T \mathbf{H}_I \mathbf{P}\right) \\ s.t. (\mathbf{P})_{i,i} \leq (\text{ub})^2, i = 0, 1, \ldots, N_p \\ \quad \text{trace}(\mathbf{P}) = 1 \\ \quad \mathbf{P} \succ= 0 \end{array} \tag{51}$$

By CVX tool [6], the optimal solution $\mathbf{P}^*$ of the optimization problem (51) is obtained. If the rank of $\mathbf{P}^*$ is equal to 1, then the dominant eigen-vector of $\mathbf{P}^*$ will be the optimal solution $\mathbf{p}^*$ for the optimization problem (48). But if the rank of $\mathbf{P}^*$ is not equal to 1, the dominant eigen-vector of $\mathbf{P}^*$ can not be treated as the optimal solution for the optimization problem (48), because of the violation of bound constraint.

So a computationally-efficient iterative algorithm is proposed to get the suboptimal solution $\mathbf{p}^*$ to the optimization problem (48) as follows.

1. Initialization: $P = 1$, $\mathbf{H}_0 = \mathbf{H}_I^T \mathbf{H}_I$ and $\mathbf{p}^*$ is set to be all-0 column vector.

2. Solve the following optimization problem to get the optimal $\mathbf{q}$.

$$\begin{array}{l} \max \mathbf{q}^T \mathbf{H}_0 \mathbf{q} \\ s.t. \|\mathbf{q}\|_2^2 = P \end{array} \tag{52}$$

3. Find i, such that $|q_i|$ is the maximal value in the set $\{|q_j| \, ||q_j| > \text{ub}\}$. If $\{i\} = \emptyset$, then the algorithm is terminated and $\mathbf{p}^* := \mathbf{p}^* + \mathbf{q}$. Otherwise go to step 4.

4. If q_i is greater than zero, then $(\mathbf{p}^*)_{i,1}$ is set to be ub. Otherwise $(\mathbf{p}^*)_{i,1}$ is set to be $-\text{ub}$.

5. $P := P - (\text{ub})^2$ and set $(\mathbf{H}_0)_{i,j}, j = 1, 2, \ldots, N_p + 1$ and $(\mathbf{H}_0)_{j,i}, j = 1, 2, \ldots, N_p + 1$ all to zeros. Go to step 2.

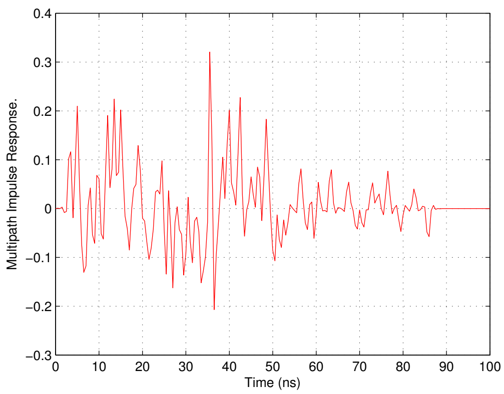

Fig. 2. Multhpath impulse response.

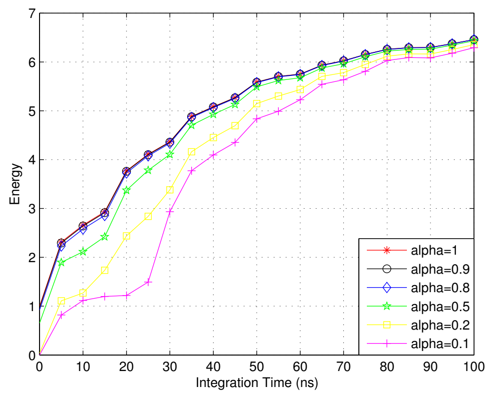

Fig. 3. The energy within the integration window.

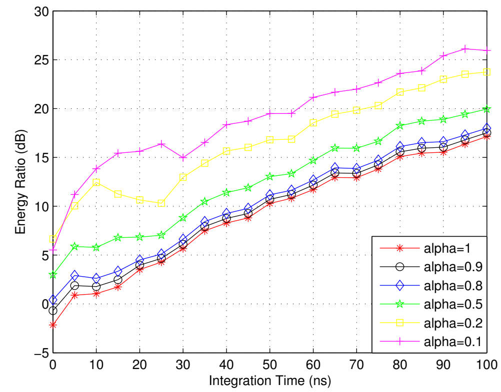

Fig. 4. The ratio of energies within and outside of integration window.

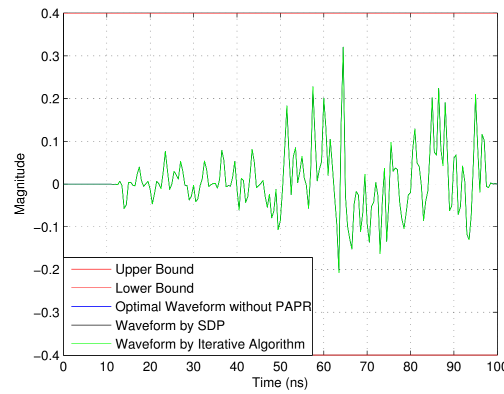

Fig. 5. The designed waveforms if $T_I \to 0$ and ub is set to be 0.4.

IV. Numerical Results

Figure 2 shows the multipath impulse response $h(t)$ under investigation in this paper and the energy of $h(t)$ is normalized. $T_s = 0.5ns$, $T_h = 100ns$, $T_p = 100ns$ and $T_{I0} + \frac{T_I}{2} = 100ns$.

Figure 3 shows the energy E_I in the integration window defined by Equation (18) when different tradeoff factor $\alpha's$ are chosen. Figure 4 shows the the corresponding ratio of energies $E_I/E_{\bar{I}}$ within and outside of integration window. When α is close to 1, the energy within integration window will have more weight. The optimization objective is to maximize the energy within integration window. Thus the larger α, the more energy within integration window. However, when α is close to 0, the optimization objective will be to minimize the energy outside of integration window. From Figure 4, the trend is, the smaller α, the bigger the ratio of energies within and outside of integration window. This results give us the hint to design the system with proper parameters if ISI is introduced.

For PAPR, if $T_I \to 0$ and ub is set to be 0.4, the designed waveforms are shown in Figure 5. In this case, the optimal waveform without consideration of PAPR, the waveform obtained by SDP and the waveform achieved by iterative algorithm are the same. The maximal eigen-value of $\mathbf{P}^*$ in the optimization problem (51) approaches 1, which means the rank of $\mathbf{P}^*$ is 1. So waveform obtained by SDP will not violate the bound constraint. Meanwhile, if $T_I \to 0$ and ub is not tight, the optimal waveform is the time reversed multipath impulse response. if $T_I \to 0$ and ub is set to be 0.15, the designed waveforms are shown in Figure 6. The optimal waveform without consideration of PAPR is still the time reversed multipath impulse response. The waveform obtained by SDP is equivalent to that achieved by iterative algorithm, which means both of two methods give the optimal solution under PAPR constraint.

If T_I is equal to $2ns$ and ub is set to be 0.15, the designed waveforms are shown in Figure 7. Because the rank of $\mathbf{P}^*$ in the optimization problem (51) is equal to 1, the waveform obtained by SDP gives the optimal solution. Meanwhile the waveform achieved by iterative algorithm gives the suboptimal solution and the ratio of energies within the integration window by the suboptimal waveform and the optimal waveform with PAPR is 0.99. If ub is set to be 0.1, the designed waveforms are shown in Figure 8. The rank of $\mathbf{P}^*$ in the optimization problem (51) is equal to 2 in this case, so the waveform obtained by SDP does not give the solution. It is easy to see from Figure 8 that this waveform violates the bound constraints in some samples.

In order to compare the performance of optimal waveform, binary waveform and ternary waveform, the down-sampled multipath impulse response in Figure 2 is used. Figure 9 shows the energies when optimal waveform, binary waveform and ternary waveform are employed. Because of SDP relaxation, the proposed algorithms to get binary waveform or ternary

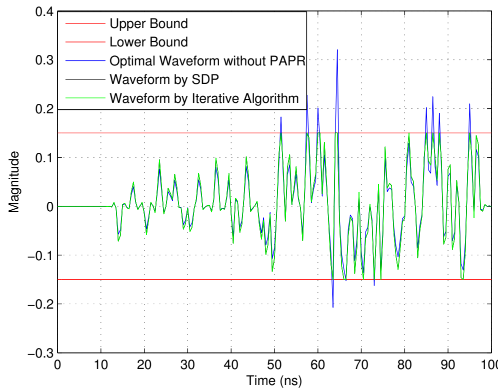

Fig. 6. The designed waveforms if $T_I \rightarrow 0$ and ub is set to be 0.15.

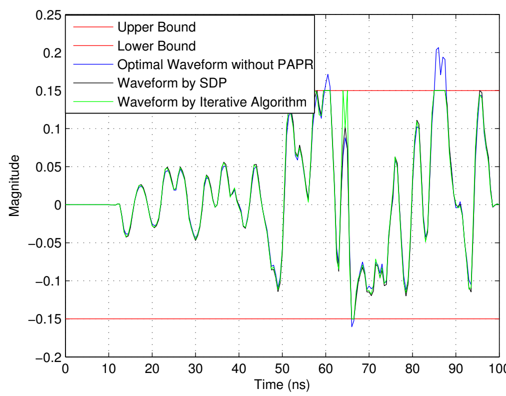

Fig. 7. The designed waveforms if T_I is equal to $2ns$ and ub is set to be 0.15.

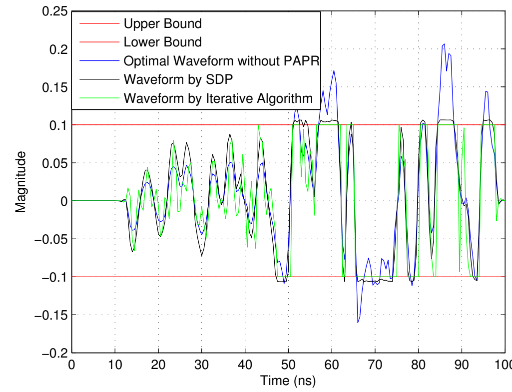

Fig. 8. The designed waveforms if T_I is equal to $2ns$ and ub is set to be 0.1.

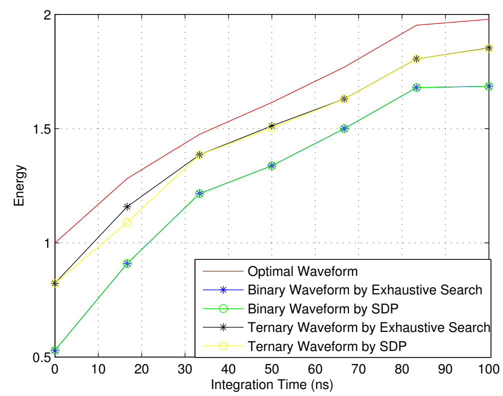

Fig. 9. The energies when optimal waveform, binary waveform and ternary waveform are employed.

waveform can not guarantee optimum from energy's point of view all the times. However, energy of the suboptimal binary or ternary waveform approaches that of the optimal binary or ternary waveform very well. Take this case as an example, the suboptimal ternary waveform catches at least 94 percent energy compared the optimal ternary waveform, while for binary waveform, this number is well above 0.99. Meanwhile, ternary waveform can obtain more than 80 percent energy from the optimal waveform, which makes ternary waveform competent in this kind of system with energy detector receiver.

V. Conclusion

Wideband waveform optimization with energy detector receiver has been studied in this paper. This work is a part of our effort in searching for simple-receiver solutions with enhanced performance. The contribution of this paper is to combine waveform design and optimization with the practical situations. Four practical cases are considered in this paper, i.e. the tradeoff between energies within and outside of integration window, binary waveform, ternary waveform and PAPR. The methods and results of this paper can give us the guidance for the real system design and implementation. More constraints posed by practical implementations will be explored.

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