

Wireless Sensor Network Localization With Imprecise Measurements Using Only a Quadratic Solver

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Abstract—*The energy constrained wireless sensor nodes need very efficient localization algorithm for event detection. We propose a method for localizing the wireless sensor nodes using the concept of Cayley-Menger Determinants, which in turn uses a quadratic solver. This is a modification of the method proposed in Ref. [5]. Cayley-Menger Determinants are used to introduce corrections to the noisy measurements, so that the measured distances meets all the Euclidean geometric properties. Ref. [7] uses semidefinite programming with L_1 norm, while Ref. [5] uses quadratic programming combined with Cayley-Menger Determinants. The proposed method is found to be computationally very much simpler and efficient than those in Ref. [7], and [5].*

Keywords: Wireless Sensor Networks, Cayley-Menger Determinants, Localization, L_2 norm approach, L_1 norm approach, Semidefinite Programming

1. Introduction

Wireless sensor networks are mainly used in hostile regions, where human intervention is very difficult. Each node will sense the environment, process the collected data, and communicate with its peers or to an external observer.

In most of the applications in wireless sensor networks, processing of the collected data needs to know the location at which the event happened. This makes the application development using wireless sensor network challenging.

Location of nodes can be found by deploying GPS in each node. But this makes the whole application a very costly approach. So different approaches has been formulated to reduce the cost involved. In those approaches, the nodes have to utilize peer nodes location estimate to find their location. In the paper [18], it discusses three main approaches that exist to determine a node's position:

- Using information about a node's neighborhood (proximity-based approaches)

- Exploiting geometric properties of a given scenario (triangulation and trilateration)
- Trying to analyze characteristic properties of the position of a node in comparison with premeasured properties (scene analysis)

The proximity based approaches are the simplest technique. It exploits the finite range of wireless communication, to decide whether a node that wants to determine its position or location is in the proximity of an anchor node, which is aware of its location.

By exploiting the geometric properties of a given scenario, the location can be found by either lateration or angulation[6]. When distances between entities are used, the approach is called lateration; when angles between nodes are used, the approach is called angulation[17]. In both cases, a system of constraints (usually) composed of polynomial equations has to be solved. It decomposes the system into irreducible subsystems, and solves them with symbolic or numerical methods. In this paper, the equations of geometric constraints are expressed using Cayley-Menger Determinants (CMDs), instead of using cartesian coordinates. The obtained system is much simpler, without spurious roots, and easily tractable by symbolic methods.

If the sensors and anchors are within the sensing range of one another, location can be estimated by direct measurement. Otherwise approximation methods like sum-dist discussed in [19], [15] and DV-hop discussed in [12], [15], can be used to estimate the sensor-anchor distances. No matter which method is used to obtain the distances, the data acquired are usually imprecise compared with the true distances, because of the measurement noise and estimation errors.

In this paper the Cayley-Menger distance concept is used to make correction in the noisy distance measurements as done in the reference paper [5]. In the paper [5], the euclidean distance geometry is made satisfied by L_2 norm optimization problem, while the proposed method uses only

a quadratic solver.

This work will be implemented with the test bed of wireless sensor networks for landslide detection¹ at Anthoniar Colony, Munnar, Idukki (Dist), Kerala (State), India. The deployment site has historically experienced several landslides, with the latest one occurring in the year 2005, which caused a death toll of 10 (people).

2. Identifying Geometric Constraints

Consider a simple wireless sensor network consisting of four sensor nodes, $i = p_0, \dots, p_3$, see Figure. 1. The anchor nodes, $k = p_1, \dots, p_3$ are aware of their location. The sensor node at p_0 is unaware of its location. For finding the location of p_0 sensor node, initially the geometric constraints between the four nodes have to be found. For this Cayley-Menger Determinants can be used. The Cayley-Menger determinant

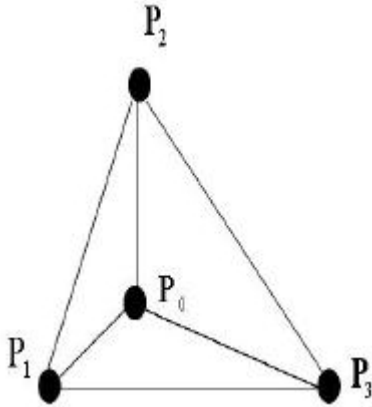


Fig. 1: Sensor node 0 with anchor node 1, 2, and 3 in 2D

of n points, $p_1, \dots, p_n \in R^n$, is defined as $D(p_1, \dots, p_n)$

$$\triangleq \begin{vmatrix} d^2(p_1, p_1) & d^2(p_1, p_2) & \dots & d^2(p_1, p_n) & 1 \\ d^2(p_2, p_1) & d^2(p_2, p_2) & \dots & d^2(p_2, p_n) & 1 \\ \dots & \dots & \ddots & \dots & \dots \\ d^2(p_n, p_1) & d^2(p_n, p_2) & \dots & d^2(p_n, p_n) & 1 \\ 1 & 1 & \dots & 1 & 0 \end{vmatrix} \quad (1)$$

where $d(p_i, p_j)$, $i, j \in 1, \dots, n$ is the Euclidean distance between the points p_i and p_j . If $n+2$ points are embeddable in E^n , Euclidean space, then their $n+1$ -dimensional volume must vanish, i.e

$$D(p_1, \dots, p_n) = 0 \quad (2)$$

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Here we are considering 2D case, with four points. So $D(p_0, \dots, p_3)$

$$\triangleq \begin{vmatrix} 0 & d^2(p_0, p_1) & d^2(p_0, p_2) & d^2(p_0, p_3) & 1 \\ d^2(p_1, p_0) & 0 & d^2(p_1, p_2) & d^2(p_1, p_3) & 1 \\ d^2(p_2, p_0) & d^2(p_2, p_1) & 0 & d^2(p_2, p_3) & 1 \\ d^2(p_3, p_0) & d^2(p_3, p_1) & d^2(p_3, p_2) & 0 & 1 \\ 1 & 1 & \dots & 1 & 0 \end{vmatrix} = 0 \quad (3)$$

Let $d_{ij} = d(p_i, p_j)$ denote the accurate Euclidean distance between nodes i and j . The distances measured using range measurements may be imprecise. So let the inaccurate distances be denoted as d_{0i} , $i = 1, 2, 3$. Let d_{0i} , $i = 1, 2, 3$ are available while the d_{ij} , with $i \neq j$, $i, j = 1, 2, 3$, are also known. Then

$$d^2_{0i} = d^2_{0i} - \epsilon_i \quad (4)$$

for some error ϵ_i . The errors ϵ_i for $i = 1, 2, 3$ should satisfy a single algebraic equality, as in Ref. [5], which is quadratic though not homogeneous in the ϵ_i 's, and whose coefficients are determined by d_{0i} for $i = 1, 2, 3$ and d_{ij} for $i, j = 1, 2, 3$ and $i \neq j$:

$$\epsilon^T A \epsilon + \epsilon^T b + c = 0 \quad (5)$$

where

$$\epsilon = [\epsilon_1 \quad \epsilon_2 \quad \epsilon_3]^T \quad (6)$$

A =

$$\begin{bmatrix} 2d^2_{23} & d^2_{12} - d^2_{13} - d^2_{23} & d^2_{13} - d^2_{23} - d^2_{12} \\ d^2_{12} - d^2_{13} - d^2_{23} & 2d^2_{13} & d^2_{23} - d^2_{12} - d^2_{13} \\ d^2_{13} - d^2_{12} - d^2_{23} & d^2_{23} - d^2_{12} - d^2_{13} & 2d^2_{12} \end{bmatrix} \quad (7)$$

$$b_1 = 4d^2_{23}d^2_{01} + 2(d^2_{12} - d^2_{13} - d^2_{23})d^2_{02} + 2(d^2_{13} - d^2_{12} - d^2_{23})d^2_{03} + 2d^2_{23}(d^2_{23} - d^2_{12} - d^2_{13}) \quad (8)$$

$$b_2 = 4d^2_{13}d^2_{02} + 2(d^2_{12} - d^2_{13} - d^2_{23})d^2_{01} + 2(d^2_{23} - d^2_{12} - d^2_{13})d^2_{03} + 2d^2_{13}(d^2_{13} - d^2_{12} - d^2_{23}) \quad (9)$$

$$b_3 = 4d^2_{12}d^2_{03} + 2(d^2_{13} - d^2_{12} - d^2_{23})d^2_{01} + 2(d^2_{23} - d^2_{12} - d^2_{13})d^2_{02} + 2d^2_{12}(d^2_{12} - d^2_{13} - d^2_{23}) \quad (10)$$

$$\begin{aligned}
c = & 2d_{12}^2 d_{13}^2 d_{23}^2 + 2d_{23}^2 d_{01}^4 + 2d_{12}^2 d_{03}^4 \\
& + 2(d_{12}^2 - d_{13}^2 - d_{23}^2) d_{01}^2 d_{02}^2 \\
& + 2(d_{13}^2 - d_{12}^2 - d_{23}^2) d_{01}^2 d_{03}^2 \\
& + 2(d_{23}^2 - d_{12}^2 - d_{13}^2) d_{02}^2 d_{03}^2 \\
& + 2d_{23}^2 (d_{23}^2 - d_{12}^2 - d_{13}^2) d_{01}^2 \\
& + 2d_{12}^2 (d_{13}^2 - d_{12}^2 - d_{23}^2) d_{02}^2 \\
& + 2d_{13}^2 (d_{13}^2 - d_{12}^2 - d_{23}^2) d_{03}^2
\end{aligned} \tag{11}$$

Furthermore, the matrix A is positive semi-definite.

3. Observations Leading to Quadratic Equations

The estimated error in the inaccurate distances can be found using the derived algebraic constraints (5).

Let ϵ_i as defined in (4) be the error in the estimated squared distances between sensor 0 and anchor i . If more than three anchor nodes are available, we take the least three distances between the anchor nodes and the sensor nodes. This is also justified from the view point that the inaccuracy in the distance measurements will be the least for the anchor nodes that are closest.

In Ref. [5], the problem formulation is, Minimize

$$\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2$$

subject to

$$\epsilon^T A \epsilon + \epsilon^T b + c = 0$$

where

$$\epsilon = [\epsilon_1 \quad \epsilon_2 \quad \epsilon_3]^T \tag{12}$$

Here the objective function is quadratic, i.e., it is a L_2 norm optimization problem. For reformulating the problem into L_1 norm approach, let $\epsilon_1 = p_1 - q_1$, $\epsilon_2 = p_2 - q_2$, and $\epsilon_3 = p_3 - q_3$, or in general $\epsilon_i = p_i - q_i$, p_i and q_i are such that

$$p_i, q_i \geq 0 \tag{13}$$

Thus the L1 norm formulation is Minimize

$$e^T (p + q)$$

subject to

$$\epsilon^T A \epsilon + \epsilon^T b + c = 0 \tag{14}$$

where e is a vector, whose elements are ones.

When we use L_1 norm, it is observed that the correction happens only in that distance measurement whose value is the greatest. If we assume only one ϵ to be non zero, the new formulation is: minimize

$$|\epsilon|$$

subject to

$$a_0 \epsilon^2 + a_1 \epsilon + a_2 = 0 \tag{15}$$

where a_0, a_1, a_2 are obtained from Equ. (5) by substituting only one ϵ as nonzero.

Set No	d _{01sq}	d _{02sq}	d _{03sq}	d _{12sq}	d _{13sq}	d _{23sq}
1	1225	1764	1849	1898	2209	65
2	1089	1764	1849	1898	2209	65
3	1296	1764	1849	1898	2209	65
4	1225	1681	1849	1898	2209	65
5	1225	1936	1849	1898	2209	65
6	1225	1764	1600	1898	2209	65
7	1225	1764	2116	1898	2209	65

Table 1: Set of Squared Distance Measurements

This result exactly agrees with the result obtained from the method in Ref. [7], whose formulation is as follows:

Let X represents the location of the node. Find X that minimizes,

$$e^T (p + q)$$

subject to

$$\begin{pmatrix} 1 \\ -a_k \end{pmatrix}^T \begin{pmatrix} Y & X \\ X^T & I \end{pmatrix} \begin{pmatrix} 1 \\ -a_k \end{pmatrix} - p + q = d_k^2,$$

$$\forall(k) \in N$$

$$Y = X^T X$$

$$p_i, q_i \geq 0 \tag{16}$$

where N is the set of closest anchor nodes. This approach uses Semidefinite Programming (SDP) with L_1 norm approach.

4. Testing

The proposed method and the method described in Ref. [7] has been tested using cvx programming [9]. The test data used are from Ref. [5].

A 2D scenario as depicted in Figure. 1, has been used for testing. The coordinates of the anchor nodes are $p_1 = (0, 0)$, $p_2 = (43, 7)$, and $p_3 = (47, 0)$ respectively. The noisy distance measurements acquired by sensor 0 are $d_{01} = 35$, $d_{02} = 42$ and $d_{03} = 43$.

The distances between beacons can be obtained using Eq. (5). The distance measurement between the anchor nodes and sensor node are unknown. Assuming that the estimated distances are error prone, the experiment was conducted with different sets of values, as shown in Table 1. The entry d_{01sq} means the squared distance between the anchor 1 and the sensor node 0. The same explanation can be extended for d_{02sq} , d_{03sq} , d_{12sq} , d_{13sq} , and d_{23sq} respectively.

The set of values from Table 1 are used for testing. Using each row entry in the Table 1, corresponding locations are found. The results are given in the Tables 2, 3, and 4. Set No is got from Table 1. The values of ϵ_1 , ϵ_2 , and ϵ_3 correspond to the values of ϵ_1 , ϵ_2 , and ϵ_3 respectively. The values of x_0, y_0 corresponds to computed location. The Table

Set No	epsi1	epsi2	epsi3	x_0	y_0
1	-21.18	-302.66	257.19	17.85	-28.60
2	-25.32	-295.84	252.45	16.27	-27.18
3	-19.17	-305.75	259.25	18.68	-29.28
4	-17.45	-254.91	215.96	18.30	-28.37
5	-29.35	-401.04	342.88	16.92	-29.05
6	-28.54	-426.73	358.84	19.19	-27.55
7	-12.50	-170.14	146.07	16.43	-29.62

Table 2: Solution for L2 norm as in Ref. [5]

Set No	epsi1	epsi2	epsi3	x_0	y_0
1	0.00	0.00	335.30	20.43	-28.42
2	0.00	0.00	310.36	18.65	-27.22
3	0.00	0.00	347.30	21.35	-28.99
4	0.00	0.00	345.32	21.29	-27.78
5	0.00	0.00	253.28	18.63	-29.63
6	0.00	0.00	167.45	20.43	-28.42
7	0.00	0.00	337.80	18.00	-28.86

Table 3: The Solution for L_1 Norm

2 gives the values of ϵ_1 , ϵ_2 , and ϵ_3 , and the locations, x_0 and y_0 computed using Ref. [5].

Table. 3 gives the values of the locations, x_0 and y_0 computed using the proposed method. Table. 4 gives the comparison results between Ref. [7], proposed method, and Ref. [5].

5. Conclusion And FutureWork

This paper estimates the location of sensor nodes from the available imprecise distance measurements between the anchor and sensor nodes. This approach uses Cayley-Menger Determinants for making corrections. This solution requires only a quadratic solver. This reduces the computational complexity and the computational time required for solving the problem. Since wireless sensor networks are highly energy-constrained, this result helps in minimum usage of energy, by reducing the computational complexity.

The approach described in Ref.[7] is more general, which considers distance between the sensor and anchor nodes as well as interdistance between the sensor nodes. In our approach we have not considered the distance between sensor nodes. So we are planning to use Cayley-Menger distance method in a general framework. We are also planning to work for 3D case.

Set No	Ref.[5] x_0	Ref.[5] y_0	New x_0	New y_0	Ref. [7] x_0	Ref. [7] y_0
1	17.85	-28.60	13.90	31.79	13.90	31.79
2	16.27	-27.18	12.46	30.14	12.46	30.14
3	18.68	-29.28	14.66	32.59	14.66	32.59
4	18.30	-28.37	14.38	31.64	14.38	31.64
5	16.92	-29.05	12.90	32.08	12.90	32.08
6	19.19	-27.55	15.39	30.98	15.39	30.98
7	16.43	-29.62	12.33	32.56	12.33	32.56

Table 4: Comparison of Ref. [7], new method, Ref.Caoetal05 while allowing all epsilons to vary

References

- [1] Xiao, B., Chen, H., Zhou, S; *A Walking Beacon-Assisted Localization in Wireless Sensor Networks*, Proceedings of ICC 2007, IEEE, 2007
- [2] Biswas, P., Lian, T., Wang, T., Ye, Y; *Semidefinite Programming Based Algorithms for Sensor Network Localization*, ACM Transactions on Sensor Networks, 2006, Vol. 2(2), pp. 188-220
- [3] Blumenthal, L. M; *Theory and Applications of Distance Geometry*, Oxford University Press, Oxford, 1953
- [4] Boyd, S., Ghauoi, L.E., Freon, E., Balakrishnan, V; *Linear Matrix Inequalities in System and Control Theory*, SIAM, 1994
- [5] Cao, M., Anderson, B.D.O., Morse, A.S; *Localization with Imprecise Distance Information in Sensor Networks*, Proceedings of the 44th IEEE Conference on Decision and Control, and the European CControl Conference 2005, IEEE, 2005, pp. 2829-2834
- [6] Cao, M., Anderson, B.D.O., Morse, A.S; *Sensor Network Localization With Imprecise Distances*, Elsevier, 2006, Vol. 55(11), pp. 887-893
- [7] Carter, M.W., Jiny, H.H., Saundersz, M.A., Yex, Y; *SPASELOC: An adaptive subproblem algorithm for scalable wireless sensor network localization*, 2005
- [8] Crippen, G.M., Havel, T. F; *Distance Geometry and Molecular Conformation*, Wiley, New York, 1988
- [9] Grant, M., Boyd, S., Ye, Y; *cvx User's Guide*, 2006
- [10] Michelucci, D., Foufouy, S; *Using Cayley-Menger Determinants for Geometric Constraint Solving*, Proceedings of ACM Symposium on Solid Modeling and Applications, 2004
- [11] Moore, D., Leonard, J., Rus, D., Teller, S; *Robust distributed network localization with noisy range measurements*, Proceedings of SenSys, 2004
- [12] Niculescu, D., Nath, B; *Localized positioning in ad hoc networks*, Journal of Ad Hoc Networks (Special issue on Sensor Network Protocols and Applications), 2003, Vol. 1, pp. 247-259
- [13] Peng, R., Sichitiu, M. L; *Angle of Arrival Localization for Wireless Sensor Networks*, Secon 2006
- [14] Rai, A., Ale, S., Rizvi, S.S., Riasat, A; *A New Methodology for Self Localization in Wireless Sensor Networks*, IEEE INMIC-08 proceeding, IEEE, 2008
- [15] Savarese, C., Rabay, J., Langendoen, K; *Robust positioning algorithms for distributed ad-hoc wireless sensor networks*, USENIX Technical Annual Conference, 2002
- [16] Savvides, A., Park, H., Srivastava, M; *The n-hop multilateration primitive for node localization problems*, ACM Mobile Networks Application, 2003, Vol. 8, pp. 443-452
- [17] *Handbook of Sensor Networks: Compact Wireless and Wired Sensing Systems*, Ilyas, M., Mahgoub, I (ed.), CRC Press, 2005
- [18] Karl, H., Willing, A., *Protocols and architectures for wireless sensor networks*, John Wiley Sons , Ltd, 2005.
- [19] Langendoen, K., Reigers, N; *Distributed localization in wireless sensor networks: a quantitative comparison*, Journal of Computer Networks, Vol. 43, pp. 499-518, 2003.