PAPER

Cross-Layer Proportional Fair Scheduling with Packet Length Constraint in Multiuser OFDM Networks*

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SUMMARY In this paper, we investigate the proportional fair scheduling (PFS) problem for multiuser OFDM systems, considering the impact of packet length. Packet length influences scheduling schemes in a way that each scheduled packet should be ensured to be completely transmitted within the scheduled frames. We formulate the PFS problem as an optimization problem. Based on the observations on the structure of optimal solutions, we propose a heuristic scheduling algorithm that consists of two stages. First, subcarriers are allocated among users without considering the packet length constraint. Then on the second stage, subcarrier readjustment is done in a way that surplus subcarriers from length-satisfied users are released and allocated among length-unsatisfied users. The objective is to provide proportional fairness among users while guaranteeing complete transmission of each scheduled packet. Simulation results show that the proposed scheme has quite close performance to the optimal scheme in terms of Multi-carrier Proportional Fairness Measure (MCPFM), throughput and average packet delay.

key words: *proportional fairness, packet scheduling, cross-layer design, packet length, OFDM*

1. Introduction

In wireless environments, one approach that has the potential of increasing system capacity is opportunistic scheduling; accordingly, it has drawn much research interests recently [1], [2]. Opportunistic scheduling exploits the multiuser diversity inherent in a wireless network with multiple users. Assuming that different users in the system experience independent channel fading conditions, there is likely to be a user whose channel is near its peak at any one time. Therefore, the system can always choose a user who has the best channel condition for data transmission. In this way, the system capacity can be greatly increased.

However, when it comes to the implementing issue, one is immediately confronted with a problem: fairness. Pure opportunistic scheduling, which always selects the user with the best channel condition for transmission, makes the fairness among the users unacceptable although it can maximize the total system throughput.

To address this issue, many fair opportunistic scheduling schemes have been proposed to provide different tradeoffs between system throughput and user level fairness [3]–[5]. Among all the fairness criterions, proportional fairness (PF), which was originally proposed by Kelly [6] as an alternative for a max-min scheduler, is one of the most commonly studied. In fact, a PF scheduling scheme has been implemented in the context of the downlink of IS-856 system (also known as Qualcomm's HDR system) and it turns out to be efficient in striking a balance between system throughput and fairness.

Previous research about proportional fair scheduling (PFS) mostly focuses on single carrier systems (e.g., HDR system). However, with the emergence of multi-carrier technologies (e.g., MC-CDMA and OFDM) and their widely application in real wireless systems, the interests in proportional fairness have shifted from single carrier systems to multi-carrier cases [7]–[9] recently. In [7], Anchun considered the PFS problem by simply applying the PFS scheme that is adopted in single carrier case to each subcarrier of the OFDM system. The scheme is so simple that it does not actually meet the optimal conditions for proportional fairness in multi-carrier case, which were not derived until recently by Kim [8]. In [8], Kim deduced the sufficient and necessary conditions for proportional fairness in multi-carrier systems. However, to achieve the optimal conditions requires prohibitively high complexity. In [9], Kaneko considered a practical system where each frame is composed of several OFDM symbols and then derived the upper and lower proportional fairness limits which bound the optimal solution by means of convex optimization. Two heuristic algorithms were further proposed in [9] to satisfy proportional fairness among multiple users.

Despite the performance gain these schemes have gotten, all of them failed to take the factor of *packet length* into account, which has an influence on the performance of scheduling schemes, especially for OFDM systems. The reasons go as follows. In wireless OFDM systems, cross-layer design methodologies are widely adopted to improve system performance. When joint MAC-layer packet scheduling and PHY-layer resource allocation (e.g. subcarrier allocation and bit loading) is considered (e.g., [9]), the scheduler will not only decide which packets are scheduled but also decide the subcarriers and the rates (e.g., bits/frame) to carry those packets. In this case, it should be assured that the scheduled packets can be *completely transmitted* within a frame, i.e., the length of each scheduled packet should be less than or equal to the rate (bits/frame) it is assigned within the frame. However, to our best knowledge, current research on cross-layer design in OFDM systems does not take note of this problem and left it untouched.

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Therefore, in this paper, we are motivated to consider proportional fair scheduling in OFDM systems, taking packet length into account. Based on the optimal conditions for proportional fairness derived in [8], we formulate the problem to an optimization problem, with the packet length being a constraint. By Lagrangian method, we can get the KKT (Karush-Kuhn-Tucker, [14]) optimality conditions. Although the optimal solution is still hard to get, the KKT conditions provide an insight into the structure of optimal solutions. Therefore, based on the observations on the structure, we proposed a heuristic algorithm. The proposed scheme is composed of two stages. First, subcarriers are allocated among users according to a criterion derived from the KKT conditions. Then on the second stage, we remove some of the surplus subcarriers from the users who have already met the packet length constraint and then, reallocate them among those users who do not satisfy the constraint. By this way, all the scheduled packets can be assured to be completely transmitted within one frame while the proportional fairness can be maintained among users as far as possible. We also evaluate the performance of the proposed scheme through extensive simulation.

The outline of this paper is as follows. In Sect. 2, we describe the cross-layer packet scheduling model for OFDM systems and then, formulate the MCPF scheduling problem in Sect. 3. In Sect. 4, we analyze the problem and develop a heuristic algorithm. Simulation results are given in Sect. 5, followed by the conclusion in Sect. 6.

2. System Model

Figure 1 illustrates the cross-layer scheduling structure for a packet-switched OFDM system. We focus on downlink transmission of non-real-time data traffic. Therefore, it is the base station (BS) that makes scheduling decisions for packet transmission. Suppose that there are N_c subcarriers in the current system. Without loss of generality, we assume that each user in the system has only one active session.

At MAC layer, upon each packet arrival, the BS puts the packet into its corresponding buffer which is assumed to have infinite space. Unlike other scheduling schemes such as [10], in which the packet length is assumed to be the same for all the packets, in our scheduling model, different packets are supposed to have different length of bits. This is a more realistic assumption since packets with different length have different requirements for system resources and will have an impact on scheduling schemes (The impact will be explained more clearly in the next section). Furthermore, we assume that ARQ (Automatic Repeat Request) is applied to recover the packet errors, and a packet is retransmitted until it is received successfully.

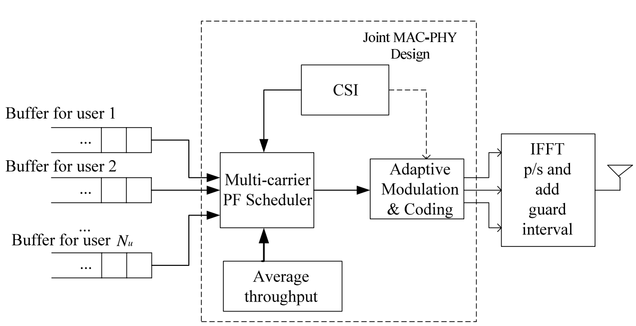

Fig. 1 Cross-layer scheduling model for OFDM systems.

At PHY layer, the channel is assumed to be reciprocal so that the BS is able to estimate the instantaneous channel characteristics for the downlinks based on the uplink transmission as long as channel varies relatively slowly. In the following, we assume perfect channel state information (CSI). With this CSI, the BS can implement adaptive modulation and coding (AMC) to maximize the throughput on each subcarrier. Furthermore, the throughput information is passed to the multi-carrier proportional fair (MCPF) scheduler to help make scheduling decision.

The MCPF scheduler, which is the core of our cross-layer scheduling design, makes scheduling decisions based on a frame-by-frame basis. At every scheduling frame, the MCPF scheduler selects packets for transmission based on the following three input parameters: packet characteristics such as packet length, achievable data rate on each subcarrier for each user, and the users' average throughput till current scheduling time. The objective of the MCPF scheduler is to provide proportional fairness among the users, which has been proven to be able to strike a good tradeoff between system throughput and fairness.

It will be seen that the MCPF scheduler not only schedules the packets for transmission but also realize subcarriers allocation for each packet, which will be further described in the following section.

3. Formulation of the MCPF Scheduling Problem

In this section, we will first review some of the concepts about proportional fairness (PF) and then explain the impact of packet length on scheduling schemes in OFDM systems. After that, we give a description of the MCPF scheduler. Finally, we formulate the MCPF scheduling problem as an optimization problem.

3.1 Review of Proportional Fairness

The concept of proportional fairness (PF) is first introduced in [6] by Kelly in the context of rate control for communication networks and has been proven to be able to achieve a sound tradeoff between system throughput and user fairness. The definition of PF for a vector is as follows.

Definition 1: For a set of vectors $C = \{x | x \in \mathbf{R}^n\}$, a vector $x^* = (x_1^*, ..., x_n^*)$ is called proportionally fair if and only if for any other feasible vector $x \in C$, the following inequality holds:

$$\sum_{p=1}^{n} \frac{x_p - x_p^*}{x_p^*} \leq 0.$$

It is further proven in [6] that for a vector x, being proportionally fair is equivalent to maximizing the following logarithmic utility function

$$x^* = \arg\max_{x \in C} \sum_{p=1}^{n} \log x_p.$$

In a wireless packet-switched network, a packet scheduling policy P selects some packet(s) for transmission, leading to a change to some system metrics of interest such as average data rate. Therefore, a scheduling policy is called proportionally fair if it makes the metrics of interest proportionally fair.

For example, in IS-856 systems, the following scheduling policy has been proven to be proportionally fair in terms of the average user data rate T_i (i.e., the average data rate is proportionally fair under the scheduling policy)

$$i^* = \arg\max_{i} \frac{R_i}{T_i} \tag{1}$$

where i is the user index, i^* denotes the selected user by the scheduling policy P, R_i is the current achievable data rate for user i and T_i is the user's average data rate.

While the scheduler P described by (1) is proportionally fair for single-carrier systems, it is not the case for OFDM networks. In an OFDM system, multiple subcarriers can transmit more than one user's packets within the same time frame, i.e., each time there may be at least two users scheduled. In this case, a scheduler which aims at achieving proportional fairness is much more complicated. In the following subsection, we will formulate the multi-carrier proportional fair (MCPF) scheduling problem and design a heuristic scheduling scheme for OFDM systems.

3.2 Impact of Packet Length on Scheduling Schemes in OFDM Systems

In a cross-layer scheduling design for OFDM systems, scheduling a user implies assigning certain number of subcarriers to him for transmitting the packet. In turn, subcarrier allocation to a user implies that the user is scheduled. For this reason, in OFDM systems, packet scheduling and subcarrier allocation are closely related and always integrated as a whole.

In this paper, we focus on time-slotted OFDM systems. In a time-slotted OFDM, the time axis is divided into frames. A frame is the basic transmission unit for packets. Every scheduled packet should be carried within only one frame. It can not be transmitted through multiple frames. On the other hand, every frame can accommodate more than one packets for transmission.

For time-slotted OFDM, packet length can impose a constraint on the scheduler. Mathematically, the constraint can be expressed by the following *packet length constraint*:

$$L_i \leq \sum_{j \in S_i} r_{ij}, \quad \text{for each scheduled user } i \tag{2}$$

where i and j are the index for users and subcarriers, respectively. L_i is the length of the HOL packet of user i and r_{ij} is the data rate of user i on subcarrier j. S_i denotes the subcarrier set allocated to user i in the current scheduling frame. The physical meaning of (2) lies in that the allocated data rate ($\sum_{j \in S_i} r_{ij}$) should be large enough to afford the HOL packet (which has L_i bits of length). In other words, this means: For each scheduled packet within the frame, its "allocated capacity" (in terms of the summation of the data rate of its allocated subcarriers) should be equal to or larger than its length. Otherwise, the packet will not be able to be transmitted successfully.

In short, packets with different length have different requirements for system resources. It should be assured that each scheduled packet can be *completely transmitted* within a frame. Otherwise, the packets will not be transmitted successfully in this scheduling frame and will be dropped or retransmitted until next scheduling frame, which will increase the packet delay and also decrease the spectrum efficiency. A packet scheduler in time-slotted OFDM systems should be aware of this factor.

3.3 Description of MCPF Scheduler

For the MCPF scheduler, the time axis is divided into frames which consist of a fixed number n_s of OFDM symbols. The scheduler makes scheduling decisions on a frame-by-frame basis. Suppose that there are $N_u(t)$ backlogged users at time t and $L_i(t)$ is the HOL packet length for user i ($i = 1, 2, ..., N_u(t)$). Since scheduling is made independently between frames, in the following we will omit the time reference t when the context is clear. At the every beginning of a scheduling frame, through the CSI and AMC technology, the BS can calculate the achievable data rate of the users for each subcarrier, denoted by r_{ij}, where $i = 1, 2, ..., N_u$ and $j = 1, 2, ..., N_c$ are user and subcarrier index, respectively. Then the rate information $\{r_{ij}\}$ is passed to the MCPF scheduler. Based on the rate information, together with the queue status (e.g., HOL packet length) and the average throughput, the MCPF scheduler will select some packets for transmission.

In time-slotted OFDM systems, a scheduling frame consists of several time-slots. For every time-slot, the profile of subcarrier distribution (i.e., subcarrier allocation) among the users can be either different or just the same. To reduce the signalling overhead, we now assume that the subcarrier distribution at different time-slots (i.e., OFDM symbols) of a frame is the same. The assumption is reasonable. The reason is that dynamic subcarrier allocation with a time-slot is enough to exploit the inherent multiuser diversity of the system. Additional system performance improvement due to extra dynamic time-slot allocation for users is negligible [11].

Under this assumption, for each frame, the scheduling problem can be treated within a time-slot. In the following formulation of the problem, we consider that the number of time-slots n_s in a frame is 1 for simplicity.

3.4 Formulation of PF Scheduling in Multi-Carrier Systems

This subsection formulates the MCPF scheduling problem as an optimization problem. The objective of the MCPF scheduler is to provide proportional fairness among users in terms of user average throughput T_i ($i = 1, ..., N_u$), while satisfying the packet length constraint. The details go as follows.

First, to make the problem more tractable, define c_{ij} to be the binary subcarrier allocation indicator. That is, $c_{ij} = 1$ indicates the jth subcarrier is allocated to user i, while $c_{ij} = 0$, otherwise.

Next, according to the definition of PF, to provide proportional fairness in terms of user average throughput, we should maximize the logarithmic utility function

$$\sum_{i \in U_s} \log T_i \tag{3}$$

where U_s is the set of selected users and T_i is the ith user's average throughput at the end of the scheduling frame.

By definition, the average throughput of user i at the beginning of the $(t+1)$th scheduling time frame $T_i(t+1)$ is given by

$$T_i(t+1) = \begin{cases} \left(1 - \frac{1}{t_c}\right) T_i(t) + \frac{1}{t_c} \sum_{j=1}^{N_c} r_{ij} c_{ij} \\ \quad \text{queue } i \text{ is served} \\ \left(1 - \frac{1}{t_c}\right) T_i(t) \quad \text{otherwise} \end{cases} \tag{4}$$

where r_{ij} is the data rate of the jth subcarrier for user i and t_c is the observation window size.

To maximize (3) is not an easy task, fortunately, in [8], Kim has proven that maximizing (3) is equivalent to maximizing the following objective function

$$\prod_{i \in U_s} \left(1 + \frac{\sum_{j \in C_i} r_{ij}}{(t_c - 1) T_i}\right) \tag{5}$$

where C_i is the set of subcarriers allocated to user i.

Finally, for each scheduled packet, it should satisfy the packet length constraint. That is to say, the length of the packet (in bits) should be less than or equal to its assigned data rate (in bits/frame) within the frame.

Based on the analysis above, the MCPF scheduling problem can now be mathematically formulated as

$$\max_{c_{ij}} \prod_{i=1}^{N_u} \left(1 + \frac{\sum_{j=1}^{N_c} r_{ij} c_{ij}}{(t_c - 1) T_i}\right) \tag{6}$$

$$s.t. \sum_{i=1}^{N_u} c_{ij} = 1, \quad \forall j = 1, 2, \ldots, N_c \tag{7}$$

$$L_i \left(\sum_{j=1}^{N_c} c_{ij}\right) \leq \left(\sum_{j=1}^{N_c} r_{ij} c_{ij}\right)\left(\sum_{j=1}^{N_c} c_{ij}\right) \quad \forall i = 1, 2, \ldots, N_u \tag{8}$$

$$c_{ij} \in \{0, 1\} \quad \forall i, j. \tag{9}$$

It can be seen that the objective function in (6) is equivalent to that in (5). Condition (7) means that every subcarrier should be exclusively assigned to only one user. Condition (8) reveals the packet length constraint. That is, if user i is scheduled, i.e., $\sum_{j=1}^{N_c} c_{ij} > 0$, then we can eliminate the $\sum_{j=1}^{N_c} c_{ij}$ from both sides of inequality (8) to get: $L_i \leq \sum_{j=1}^{N_c} r_{ij} c_{ij}$, which indicates the packet length constraint, i.e., the length of a scheduled packet should be less than or equal to its assigned data rate (in bits/frame) within the frame. On the other hand, if user i is not scheduled, i.e., $\sum_{j=1}^{N_c} c_{ij} = 0$, then (8) always holds. That is, no constraint is imposed on unscheduled users.

4. Our Proposed Heuristic Suboptimal MCPF Scheduling Algorithm

4.1 Analysis of the MCPF Optimization Problem

We use a technique called convex relaxation to analyze the MCPF scheduling problem (6)–(9). The technique is to relax the constraints $c_{ij} \in \{0, 1\}$ to $0 \leq c_{ij} \leq 1$. Such approach is first adopted in [12] and has been frequently used in the context of subcarrier allocation for OFDM systems.

In this way, the original problem in (6)–(9) can be further transformed into:

$$\min_{c_{ij}} - \sum_{i=1}^{N_u} \log\left(1 + \frac{\sum_{j=1}^{N_c} r_{ij} c_{ij}}{(t_c - 1) T_i}\right) \tag{10}$$

$$s.t. \sum_{i=1}^{N_u} c_{ij} = 1, \quad \forall j = 1, 2, \ldots, N_c \tag{11}$$

$$L_i \left(\sum_{j=1}^{N_c} c_{ij}\right) \leq \left(\sum_{j=1}^{N_c} r_{ij} c_{ij}\right)\left(\sum_{j=1}^{N_c} c_{ij}\right) \quad \forall i = 1, 2, \ldots, N_u \tag{12}$$

$$0 \leq c_{ij} \leq 1 \quad \forall i, j. \tag{13}$$

Using standard optimization technique in [14], we obtain the Lagrangian

$$\begin{aligned} L = &- \sum_{i=1}^{N_u} \log\left[1 + \frac{\sum_{j=1}^{N_c} r_{ij} c_{ij}}{(t_c - 1) T_i}\right] \\ &+ \sum_{j=1}^{N_u} \lambda_i \left[L_i \left(\sum_{j=1}^{N_c} c_{ij}\right) - \left(\sum_{j=1}^{N_c} r_{ij} c_{ij}\right)\left(\sum_{j=1}^{N_c} c_{ij}\right)\right] \\ &+ \sum_{j=1}^{N_c} \beta_j \left(\sum_{i=1}^{N_u} c_{ij} - 1\right) \end{aligned} \tag{14}$$

where λ_i ($i = 1, 2, ..., N_u$) and β_j ($j = 1, 2, ..., N_c$) are the Lagrangian multipliers for the constraints (12) and (11), respectively.

Let c_{ij}^* denote the optimal solution for (10)–(13). Applying the KKT conditions, we obtain the necessary conditions for c_{ij}^*. Specially, differentiating L with respect to c_{ij}, we have

$$\frac{\partial L}{\partial c_{ij}} = \frac{-r_{ij}}{(t_c-1)T_i + \sum_{j=1}^{N_c} r_{ij}c_{ij}} + \lambda_i\left(L_i - \sum_{j=1}^{N_c} r_{ij}c_{ij} - r_{ij}\sum_{j=1}^{N_c} c_{ij}\right) + \beta_j \tag{15}$$

and c_{ij}^* should satisfy the following

$$\left.\frac{\partial L}{\partial c_{ij}}\right|_{c_{ij}=c_{ij}^*} \begin{cases} > 0, & \text{if } c_{ij}^* = 0, \\ = 0, & \text{if } 0 < c_{ij}^* < 1, \\ < 0, & \text{if } c_{ij}^* = 1. \end{cases} \tag{16}$$

From (15) and (16), it follows that

$$c_{ij}^* = \begin{cases} 0, \text{if } \beta_j > H_{ij}(\lambda), \\ 1, \text{if } \beta_j < H_{ij}(\lambda) \end{cases} \tag{17}$$

where

$$H_{ij}(\lambda) = \frac{r_{ij}}{(t_c-1)T_i + \sum_{j=1}^{N_c} r_{ij}c_{ij}^*} - \lambda_i\left(L_i - \sum_{j=1}^{N_c} r_{ij}c_{ij}^* - r_{ij}\sum_{j=1}^{N_c} c_{ij}^*\right). \tag{18}$$

If $H_{ij}(\lambda)$ is independent of c_{ij}^*, then the optimal solution c_{ij}^* is obtained through (17), (18)†. However, from (18), we see that $H_{ij}(\lambda)$ contains c_{ij}^* in its expression. This means it is not easy to obtain c_{ij}^* by (17). We need to look for another way.

4.2 Our Heuristic Suboptimal MCPF Scheduling Algorithm

The reason that makes c_{ij}^* difficult to get is that $H_{ij}(\lambda)$ contains the term of c_{ij}^*, which in fact is caused by packet length constraint (12). If we can drop the constraint (12) temporarily and compensate for it afterwards, the complexity of searching c_{ij} will be greatly reduced, while the cost is that the solution we got may be suboptimal.

Our heuristic MCPF scheduling algorithm consists of two stages: initial subcarrier allocation and compensation stage. In the first stage of initial subcarrier allocation, we drop the packet length constraint (12) temporarily to get the subcarrier allocation scheme which can assure proportional fairness. Then, at the second stage, which we call compensation stage, we will inspect the packet length constraint again and by releasing "surplus" subcarriers from those users who have already met the constraint to those who have not, we assure that the constraint can be satisfied by all scheduled packets. The details go as follows.

4.2.1 Initial Subcarrier Allocation

By dropping the packet length constraint (12), we obtain

$$\min_{c_{ij}} -\sum_{i=1}^{N_u} \log\left(1 + \frac{\sum_{j=1}^{N_c} r_{ij}c_{ij}}{(t_c-1)T_i}\right) \tag{19}$$

$$s.t. \sum_{i=1}^{N_u} c_{ij} = 1, \quad \forall j = 1, 2, \ldots, N_c \tag{20}$$

$$0 \le c_{ij} \le 1 \quad \forall i, j. \tag{21}$$

Using standard Lagrangian method and following the same procedures as above, we obtain

$$\frac{\partial L}{\partial c_{ij}} = \frac{-r_{ij}}{(t_c-1)T_i + \sum_{j=1}^{N_c} r_{ij}c_{ij}} + \beta_j \tag{22}$$

and

$$c_{ij}^* = \begin{cases} 0, \text{if } \beta_j > H'_{ij}, \\ 1, \text{if } \beta_j < H'_{ij} \end{cases} \tag{23}$$

where

$$H'_{ij} = \frac{r_{ij}}{(t_c-1)T_i + \sum_{j=1}^{N_c} r_{ij}c_{ij}^*}. \tag{24}$$

We find that the H'_{ij} in (24) still contains the variables $\{c_{ij}^*\}$. However, from (24), we make a key observation. $\sum_{j=1}^{N_c} r_{ij}c_{ij}^*$ is the assigned data rate for user i within a frame and it should be larger than or equal to the packet length L_i. The larger the L_i is, the larger the assigned data rate $\sum_{j=1}^{N_c} r_{ij}c_{ij}^*$ should be. Therefore, we make a bold replacement. We substitute L_i for $\sum_{j=1}^{N_c} r_{ij}c_{ij}^*$ to obtain

$$H''_{ij} = \frac{r_{ij}}{(t_c-1)T_i + L_i} \tag{25}$$

and

$$c_{ij}^* = \begin{cases} 0, \text{if } \beta_j > H''_{ij}, \\ 1, \text{if } \beta_j < H''_{ij}. \end{cases} \tag{26}$$

Since the constraint (20) must be satisfied, i.e., each subcarrier should be exclusively assigned to one user, from (26), we conclude that for each subcarrier j, if H''_{ij}s for $i = 1, 2, ..., N_u$ are all different, then only the user with the largest H''_{ij} can use the subcarrier. In other words, for each subcarrier j, if H''_{ij}s are all different for all i, then

$$c_{i'j}^* = 1, \quad c_{ij}^* = 0, \qquad \text{for all } i \neq i' \tag{27}$$

where

$$i' = \arg\max_i H''_{ij} = \arg\max_i \frac{r_{ij}}{(t_c-1)T_i + L_i}. \tag{28}$$

Equations (27) and (28) constitute the algorithm for the initial subcarrier allocation. That is, for each subcarrier j, find the user with the largest H''_{ij}, then allocate the jth subcarrier

†When $H_{ij}(\lambda)$ is independent of c_{ij}^*, we just need one more step to get the optimal solution c_{ij}^* for subcarrier allocation. Readers can be referred to [12] and [13] for such kind of technique.

to him. In this way, we finish the first stage of our heuristic scheduling algorithm.

Remark: If all the packets have the same length, i.e., $L_i = L$ where L is constant, then (28) can be reduced to

$$i' = \arg\max_i \frac{r_{ij}}{(t_c-1)T_i+L} = \arg\max_i \frac{r_{ij}}{(t_c-1)T_i}. \qquad (29)$$

Equation (29) is in fact the scheduling scheme proposed in [7]. Furthermore, if the system is a single-channel sytem, i.e., $j = 1$, then (29) can be further reduced to

$$i' = \arg\max_i \frac{r_i}{(t_c-1)T_i} = \arg\max_i \frac{r_i}{T_i}. \qquad (30)$$

Note that (30) is the PF scheduler adopted in HDR system. Therefore, (29) and (30) indicate that both the HDR scheduler and the scheduling scheme in [7] are special cases of the *Initial subcarrier allocation* of our scheme.

4.2.2 Compensation Stage

The first stage has indeed provided an initial scheduling result for users. That is, the users who have been assigned at least one subcarrier will be scheduled, while others without any allocated subcarrier are still left in their queues. However, since initial subcarrier allocation does not account for the packet length constraint (12), it is very likely that some scheduled users can meet the constraint (12) while others not. In the following, for convenience, we call the scheduled users who have met the packet length constraint (12) the length-satisfied users, while calling other scheduled users having not met the constraint the length-unsatisfied users. In order to ensure all the scheduled packets to be transmitted completely within a scheduled frame, all the scheduled users must be length-satisfied. To do this, we need to adjust the subcarrier distribution among length-satisfied and length-unsatisfied users after the first stage, which is what the second stage deals with.

The second stage is called compensation stage. The objective of the stage is to guarantee that every scheduled user satisfies the packet length constraint (12). In other words, this means that every scheduled user is length-satisfied. The basic idea for compensation stage is that we remove the *surplus* subcarriers from the length-satisfied users and reallocate them among the length-unsatisfied users. Here, a surplus subcarrier of a length-satisfied user refers to the subcarrier without which the user still remains length-satisfied. In this way, we can compensate for length-unsatisfied users. If a user is compensated successfully, he will become length-satisfied and therefore, his packet can be transmitted completely. Otherwise, we should release all of his resources (i.e., subcarriers) to those who are still waiting for compensation.

Let R_i denote the data rate assigned to the ith user within a frame, that is, $R_i = \sum_{j=1}^{N_c} r_{ij}c_{ij}$. Let U_1 and U_2 denote the set of length-satisfied and length-unsatisfied users, respectively. Then, the second stage, i.e., compensation stage can be described in details as follows.

1. while $U_2 \neq \emptyset$,
   a. find the user i_2 satisfying $i_2 = \arg\min_{i \in U_2} (L_i - R_i)$ and set subcarrier index $j = 1$;
   b. while $i_2 \in U_2$ and $j \leq N_c$
      i. if the jth subcarrier is a surplus subcarrier of some user, then release it, and allocate to the user i_2;
      ii. get the next subcarrier $j = j + 1$;
   c. if $i_2 \in U_2$, i.e., compensation for i_2 failed, release all of his subcarriers and reallocate them among other users in U_2 according to (27) and (28);
   d. update U_1, U_2 and $\{R_i\}$.

The principle of the compensation stage is that the user with the least stringent packet-length constraint, i.e., minimum difference between packet length L_i and data rate R_i, is to be compensated first. If compensation for this user succeeds, the user will become length-satisfied, otherwise, we should release all of his resources and delete him from the set U_2. Combined the first and the second stage, it can be seen that the complexity of our scheme is $O(N_cN_u)$.

5. Performance Evaluation

In this section, we present simulation results to show the performance of the proposed heuristic MCPF scheduling scheme. An OFDM system with AMC technique adopted at PHY layer is considered. We assume there are 6 different constellations available. They are, BPSK, 4QAM, 8QAM, 16QAM, 64QAM and 256QAM. The target BER is set to be 10^{-3}. The transmitted SNR on each subcarrier, i.e., the ratio of the transmitted power to the noise power, $P_t/(N_0B)$ is normalized to 1, where P_t, N_0 and B are the transmitted power on each subcarrier, power spectral density of noise and the signal bandwidth of each subcarrier, respectively.

At MAC layer, packet arrival is modelled as a Poisson process with arrival rate 0.5 packets per OFDM symbol (i.e., time-slot) while 4 OFDM symbols constitute a transmission frame.

Throughout the simulations, the wireless channel is modelled as a frequency-selective fading channel consisting of 3 independent Rayleigh multipath components. We assume that the delay spread for each multipath component is uniformly distributed between 0 and T_s with T_s the OFDM symbol duration.

To our best knowledge, existing work about PF scheduling in multi-carrier systems (e.g. [8], [9]) does not take packet length into account. Therefore, it is difficult to compare our scheme with them. Instead, we will compare the proposed MCPF scheduling scheme with the optimal

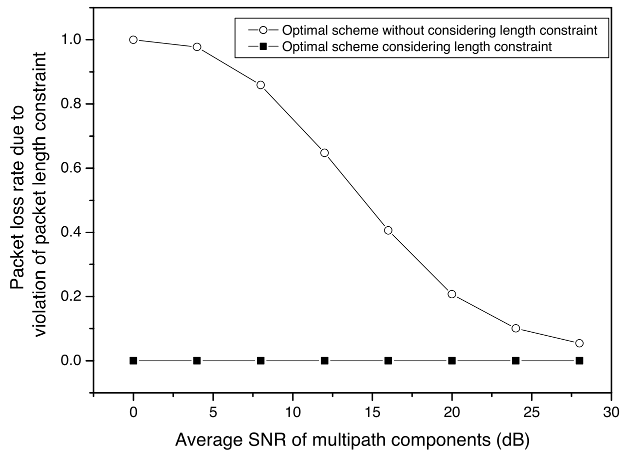

Fig. 2 Packet loss rate due to violation of length constraint versus average SNR.

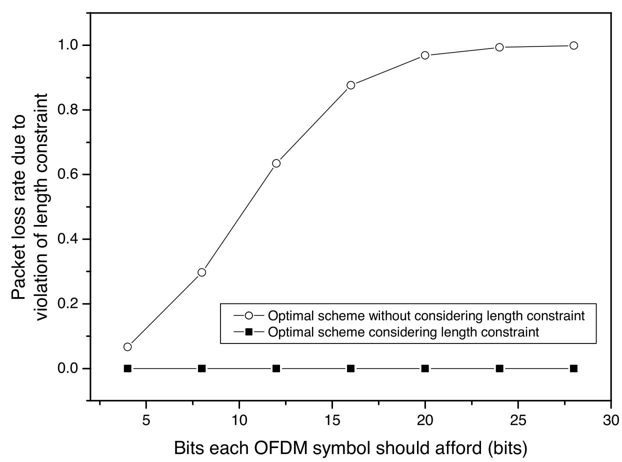

Fig. 3 Packet loss rate due to violation of length constraint as a function of the number of bits each OFDM symbol should afford.

solution. The optimal scheme, i.e., the optimal subcarrier allocation which meets the packet length constraint (8) is derived by searching all the possible solutions. Due to the extremely long computational time required, we only consider a small number of users and subcarriers for the OFDM system.

5.1 Verification of the Impact of Packet Length

In Sect. 3, we have explained the impact of packet length on scheduling schemes in OFDM systems. If the length of a scheduled packet exceeds the rate (in bits/frame) it is assigned within the scheduling frame, the packet will not be transmitted completely and will be dropped. Figure 2 and Fig. 3 demonstrate the impact of packet length by plotting the packet loss rate due to violation of packet length constraint (8) for a system with 3 users and 8 subcarriers[†]. Two schemes are considered here. One is the optimal scheme to the MCPF optimization problem (6)–(9). The scheme accounts for the length constraint. The other is almost the same, except that it does not consider the length constraint (8). Both schemes are obtained through exhaustive search.

From Fig. 2, it can be seen that if the constraint (8) is taken into account, then each scheduled packet can be ensured to be transmitted completely. Then the packet loss due to violation of length constraint can be avoided and the packet loss rate is kept at 0. Otherwise if not, then the packet loss rate is high, especially at low SNR region. This is not surprising since although the optimal scheme can achieve optimal subcarrier allocation, it can not ensure the rate of the subcarriers larger than the length of their scheduled packet. The poorer the channel is, the more likely the packet length constraint is violated. Therefore, at low SNR region, the packet loss rate is high. As the average SNR increases, the packet loss rate decreases accordingly.

Figure 3 shows the packet loss rate as a function of the number of bits each OFDM symbol should afford, i.e., packet length (recall that a packet is carried through several OFDM symbols). As expected, if the optimal scheme does not consider the length constraint, then when a packet becomes longer, it is more possible that the assigned subcarriers can not afford the packet. Therefore, the packet will be dropped, which leads to high packet loss rate. The longer a packet is, the higher the packet loss rate is. On the other hand, a scheme with considering length constraint can keep packet loss rate at zero.

5.2 Comparison with the Optimal Solution

This subsection compares the performance between the proposed heuristic MCPF scheduling scheme and the optimal solution. In this subsection, we assume that all the packets have the same length of 80 bits. Therefore, every OFDM symbol within a frame needs to afford 20 bits.

Since either the proposed scheme or the optimal solution aims at maximizing the objective function of (5), a natural performance comparison is to compare the objective function that each scheme can achieve. We call the objective function in (5) Multi-carrier Proportional Fairness Measure (MCPFM). Figure 4 plots the MCPFM versus the average SNR of multipath components for the case of 3 users and 8 subcarriers[††]. From Fig. 4, we can see that MCPFM increases with average SNR. This is not surprising since when SNR becomes higher, the achievable data rate for each user increases accordingly, which contributes to the increase of MCPFM. Furthermore, Fig. 4 shows that the proposed heuristic algorithm has almost the same performance as the optimal scheme, especially at low and medium SNR region. Even when the SNR is high, e.g., 20 dB in this figure, the proposed scheme can still achieve more than 93% of the optimal performance.

While Fig. 4 demonstrates the performance of the proposed scheme in terms of MCPFM, the variation of user's

[†]In the following, for convenience, we simply use packet loss rate to denote the packet loss rate due to violation of length constraint

[††]The average SNR denotes the expectation of the SNR, which is a Rayleigh random variable.

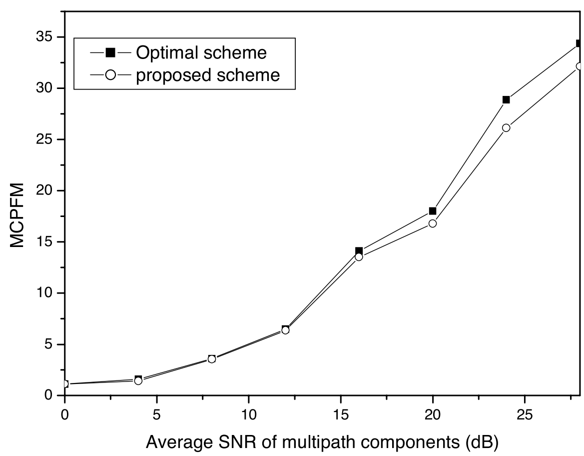

Fig. 4 MCPF measure versus average SNR.

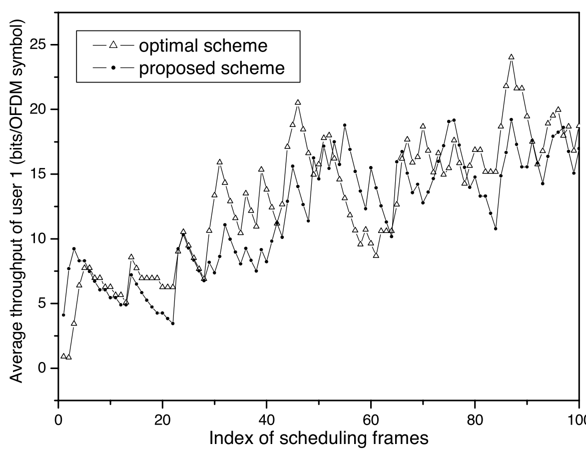

Fig. 5 User 1's average throughput with simulation time for the case of $\overline{SNR}_1 = \overline{SNR}_2 = 10\,\text{dB}$.

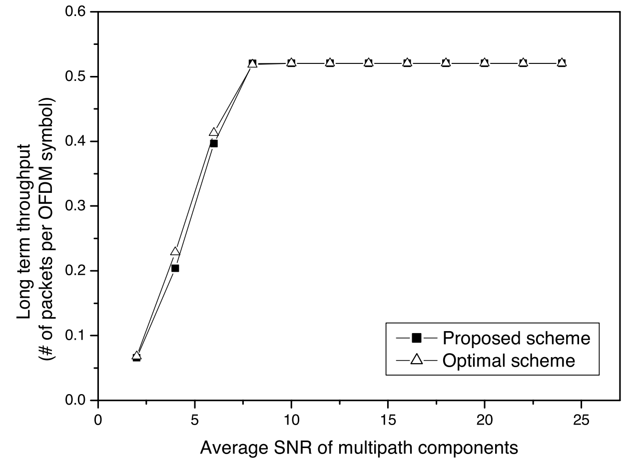

Fig. 6 User 1's long-term throughput versus average SNR.

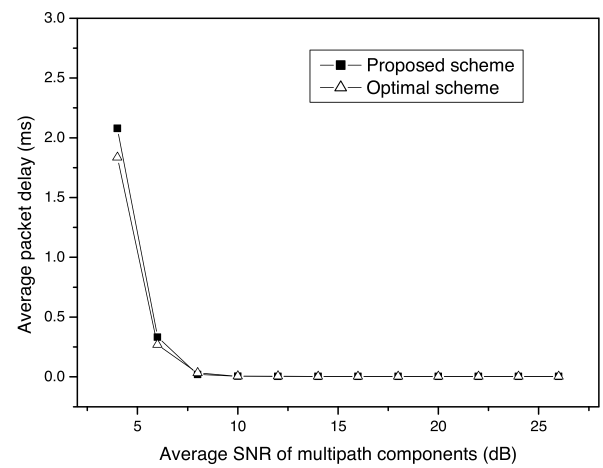

Fig. 7 Average packet delay versus average SNR.

average throughput with time, which is given by (4) provides another way for easier understanding of the performance of the proposed scheduling scheme, which is depicted in Fig. 5 for a system with 2 users and 16 subcarriers. In Fig. 5, we consider the case that both users have the same average SNR: $\overline{SNR}_1 = \overline{SNR}_2 = 10\,\text{dB}$ and plot the average throughput T_1 of user 1 versus scheduling frames (i.e., simulation time) for both the proposed and optimal scheduling schemes. From Fig. 5, we can see that as expected, the average throughput T_1 fluctuates with time. Furthermore, a closer observation indicates that the average throughput undergoes valleys and peaks repeatedly, which can be explained by the "proportional fairness" principle. For example, when the average throughput is at some local peak value, then in the next few scheduling frames, some subcarriers may not be allocated to the user, according to the initial subcarrier allocation stage, which uses $H''_{ij} = \frac{r_{ij}}{(t_c-1)T_i+L_i}$ as the allocation metric. Therefore, without being scheduled, the average throughput decreases until it gets at some valley value, from which the throughput begins to increase because of being scheduled again.

It can also be seen from Fig. 5 that the average throughput for the proposed and optimal schemes has similar variation profile. In other words, the proposed scheme can trace the variation of the average data rate under optimal scheduling scheme. Therefore, it can be said that the proposed scheme has approximate performance with the optimal one.

While Fig. 5 describe the average throughput with simulation time, Fig. 6 investigates the long-term throughput of user 1 as a function of average SNR. The long-term throughput is derived through average the total successfully transmitted packets over the simulation time. From this figure, it can be seen that at low SNR region, our scheme is worse than the optimal scheme. However, the difference gap is very small. As SNR increases, the throughput increases accordingly until it reaches a constant which is not affected by SNR. The reason is that at high SNR region, the system service rate (i.e., channel capacity) is higher than the packet arrival rate. In this case, the throughput is mainly determined by the amount of input traffic. Therefore, in this case our scheme has the same performance as the optimal scheme.

Figure 7 plots the average packet delay of user 1 versus the average SNR. It can be seen that at low SNR region, packet delay is large. The reason is that when the channel condition is poor, the assigned subcarriers for user 1 do not have enough capacity to carry his scheduled paket. That is, length constraint (8) can not be met in this case. Thus, the user will not be scheduled and have to wait until channel becomes better, which leads to large delay for the packet. Figure 7 also shows that packet delay decreases dramatically with the increase of SNR, as expected. At high SNR, when the system service rate is high enough, each incoming packet can be scheduled quickly and thus, the packet delay is low and almost constant. Finally, as in Fig. 6, Fig. 7 also shows that our proposed scheme has almost the same delay characteristic as the optimal scheme, which strongly demonstrates the performance of our proposed scheme.

6. Conclusions

For OFDM systems, scheduling problems are always integrated with subcarrier allocation. Packet length has an impact on the performance of scheduling schemes. In this paper, we investigate the proportional fair scheduling problem for OFDM systems, taking the impact of packet length into account. The PFS problem is formulated as an optimization problem and a heuristic MCPF scheduling algorithm is further proposed. First, by dropping the packet length constraint temporarily the subcarriers are allocated among users to provide proportional fairness. Then at the second stage, subcarrier readjustment is done in a way that surplus subcarriers from length-satisfied users are released and allocated among length-unsatisfied users. In this way, each scheduled packet can be guaranteed to be transmitted completely. Simulation results show that the proposed scheme has almost the same performance as that of the optimal scheme in terms of MCPF measure, throughput and packet delay. In addition, both schemes have similar variation profile of average throughput.

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References

[1] X. Liu, E.K.P. Chong, and N.B. Shroff, "A framework for opportunistic scheduling in wireless networks," Comput. Netw. J. (Elsevier), vol.41, no.4, pp.451–474, 2003.

[2] Z. Zhang, Y. He, and E.K.P. Chong, "Opportunistic downlink scheduling for multiuser OFDM systems," IEEE WCNC'05, vol.2, pp.1202–1212, March 2005.

[3] D. Yang, D. Shen, W. Shao, and V.O.K. Li, "Towards opportunistic fair scheduling in wireless networks," IEEE ICC'06, vol.11, pp.5217–5221, June 2006.

[4] Y. Liu and E. Knightly, "Opportunistic fair scheduling over multiple wireless channels," IEEE INFOCOM'03, pp.1106–1115, April 2003.

[5] Z. Shen, J.G. Andrews, and B.L. Evans, "Adaptive resource allocation in multiuser OFDM systems with proportional fairness," IEEE Trans. Wireless Commun., vol.4, no.6, pp.2726–2737, Nov. 2005.

[6] F.P. Kelly, A.K. Maulloo, and D.K.H. Tan, "Rate control for communication networks: Shadow prices, proportional fairness and stability," J. Operational Research Society, vol.49, pp.237–252, April 1998.

[7] W. Anchun, X. Liang, Z. Shidong, X. Xibin, and Y. Yan, "Dynamic resource management in the fourth generation wireless system," IEEE ICCT03, vol.2, pp.1095–1098, April 2003.

[8] H. Kim and Y. Han, "A proportional fair scheduling for multicarrier transmission systems," IEEE Commun. Lett., vol.9, no.3, pp.210–212, March 2005.

[9] M. Kaneko and P. Popovski, "Heuristic subcarrier allocation algorithms with multi-slot frames in multi-user OFDM systems," IEEE ICC06, vol.10, pp.4391–4396, June 2006.

[10] Y. Zhang and S.C. Liew, "Link-adaptive largest-weighted-throughput packet scheduling for real-time traffics in wireless OFDM networks," Proc. IEEE GLOBECOM'05, vol.5, pp.2490–2494, Nov. 2005.

[11] A. Todini, M. Moretti, A. Valletta, and A. Baiocchi, "A modular cross-layer scheduling and resource allocation architecture for OFDMA systems," IEEE GLOBECOM 2006, 2006.

[12] C.Y. Wong, R.S. Cheng, K.B. Letaief, and R.D. Murch, "Multicarrier OFDM with adaptive subcarrier, bit, and power allocation," IEEE J. Sel. Areas Commun., vol.17, no.10, pp.1747–1758, Oct. 1999.

[13] J. Jang and K.B. Lee, "Transmit power adaptation for multiuser OFDM systems," IEEE J. Sel. Areas Commun., vol.21, no.2, pp.171–178, Feb. 2003.

[14] S. Boyd and L. Vandenberghe, Convex Optimization, Cambridge University Press, 2004.

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