Optimization of Transmission Strategies for Opportunistic Access in Cognitive Radio Networks

Abstract—**Cognitive radio is a promising technology to mitigate spectrum shortage in wireless communications. It enables secondary users (SUs) to opportunistically access low-occupancy primary spectral bands as long as the primary user (PU) access is protected. Such a protection requirement is particularly challenging for multiple SUs over a potentially wide geographical area. In this paper, we study the fundamental limit on the throughput performance of cognitive networks under the constraint of packet collision probability with the PU. With perfect sensing, we develop an optimum spectrum access strategy suitable under generic PU traffic. Without perfect sensing, we quantify the impact of missed detection and false alarm, and propose a modified threshold-based spectrum access strategy that achieves close-to-optimal performance. Moreover, we develop and evaluate a distributed access scheme that enables multiple SUs to collectively protect the PU while adapting to changes in PU activity patterns. Our results in this paper provide useful insight on the trade-off between the protection of the primary user and the throughput performance of cognitive radios.**

I. Introduction

Cognitive radio technology can potentially mitigate spectrum shortage in wireless communications by letting SUs to opportunistically utilize spectral white spaces of PUs. Because legacy users have access priority, secondary cognitive radio networks are required to exert minimal effect on PUs if and when they become active. For example, in the DARPA XG project [1], one of the three major test criteria in the field test is "to cause no harm" [2]. The protection of PUs is vital to the success of cognitive radio system because no PU would be inclined to accommodate secondary cognitive networks without such assurance. Thus, the spectrum access strategy of the SU should aim to maximize the performance of SUs while operating under the protection guarantee of PUs.

In some deployment, PUs may have much larger transmission power compared to SUs, i.e., TV stations vs. unlicensed devices, as shown in Figure 1. In such cases, the protection of PUs is more challenging when there are multiple SUs accessing a PU channel. Because these SUs are widely spread in the area, one SU may not be able to sense all other SUs' transmission. On one hand, this enables spatial reuse among secondary users, which increases SUs' capacity. On the other hand, the collective protection of PU's becomes more challenging because individual SU's intrusion/interference can accumulate to an unacceptable degree to the PU. To address this issue, one may consider a centralized SU controller to coordinate all SUs. However, a centralized controller may not always be available or practical. Additionally, the communication and coordination among all SUs will incur much overhead and delay, even if such inter-cooperation is possible. Thus, one

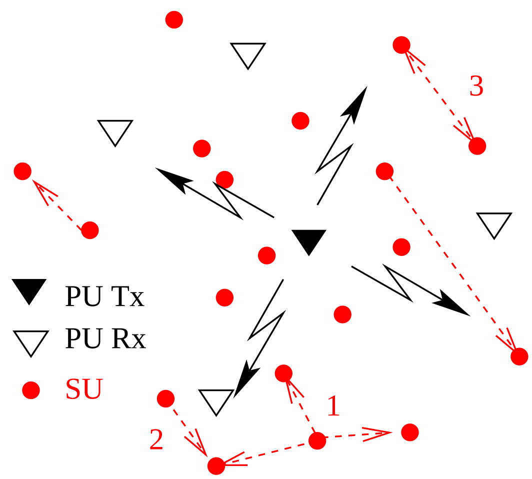


Fig. 1. Overlaying a cognitive radio network to top of a legacy primary network

of our objectives is to achieve the collective protection of the PU without a centralized controller while still enabling spatial reuse.

In this work, we use PU packet collision probability as the metric for PU protection. SUs access potential idle PU channels using a sensing-based access scheme. Multiple SUs can consider access to the idle primary band independently or collaboratively. Regardless, in order not to upset the PU performance, the total collision probability from all SU access must be restricted to satisfy the collision probability constraint set by the PUs. Under such a protection requirement, we investigate the fundamental limit on the throughput performance of opportunistic spectrum access by SUs. Our contributions are:

- Development of an optimal spectrum access policy under the assumption of perfect sensing by SUs.
- Derivation of (tight) lower and upper bounds on the throughput of the SU.
- Analyzing the impact of imperfect sensing on the throughput performance of the SU and the development of a modified spectrum access policy.
- Development of a distributed access scheme that enables multiple SUs to operate under the PU protection requirement and to adapt to changes in the PU traffic pattern.

In summary, our results illustrate and quantify the effect of the PU traffic pattern, the packet collision constraint, and SU sensing capability, on the performance of multiple SUs.

We organize our paper as follows. Section II discusses related work. Section III introduces our system model, the protection metric for the PU, and the performance metric for the SU. Section IV presents an optimal threshold-based spectrum access policy under generic PU idle time distributions under the assumptions of perfect sensing. The impact of imperfect sensing is studied in Section V. Section VI presents and evaluates a distributed access scheme that adapts to changes in the PU traffic pattern. The paper is concluded in Section VII.

II. Background and Related Work

Cognitive radio has attracted much research attention. Both distributed and centralized schemes have been proposed to facilitate the spectrum sharing between SUs and PUs. In [3]-[4], centralized mechanisms are proposed to implement spectrum leasing and real time spectrum auction of unused PU bands in which spectrum brokers and infrastructure may need to match "white space" between providers and consumers. Spectrum sharing by cognitive radio devices in unlicensed bands is also considered in [5]-[6]. Our works differ from the above in that we address the asymmetrical relationship in access priority and protection requirement of the PU and SUs when designing spectrum access schemes of the SUs.

In [7], the multi-armed bandit model is used to describe the spectrum access problem of the SU. However, the primary users in this case do not consider interference from the SUs. In [8], a CSMA/CA based spectrum access scheme is proposed by exchanging the channel reservation information on a separate control channel. In [9], a distributed dynamic spectrum allocation, *b-smart* is developed to allocate different sizes of time-frequency blocks available in TV bands to multiple SUs with different traffic demands. Still, neither work studies the impact of the PU's behavior on the performance and access protocol of the SU's.

Other related works include designs of channel sensing/probing, channel selection [10], flow control and scheduling [11], and joint PHY/MAC design [12]. For example, in [10], authors show that the optimal joint channel probing and transmission strategy has a threshold based structure. However, there is no explicit consideration of the protection of the PU.

The work in [12] formulates the joint PHY-MAC design of the opportunistic spectrum access with imperfect sensing as a constrained partially observable Markovian decision process, and exploits the whole observation history to decide the optimal action. The optimal access policy obeys a separation principle, which simplifies the joint design of the spectrum sensor and MAC protocol. In comparison, in [12], it is assumed that the primary channel has a slot structure with which the SU's can synchronize and access while we do not assume a slotted structure in this paper. In addition, the authors assume Markovian transition between idle and busy states in [12] while we consider general PU traffic patterns.

In [13], the authors show that the maximum usable time proportion for the SU cannot exceed the product of the collision probability constraint and idle percentage of the PU's traffic under the assumption that the PU's idle time distribution is exponential. For small value of collision probability constraint, this result is pessimistic. In contrast to these related works, we require neither the restrictive slot synchronization assumption, the Markovian transition model, nor the exponential idle time distribution assumption. Our model is more general and emphasizes the interaction between the SU's performance and the characteristics of the primary user's behaviors.

III. System Model

Consider the scenario in which one spectrum band is licensed to PUs while multiple SUs try to exploit spectrum opportunities vacated by the PUs under the protection requirements of the PUs. We assume that a SU does not distinguish different PUs, and can only access the channel when no PU is active. Thus, SUs treat the collective of all PUs as one "aggregated" PU in designing SUs' spectrum access schemes. SUs consider the PU channel as either idle or busy. Multiple SUs can have access to the idle primary band independently or collaboratively. Regardless, in order not to upset the PU performance, the sum effect from all SU access must be constrained to satisfy the protection requirement of the PU.

A. PU activity model

Because PUs do not operate at full capacity, the idle interval of the PU band is random and depends on the PU traffic pattern. Here, we denote the length of a PU's idle interval as V_p, its probability density function as $f_{V_p}(\cdot)$, its cumulative distribution function as $F_{V_p}(\cdot)$, and its mean as v_p. Denote the busy interval of the PU as L_p with mean l_p. The idle percentage of the primary channel is then $\alpha = \frac{v_p}{v_p+l_p}$. In each busy period, there are N_p consecutive PU packets. Each packet has a fixed length L. We let N_p be a random variable with mean n_p. Thus, $l_p = n_p L$. Note that here we loosely use the term "packet" to denote the time granularity of the PU's traffic. For PUs which do not use packet-based transmission scheme, they may be more interested in the amount of overlapping time between them and secondary users. It requires minor alteration to transform the packet collision probability to the overlapping time constraint.

We assume that the aggregated behavior of PUs is ergodic and stationary for the concerned time-scale of secondary users. We also assume that the PU's channel access is not affected by SUs' behavior. Whenever the PU has traffic to send, it will access the channel without sensing. Additionally, even though on rare occasions, collisions between the PU and SUs may lead to retransmission by the PU, we ignore such impact on the distribution of V_p and L_p. (We note that such retransmission can be accounted for in the PU behavior easily.)

Since the PU is the high priority legacy user within the band, the SU's transmissions must be conservative enough to protect the communication quality of the PU. One way to quantify this objective is to limit the average collision probability "perceived" by the PU, where the average collision probability is defined as follows:

$$p_p^c = P\{\text{PU packet collision}\} \tag{1}$$

Relying on the stationarity and ergodicity of the PU's access time, we have

$$p_p^c = \lim_{K\to\infty} \frac{\sum_{k=0}^{K} N_c(k)}{\sum_{k=0}^{K} N_p(k)}, \tag{2}$$

where $N_c(k)$ is the total number of collided packets of the PU in the kth busy-idle cycle. The collective access activities from all SUs have to satisfy the following constraint:

$$p_p^c \le \eta. \tag{3}$$

The constraint can be imposed by either the PU or the regulators and is known to the SU *a priori*. When there are multiple SUs, the overall collision probability at the PU needs to be bounded.

Note that when the SU and the PU transmit simultaneously, it may still be possible for the primary receiver to decode its signal if it is comparably strong. In addition, advanced signal (and array) processing techniques can also be explored to improve the interference tolerance capability of the PUs. In this case, the PU may provide a looser collision probability constraint η. Otherwise, the result here is a conservative estimation of the true collision probability, and thus results in better protection of the PU.

B. Access behaviors of SUs

For a group of SUs that want to access the spectral opportunity evacuated by inactive PUs, the goal is to maximize the time proportion they can transmit successfully while limiting their aggregated intrusion to an acceptable threshold. We let the SUs' transmission be packet-based, as is commonly practiced today. For simplicity, the length of the packet, Δ, is assumed to be fixed as a small value $\Delta \ll v_p$, and $\Delta \ll L_p$. The results in this paper can be directly generalized to other distributions of the packet length. We also assume that SUs always have packets to transmit, consistent with most related works.

Each SU is equipped with a cognition engine to facilitate the opportunistic spectrum access on the primary band. The cognition engine makes decisions for the SU to interact with the PU, but do not participate in channel competition or/and cooperation among SUs. The introduction of cognition engine divides the medium access protocol of SUs into two separate domains: an outer state machine focus on the spectrum opportunities detection and assessment; and inner state machine focus on the opportunities sharing among SUs. In other words, the cognition engine constructs a virtual channel over the spectrum holes in the primary band. Multiple SUs in vicinity share the virtual channel using existing well-designed protocols. This greatly simplifies the design of cognitive radio devices since the design of the inner state machine can inherit from the existing MAC protocols easily.

As illustrated in Fig. 2, the cognition engines includes the following three components: spectrum sensor, decision maker, and transmitter. The spectrum sensor listens to the channel and detects whether the primary user is present or absent.

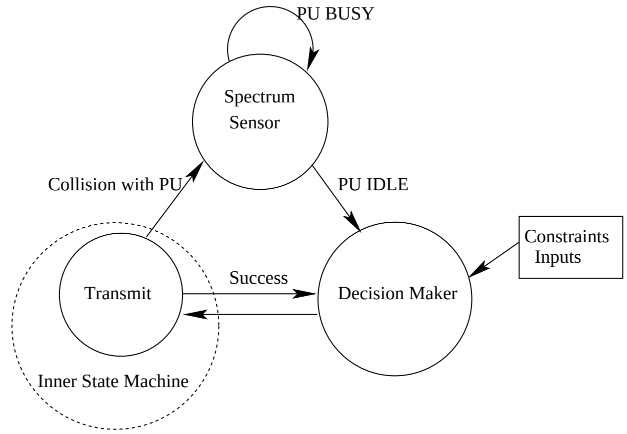


Fig. 2. Cognitive Components of Secondary Users

The decision maker is the focus of this paper. It determines whether to transmit based on the protection constraint from the PU, sensing results, and previous transmission history at the instance. ¿From the viewpoint of the practical protocol stack, sensing and transmission can be implemented at PHY layer while the decision maker can be implemented at MAC layer.

We assume that a SU can sense its collision with the PU, and can distinguish collisions with the PU from collisions with other SUs. Several mechanisms can be used to detect collisions with the PU. For example, the SU can perform sensing after each packet transmission. Since we have $\Delta \ll L_p$, if the SU senses the presence of PU signal after the SU's transmission, a collision is detected. Omitting lengthy discussion on this matter, we assume that the SU can detect the collision perfectly.

The cognition engine follows the listen-before-talk principle. Basically, as illustrated in Fig. 3, before the SU accesses the channel, it senses the channel to detect the activities of the PU. If the PU is active, the virtual channel is suspended, and the SU should freeze its inner state machine. The cognition engine continues to assess the PU channel. After detecting the PU is idle, the SU will make access decision. If the decision is to have access to the spectrum band, it will reactivate the virtual channel, and the inner state machine will control the access activities over the virtual channel. When a collision with the PU happens, the inner state machine informs the decision maker, and switch back the control flow to the cognition engine. Consider the example illustrated in Fig. 1. When the PU is idle, the cognition engines at SUs 1, 2, and 3 detect the opportunity, declare the virtual channel, and reactivate the SUs' inner state machines. SU 1 and SU 2 may interfere with each and thus they use their inner state machine to decide their channel sharing (such as using CSMA/CA). On the other hand, they are not interfering with SU 3, which enables the spatial reuse. The synchronization among SUs due to imperfect sensing will be considered in Section VI.

We consider both perfect and imperfect sensing of the SU's spectrum sensor. The perfect sensing means that there is no sensing error and the sensing time is negligible; while imperfect sensing consider both the impact of sensing error and sensing time. There are two types of collisions with the PU in the opportunistic spectrum access. As illustrated in Fig. 3, type-I collisions happen when the SU does not finish its transmission when the PU returns the channel. This type of collisions happens because wireless radio cannot receive and transmit on the same spectrum at the same time. Even if the sensing at the SU is perfect at the beginning of the transmission, type-I collisions exist unless that the SU can predict when the PU returns to the channel. Type-II collisions are caused by the imperfect detection of the spectrum sensor at the SU. Specifically, the SU may mistakenly detect a busy channel as idle and transmit its packet; thereby causing additional collisions to the PU. In order to satisfy the protection constraint from the PU, the SU may not transmit even when the PU is inactive at current time instance due to the potential collision. The constraint reduces the available transmission time (opportunities) of the SU.

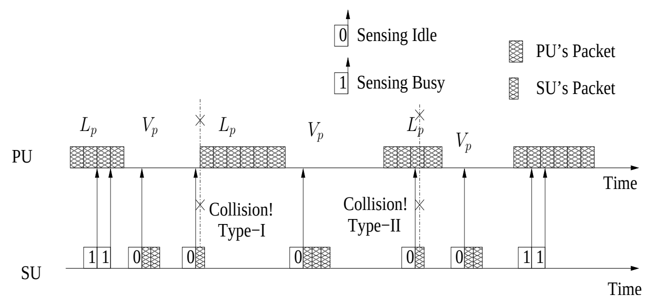


Fig. 3. Illustration of Opportunistic Spectrum Access Schemes

The performance metric of the SU is the percentage of time during which SUs can transmit without colliding with the PU. Specifically, the throughput of each SU is defined by

$$C_s = \lim_{T\to\infty} \frac{\text{SU's successful access time in } [0,T]}{T}. \quad (4)$$

Obviously, the throughput has an upperbound: $C_s \leq \alpha$ (where α is the percentage of PU idle time). This capacity upper bound is achievable if the SU knows exactly when PU starts and ends its transmissions, which is not realistic in general. Our object here is to study the fundamental limits on the achievable throughput of the SUs and to develop access schemes for SUs to maximize the throughput, C_s, while satisfying the constraint $p_p^c \leq \eta$.

IV. Optimal Access for Primary Idle Time of General Distribution

In this section, we reveal the close relationship between the optimal throughput performance of SUs and the idle time distribution of the PU. We propose a threshold-based spectrum access policy for SUs to achieve the maximum throughput performance under the packet collision constraint imposed by the PU. Bounds on the achievable throughput performance with regard to the idle time distribution are presented. Several examples are used to demonstrate the optimal scheme for various idle time distributions. We also discuss the implications of these results on practical system designs. We assume perfect sensing and focus on the single SU case in this section, and extend the results in Section Section V and VI.

A. Definition of spectrum access policy

In general, the optimal policy at time instant h depends the current and the historic channel activities of the PU. Therefore, we express the spectrum access policy as a function of h and Λ, where h is the current (absolute) time, and Λ is the observation history of PU's activities until now. Since the PU can be either busy or idle, we have $\Lambda = \{\tau : \Phi(\tau) = \text{Busy}, \tau \leq h\}$, where $\Phi(\tau)$ is the sensing outcome at time τ. Under perfect sensing, the sensing result always reflects the real condition of the PU channel. Due to the uncertainty in the PU's traffic activities, the optimal spectrum access policy for the SU is generally a probabilistic approach, rather than a deterministic policy. Thus, we use transmission probability of the SU, denoted by $q(h,\Lambda)$, to model the spectrum access policy. Obviously, we have $0 \leq q(h,\Lambda) \leq 1$. The listen-before-talk principle imposes the following restriction on the spectrum access policy: $q(h,\Lambda) = 0$ when $\Phi(h) = \text{Busy}$.

Let τ_{max} be the beginning of the latest idle period; i.e.,

$$\tau_{max} = \max_{\tau \leq h}\{\tau : \quad \Phi(\tau) = \text{Busy}\}.$$

We define a time variable t that reflects the time elapsed since the beginning of the latest idle period; i.e.,

$$t = h - \tau_{max}.$$

In particular, $t = 0$ indicates the beginning of the latest idle period.

With perfect sensing, the SU only accesses the spectrum when $t \geq 0$. Then, due to the stationarity and ergodicity of the PU's behavior, the expected number of collisions caused by the opportunistic access of the SU in each PU's busy-idle period is the same. The expected gain/reward (here throughput) for the SU is also the same for each PU's busy-idle period. Therefore, the policy/action space of the SU becomes irrelevant to the observation history before τ_{max} (the last time instant the spectrum band is determined to be idle). Thus, the problem reduces to determining when and how the SU should transmit after τ_{max}. In other words, the spectrum access action taken by the SU becomes a function of $t = h - \tau_{max}$. To simplify the notations, we use $q(t)$ instead of $q(h,\Lambda)$ in the rest of the paper. Whenever it is not confusing, we also use $\Phi(t)$ and $\Phi(\tau)$ interchangeably.

Consider the case that the SU transmits at several sequence time instants from the end of the latest PU transmission, $0 \leq t_0, t_1, \cdots, t_k, \cdots \leq V_p$, where there is no packet overlap, i.e., $[t_i, t_i+\Delta] \bigcap [t_j, t_j+\Delta] = \emptyset$ for any $i \neq j$. The transmission strategies at different time instants may be dependent. Since the sensing is perfect, we have only type-I collisions with the PU, which only happens at the beginning of the PU's busy period. Because $\Delta \leq L$, at most one PU's packet is collided in each PU busy period. Note that if $\eta \geq \frac{1}{n_p}$, the strategy of the SU is trivial; the SU always transmits until the PU returns. Therefore, we consider the non-trivial case when $\eta < \frac{1}{n_p}$. Let

$Z_p(k) =$ Event that the PU returns to transmit in $[t_k,\, t_k + \Delta)$.

Then, the average number of collided packets in an idle-busy period can be obtained as:

$$\begin{aligned} n_c &= E_{V_p}[N_c] \\ &= \sum_{k=0}^{\infty} 1 \cdot Pr[Z_p(k), \text{SU transmits at } t_k] \\ &= \sum_{k=0}^{\infty} q(t_k) Pr[Z_p(k)]. \end{aligned} \tag{5}$$

where, $Pr[Z_p(k)] = Pr[t_k < V_p \leq t_k + \Delta]$. For a given policy $q(t)$, we obtain the average packet collision probability observed by the PU as:

$$p_p^c(q) = \frac{n_c}{n_p} = \frac{1}{n_p} \sum_{k=0}^{\infty} q(t_k) Pr[Z_p(k)]. \tag{6}$$

Note that, by setting $n_p = 1$, the packet collision probability defined here coincides with the collision probability defined in [13]. Our definition in (2) is more general, because it can capture the additional collision caused by sensing error on the PU's traffic, as shown in Section V .

The average achievable throughput of the SU, which is the normalized time that SUs successfully access the channel during an idle-busy cycle of the PU, can be written as below:

$$C_s(q) = \frac{\sum_{k=1}^{\infty} \left(\sum_{l=0}^{k-1} \Delta \cdot q(t_l)\right) \cdot P[Z_p(k)]}{l_p + v_p}, \tag{7}$$

where $\sum_{l=0}^{k-1} \Delta \cdot q(t_l)$ is the average total number of transmission time without colliding with the PU when the PU returns during $[t_k,\, t_k + \Delta)$.

We have the following result

Proposition 1. *The throughput of the SU $C_s(q)$ increases as Δ decreases.*

The statement is quite intuitive. Whenever we split the packet of the SU into two halves, the first half of the packet has a higher successful rate while the total collision observed by the PU remains unchanged. Therefore, $C_s(q)$ increases as Δ decreases. We omit the proof here.

Letting $\Delta \to 0$, we can convert the summation into Riemann integral:

$$p_p^c(q) = \frac{1}{n_p} \int_0^{\infty} f_{V_p}(\tau) \cdot q(\tau) d\tau. \tag{8}$$

Correspondingly, the maximum achievable throughput of the SU can be expressed as:

$$\bar{C}_s(q) = \frac{\int_0^{\infty} f_{V_p}(t) \cdot \int_0^t q(\tau) d\tau dt}{v_p + l_p} = \frac{G_s(q)}{v_p + l_p}, \tag{9}$$

where $G_s(q) = \int_0^{\infty} f_{V_p}(t) \int_0^t q(\tau) d\tau dt$.

Therefore, we can maximize an upper bound on the throughput performance by solving the following optimization problem:

$$\begin{aligned} &\max_{q(t): 0 \leq q(t) \leq 1} \quad \bar{C}_s(q) \\ &\text{subject to} \\ &p_p^c(q) \leq \eta, \end{aligned} \tag{10}$$

where the expressions of $\bar{C}_s(q)$ and p_p^c are given in (9) and (8) respectively. The dimension of the optimizing variable $q(t)$ in (10) is infinite, which renders the problem highly difficult, if not impossible, to solve. However, we will exploit the structure of the problem (in both objective function and constraint), and present a threshold-based optimal spectrum access policy for the SU next.

B. Optimal spectrum access policy

First, to facilitate the decision maker to efficiently use the spectrum opportunities in a non-intrusive way, we define a time-related decision metric for $t = h - \tau_{max}$ as follows:

$$g(t) = \frac{1 - F_{V_p}(t)}{f_{V_p}(t)}. \tag{11}$$

Since $\frac{f_{V_p}(t)}{1 - F_{V_p}(t)}$ can be regarded as the likelihood of the conditional collision probability given that the SU transmit at t, $g(t)$ indicates the successful possibility when the SU transmits a packet. We will show in the following that this metric captures the connection between the packet collision probability constraint and the throughput performance.

We have the following theorem on the optimal spectrum access policy under a given distribution $f_{V_p}(\cdot)$.

Theorem 1. For a given distribution $f_{V_p}(t)$ for the PU's idle time, the following listen-before-talk spectrum access policy is optimal under the collision probability constraint $p_p^c \leq \eta$:

$$q^*(t) = \begin{cases} 1, & \text{if } \quad g(t) > \gamma^*, \Phi(t) = Idle \\ p^*, & \text{if } \quad g(t) = \gamma^*, \Phi(t) = Idle \\ 0, & \text{otherwise}, \end{cases}$$

where the values of γ^* and p^* are determined by

$$\int_{\tau: g(\tau) > \gamma^*} f_{V_p}(\tau) d\tau + p^* \int_{\tau: g(\tau) = \gamma^*} f_{V_p}(\tau) d\tau = n_p \eta. \tag{12}$$

A random strategy is required when $g(t) = \gamma^*$.

The proof of the above theorem is given here.

Proof: Consider any policy q, according to (9), and $\Phi(t) = Idle$ if $V_p > t$, we have:

$$\begin{aligned} G_s(q) &= \int_0^{\infty} q(\tau) \int_{\tau}^{\infty} f_{V_p}(t) dt d\tau \\ &= \int_0^{\infty} q(\tau)(1 - F_{V_p}(\tau)) d\tau \\ &= \int_0^{\infty} (\tau) f_{V_p}(\tau) \frac{(1 - F_{V_p}(\tau))}{f_{V_p}(\tau)} d\tau. \end{aligned}$$

We want to show that $C_s(q^*) \geq \bar{C}_s(q)$ holds. Since $\forall t, 0 \leq q(t) \leq 1$, we have the following relations:

$$q^*(t) - q(t) > 0, \qquad t : g(t) > \gamma^*;$$
$$q^*(t) - q(t) < 0, \qquad t : g(t) < \gamma^*.$$

The above relations are used in (13) to prove that $G_s(q^*) \geq G_s(q)$.

Because $\bar{C}_s(q) = \frac{G_s(q)}{v_p + l_p}$, we have $C_s(q^*) \geq C_s(q)$. The threshold γ^* and the randomized factor p^* can be obtained from the packet collision probability constraint (12). ■

We have shown that a threshold-based policy (1) is at least one (not necessarily unique) optimal access policy for the SU. This result is intuitive in that the SU should transmit when the conditional collision probability is low. Interestingly enough, the decision variable $g(t)$ in the optimal transmission strategy, has the same form as the inverse of the "failure rates" function, which is commonly used in the reliability study of a system with multiple components.

For practical cognitive radio systems, Theorem 1 implies that the optimal listen-before-talk spectrum access policy for the SU is threshold-based, which greatly simplifies the MAC layer implementation for the cognitive devices. Using a reliable spectrum sensor at the PHY layer, the SU can detect previous transmissions of the PU with high accuracy. When the PU switches from busy to idle, the SU initializes an internal timer $t = 0$, compute the value of $g(t)$, and then compares $g(t)$ with the calculated threshold γ^*. Finally, the SU decides whether to access based on the comparison result.

The values of threshold γ^* and parameter p^* are determined by the protection requirement and the idle time distribution as in (12). Next, we consider a few examples of the idle time distribution to illustrate how to obtain the values of γ^* and p^* in the optimal spectrum access policy and demonstrate its impact on SU performance.

In particular, when the idle time of the PU is deterministic and known to the SU, i.e.,

$$f_{V_p}(t) = \delta(t - v_P), \qquad \text{where } \delta(t) = \text{impulse}.$$

Then, we have

$$g(t) = \begin{cases} \infty, & t < v_p \\ 0, & t = v_p \end{cases}$$

Referring to Theorem 1, we have $\gamma^* = 0$ and $p^* = n_p\eta$ in the optimal transmission policy. In other words, the optimal strategy for the SU is

$$q^*(t) = \begin{cases} 1, & t < v_p \\ \eta, & t = v_p. \end{cases}$$

In fact, $p^* = 0$ can also achieve the maximum throughput, since the measure of the set $\{f_{V_p}(t) > 0\}$ is 0. The corresponding maximum achievable throughput for SUs is α. This is intuitive, since collisions between SUs and PU can only happen at time $t = v_p$, which is known to SU. SUs can transmit as much as α fraction of the time (which is the entire portion of white space) without violating the collision probability constraint.

When the PU has a slot-structured transmission, and the transition between idle and busy is Markovian, as assumed in [12], [14], the idle time distribution follows $f_{V_p}(t) = \sum_{k=1}^{\infty} p_k \delta(t - k\sigma)$, where σ is the length of the slot of the primary transmission. Then, the optimal transmission policy will be to transmit during the slot interval with probability 1, and transmit at the boundaries of slots with probability η. The threshold can be obtained as $\gamma^* = 0$, and the maximum throughput is α. Note that this does not contradict with the results in [12][15] and serves as an upper bound on the throughput performance when the sensing is imperfect.

When the idle time of the PU is exponentially distributed, we have $g(t) = v_p$, which is a constant, i.e,. the conditional collision probability given that the SU transmits does not depend on the length of idle time elapsed. In this case, we have the threshold is $\gamma^* = v_p$, and $p^* = n_p\eta$. Note that the randomization is important here — SU transmits with probability p^* when it senses the channel being idle. The corresponding maximum throughput of the SU is $C_s^{max} = \alpha n_p \eta$, which is the same as in [13] when the SU uses VX scheme under busy period collision constraint $p_{pb}^c \leq n_p\eta$.

Weibull distribution is commonly used in the reliability study. When the idle time of the PU follows a two-parameter Weibull distribution with parameters β, and μ, i.e.,

$$f_{V_p}(t) = \frac{\beta}{\mu}(\frac{t}{\mu})^{\beta-1} e^{-\frac{t}{\mu}^{\beta}},$$

where, $\beta > 0$ is the shape parameter, $\mu > 0$ is the scale parameter. The mean of the Weibull distribution is $\mu \cdot \Gamma(\beta^{-1} + 1)$. When $\beta > 1$, the "failure rate" is an increasing function of t, while $g(t)$ is a decreasing function of t, then the optimal transmission strategy is given by:

$$q^*(t) = \begin{cases} 1, & \text{if } t \leq T^*, V_p > t \\ 0, & \text{otherwise}, \end{cases}$$

The maximum throughput is $\frac{\int_0^{T^*}(1 - F_{V_p}(t))dt}{v_p + l_p}$, where $T^* = \mu \cdot (-\ln(1 - n_p\eta))^{\frac{1}{\beta}}$. The result for $\beta \leq 1$ can be obtained similarly.

The above example for Weibull distribution implies that an optimal transmission policy with simpler form exists for a class of idle time distributions. Following Theorem 1, we have the following result:

Corollary 1. For a given distribution $f_{V_p}(\cdot)$ for the idle time of the PU, when $g(t)$ is monotonically decreasing and continuous, the following spectrum access policy is optimal under the collision probability constraint $p_p^c \leq \eta$:

$$q^*(t) = \begin{cases} 1, & \text{if} \quad t \leq T^*, \Phi(t) = \text{Idle} \\ 0, & \text{otherwise}, \end{cases}$$

where T^* can be obtained from the collision probability constraint $\int_0^{T^*} f_{V_p}(t)dt = n_p\eta$. Specifically, the achievable throughput is:

$$C_s^{max} = C_s(q^*) = \frac{\int_0^{T^*} t f_{V_p}(t)dt + T^* \int_{T^*}^{\infty} f_{V_p}(t)dt}{v_p + l_p}.$$

$$
\begin{aligned}
&G_s(q^*) - G_s(q) \\
&= \int_0^{\infty} [q^*(\tau) - q(\tau)] f_{V_p}(\tau) g(\tau) d\tau \\
&= \int_{\tau: g(\tau) > \gamma^*} [q^*(\tau) - q(\tau)] f_{V_p}(\tau) g(\tau) d\tau + \int_{\tau: g(\tau) < \gamma^*} [q^*(\tau) - q(\tau)] f_{V_p}(\tau) g(\tau) d\tau + \gamma^* \int_{\tau: g(\tau) = \gamma^*} [q^*(\tau) - q(\tau)] f_{V_p}(\tau) d\tau \\
&\geq \gamma^* \{ \int_{\tau: g(\tau) \geq \gamma^*} [q^*(\tau) - q(\tau)] f_{V_p}(\tau) d\tau + \int_{\tau: g(\tau) < \gamma^*} [q^*(\tau) - q(\tau)] f_{V_p}(\tau) d\tau \} \\
&= \gamma^* \{ \int_0^{\infty} q^*(\tau) f_{V_p}(\tau) d\tau - \int_0^{\infty} q(\tau) f_{V_p}(\tau) d\tau \} \\
&= \gamma^* (n_p \eta - n_p \eta) = 0,
\end{aligned}
\tag{13}
$$

Proof: Since $g(t)$ is monotonically decreasing, relying on Theorem 1, there exists a T^*, such that for any $0 \leq t \leq T^*$, $g(t) \geq \gamma^*$, and $g(T^*) = \gamma^*$. Then, the Corollary follows. ■

We use two examples to illustrate the optimal time threshold-based transmission policy for the SUs. For uniformly distributed idle time of the PU, we have $f_{V_p}(t) = \frac{1}{2v_p}(u(t) - u(t - 2v_P))$. Since $g(t) = 2v_p - t$, $t \in [0, 2v_p)$ is strictly decreasing, relying on Corollary 1, we have the optimal T^* as:

$$T^* = 2v_p n_p \eta.$$

The corresponding maximum throughput of the SU is

$$C_s^{max} = 2\alpha n_p \eta - \alpha n_p^2 \eta^2. \tag{14}$$

As another example, for exponentially distributed idle time of the PU, $g(t) = v_p$ is flat, relying on Corollary 1, we have the optimal T^* as:

$$T^* = -v_p \ln(1 - n_p \eta).$$

The corresponding maximum throughput of the SU is $C_s^{max} = \alpha n_p \eta$. This policy results in the same performance as the randomized strategy indicated by Theorem 1 with $p^* = n_p \eta$.

C. Optimal SU's throughput for different idle time distributions

We note that the distribution of the PU's idle time is determined by the PU's behavior. It has significant impact on the throughput performance of the SU. Therefore, it is interesting to compare the maximum throughput of different distributions of the PU's idle time. The following corollary describes the impact:

Corollary 2. The optimal throughput performance of the SU under the packet collision probability constraint, η, is between $[\alpha n_p \eta, \alpha]$. The upper bound α is achieved if $\int_0^{\infty} f_{V_p}(t) \mathbb{I}\{f_{V_p}(t) > 0\} \leq \eta$; the lower bound $\eta\alpha$ is achieved if the idle time follows the exponential distribution.

Proof: When the distribution of the idle time satisfies $\int_0^{\infty} f_{V_p}(t) \mathbb{I}\{f_{V_p}(t) > 0\} \leq \eta$, by setting $\gamma^* = 0$, $p^* = 0$, i.e., the SU does not transmit when $f_{V_p}(t) > 0$, then the SU achieves the upper-bound. Example of such distributions include deterministic distribution, and the slotted-structure, as discussed in Section IV-B.

Next, we prove the lower bound $\alpha n_p \eta$. According to Theorem 1, for any transmission policy q, we have $G_s(q^*, f_{V_p}) \geq G_s(q, f_{V_p})$. Then, we construct a random access policy $\hat{q}$ for the SU when the distribution of the idle time is $f_{V_p}(t)$ as:

$$\hat{q}(t, f) = \begin{cases} n_p \eta, & \text{if } V_p > t \\ 0, & \text{otherwise} \end{cases}. \tag{15}$$

The collision probability observed by the PU is:

$$p_p^c = \frac{1}{n_p} \int_0^{\infty} \hat{q}(t, f) f_{V_p}(t) dt = \eta.$$

We have

$$
\begin{aligned}
G_s(\hat{q}, f) &= \int_0^{\infty} \hat{q}(t, f)(1 - F_{V_p}(t)) dt \\
&= n_p \eta \int_0^{\infty} (1 - F_{V_p}(t)) dt \\
&= n_p \eta v_p.
\end{aligned}
$$

It follows that $G_s(q^*, f_{V_p}) \geq G_s(\hat{q}, f_{V_p}) = n_p \eta v_p$. Therefore, $C_s(q^*, f_{V_p}) \geq C_s(\hat{q}, f_{V_p}) \geq n_p \eta \alpha$. Since the optimal transmission strategy for exponentially distributed idle time achieve the throughput as $\alpha n_p \eta$, it is the least favorable distribution of the idle time of the primary channel for the SU. ■

The random access policy in (15) has interesting properties. In the absence of idle time distribution information, for any distribution, the scheme in (15) achieves a capacity of $n_p \eta \alpha$, which is the optimal throughput for exponential distribution. The implication is multifold. First, the knowledge of distribution is important. Knowing the distribution and exploring the distribution may significantly improve the SU's throughput. For example, the maximum throughput obtained in Eq. 14 under uniform distribution is much larger than $n_p \eta \alpha$. Therefore, in order to fully utilize the spectrum opportunities in the primary band, the SU should acquire knowledge of the PU behavior. We propose such an adaptive scheme in Section VI when PU idle time distribution is not available *a prior*. Second, when the distribution information is absent, a SU can exploit the random policy in (15) to lower bound its throughput. Not knowing the distribution is equivalent to the memoryless exponential distribution. The random access policy in (15) is similar to the VX scheme proposed in [13] in essense.

In summary, when we quantify the usefulness of the spectrum opportunities, important factors are the percentage of the idle time (α), the distribution of the idle time $f_{V_p}(\cdot)$, the availability of such information, and the protection requirement of the channel. When there are multiple spectrum bands available for SUs to choose from, SUs should consider all these factors.

V. Opportunistic Spectrum Access with Imperfect Sensing

In this section, we consider the impact of imperfect sensing. As in previous section, we assume that the SU can always detect the collision with the PU accurately through the use of ACK, combination of ACK loss or/and additional sensing. We also assume that the SU has knowledge of the idle time distribution $f_{V_p}(\cdot)$.

We denote the sensing time as σ_s. When the SU performs sensing, it cannot transmit at the same time. In General, the greater the σ_s, the more reliable the spectrum sensing, and, unfortunately, the more opportunities the SU wastes. Therefore, there exists a trade-off between the sensing time and detection performance with respect to the throughput of the SUs.

We model the sensing performance by the ROC (receiver-operating characteristic) curve, i.e., (P_f, P_m) pair, where P_f is the false-alarm probability, and P_m is the missed detection probability. Both missed detection and false alarm pose new challenges. As mentioned in Section III, missed detections cause type-II collisions with the PU. On othe other hand, reducing the missed detection probability often increases the false alarm probability, which will lead to a waste of the spectrum opportunities.

We first discuss the impact of missed detection. As illustrated in Fig. 4, the SU keeps sensing if it finds the channel busy. The SU can only transmit packets after it senses the channel being idle. Packet collisions happen after each missed detection. Multiple packets may collide in a single busy period due to multiple missed detection.

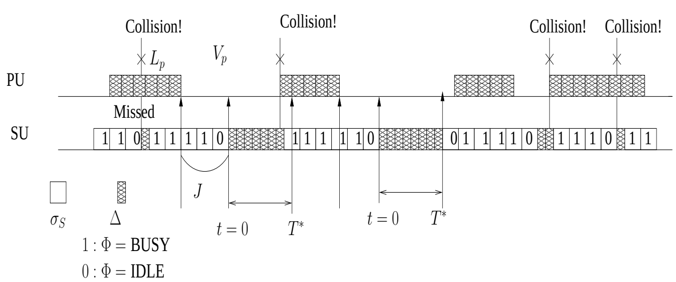


Fig. 4. Illustration of Imperfect Sensing

Assuming that $\Delta \le L$ and the sensing performance is independent of PU's behavior, we have the following expression of packet collision probability due to P_m:

$$p^c_{pm} = \frac{P_m E[\lceil \frac{N_p L}{(1-P_m)\sigma_s + P_m(\sigma_s+\Delta)} \rceil N_c]}{n_p}, \qquad (16)$$

where $P_m \lceil \frac{N_p L}{(1-P_m)\sigma_s + P_m(\sigma_s+\Delta)} \rceil$ is the average number of missed-detections during a busy period of length $N_p L$. Let X denote the residual time of the PU's packet when the SU starts transmitting, which is uniformly distributed over $[0, L]$. Since $\Delta \le L$, we have the number of collided primary packets as:

$$N_c(X) = \begin{cases} 1, & \text{if } X > \Delta, \\ 2, & \text{if } X \le \Delta, \end{cases}$$

with its mean as:

$$n_c = E_X[N_c(X)] = \frac{L+\Delta}{L}.$$

In order to satisfy the collision probability constraint imposed by the PU, we require that the sampling length of the spectrum sensor, σ_s, be large enough to guarantee $p^c_{pm} \le \eta$. Therefore, the allowance for type-I collision is:

$$\hat{\eta} = \eta - p^c_{pm}.$$

Next, we focus on the impact of false alarm. As observed in the previous section, the optimal transmission strategy is a threshold based policy, which depends on the true status of the channel and the knowledge of the beginning of the idle period. However, because of imperfect sensing, the SU knows neither the true status nor the beginning of the idle period accurately. Since the false-alarm event is random, the channel access of the SU has to guarantee that the average collision probability observed by the PU is less than $\hat{\eta}$. Let J be the number of consecutive sensing slots that the SU takes to detect the idle period after the PU channel becomes idle. Assume that the sensing result from each sensing period is independent and has the same false-alarm probability, J follows a geometric distribution: $Pr[J = j] = P_f^{j-1}(1 - P_f), j = 1, 2, \cdots$.

Let p^c_{pf} be the packet collision probability when the primary returns before the SU can finish transmission, counting the effect of false alarm. Note that since $\Delta \le L$, this type of collision only leads to in one packet collision for the PU. Define the event that the PU returns during $t_k + j\sigma_s < V_p \le t_k + j\sigma_s + \Delta$ as $Z_p(k, j)$. Then, we have the following expressions of p^c_p and C_s:

$$\begin{aligned} p^c_{pf} &= \frac{1}{n_p} E_J \left\{ \sum_{k=0}^{\infty} q(t_k) P[Z_p(k, J)] \right\} \\ &= \frac{1 - P_f}{n_p} \sum_{j=1}^{\infty} P_f^{j-1} \sum_{k=0}^{\infty} q(t_k) P[Z_p(k, j)]; \qquad (17) \\ C_s &= \frac{E_J[\sum_{k=0}^{\infty} \Delta q(t_k)(1 - F_{V_p}(t_k + \Delta + J\sigma_s))]}{v_p + l_p} \end{aligned}$$

where, $t_k, k = 0, 1, \cdots$, is the amount of time elapsed after the spectrum sensor detects the channel being idle.

When $\Delta \to 0$, we have:

$$p_{pf}^c = \frac{1}{n_p} \int_0^\infty E_J(f_{V_p}(t + J\sigma_s))q(t)dt$$
$$C_s = \frac{\int_0^\infty E_J[(1 - F_{V_p}(t + J\sigma_s))]q(t)dt}{v_p + l_p}.$$

Following similar arguments as in Theorem 1, for a given practical spectrum detector with (P_f, P_m), define the following decision metric:

$$g(t) = \frac{E_J[(1 - F_{V_p}(t + J\sigma_s))]}{E_J[f_{V_p}(t + J\sigma_s)]}.$$

We then propose the spectrum access strategy for the SU when $\Phi(t) = \text{Idle}$

$$q^*(t) = \begin{cases} 1, & \text{if } g(t) > \tilde{\gamma}^* \\ \tilde{p}^*, & \text{if } g(t) = \gamma^* \\ 0, & \text{otherwise}, \end{cases}$$

where $\tilde{\gamma}^*$ and $\tilde{p}^*$ are determined by:

$$\int_{\tau : g(\tau) > \tilde{\gamma}^*} E_J[f_{V_p}(t + J\sigma_s)]d\tau + \tilde{p}^* \int_{\tau : g(\tau) = \tilde{\gamma}^*} E_J[f_{V_p}(t + J\sigma_s)]d\tau = n_p \hat{\eta}.$$

Next, we examine the trade-off between P_f and P_m more carefully. As in [16], relying on central limit theorem, we use Gaussian distribution to approximate the detection statistic of the spectrum sensor deployed at SUs. Specifically, we have

$$P_f = \mathbb{Q}\left(\frac{\zeta - N_s\sigma_w^2}{\sqrt{2N_s\sigma_w^4}}\right) \quad P_M = 1 - \mathbb{Q}\left(\frac{\zeta - N_s(\sigma_w^2 + \sigma_x^2)}{\sqrt{2N_s(\sigma_w^2 + \sigma_x^2)^2}}\right),$$

where $\mathbb{Q}(x)$ is defined as $\int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\tau^2/2} d\tau$, ζ is the detection threshold, $N_s = \sigma_s f_c$ is number of samples (f_c is the sampling frequency), and σ_w^2 and σ_x^2 are the power of noise and primary signal, respectively. The ROC curve is jointly determined by σ_s, f_c, and SNR. By adjusting the value of the detection threshold ζ, different trade-off between P_f and P_m on the ROC curve can be achieved. For a given spectrum sensor (surely the length of sensing should be large enough to guarantee that $\hat{\eta} > 0$), the optimal operating point (P_f, P_m) can be obtained by solving a non-linear optimization problem.

Last, we present numerical results on the SU performance of with imperfect sensing. Assume that the distribution of the idle time is uniformly distributed in $[0, 2v_p]$ and the number of packets in a busy period N is a constant. We use the following parameters in the numerical calculation: $L = \Delta = 1ms$, $v_p = 200ms$, $l_p = 100ms$, $N_p = 100$, $f_c = 10MHz$, and SNR $= \frac{\sigma_x^2}{\sigma_w^2} = -3$dB, detector threshold ζ is adjusted to achieve trade-off between P_f and P_m on the same ROC curve. We set $\sigma_s = 20, 30, 40, 50\mu s$ in our simulations, with their corresponding ROC curves in Fig. 5(a).

Recall that P_m has to be small enough to satisfy that $\hat{\eta} > 0$. It is easy to observe that, for smaller packet collision probability constraint, the range of the feasible operating points is smaller. In Fig. 5(b) and Fig. 5(c), the achieved throughput for the modified threshold-based spectrum access scheme is presented when $\eta = 10^{-3}$, and $\eta = 8 \times 10^{-3}$, respectively. For comparison, we also show the throughput performance assuming perfect sensing. We observe that, there is an optimal operating point of the spectrum sensor to achieve the maximum throughput. This optimal point represents the trade-off between P_m and P_f. For a given P_f, the longer the sensing time, the larger the throughput (within a reasonable range). This implies in practical systems, since the access of the SU has to be conservative to protect the PU, a longer sensing time is desirable. In addition, for the benefit of the SU, the transmission given a missed-detection contributes nothing to its throughput, but "consumes" the allowed collision probability. So it is much more important to ensure a small missed detection probability than false-alarm probability. In other words, the capability to protect the PU (measured by smaller missed-detection probability) can benefit the SU's throughput performance. We also observe that when σ_s is large enough, the performance of the optimal operating point is very close to that of the perfect sensing. The larger the value of σ_s, the larger the region where the performance is close to optimal.

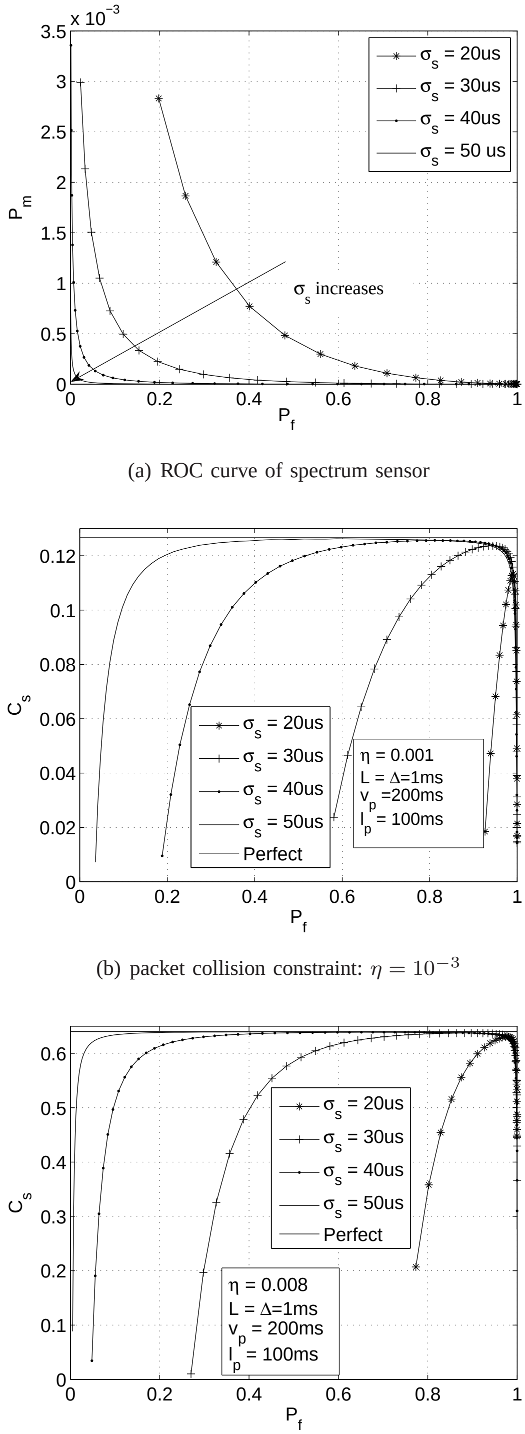
VI. Adaptive Opportunistic Spectrum Access

In Section IV, we presented an optimal spectrum access scheme assuming perfect sensing, single SU, and knowledge of the idle time distribution. In this section, we consider multiple SUs without *a prior* knowledge of the distribution of PU channel idle time.

As shown in Figure 1, the protection of PUs is more challenging when there are multiple SUs accessing a PU channel. Because SUs are widely spread in the area, one SU may not be able to sense all other SUs' transmissions or their collisions with the PU. While this enables spatial reuse among secondary users, the protection of the PU is more difficult because the SUs' disruption on the PU is cumulative. Consider the case where two SUs cause random collisions with the PU. Assume that the two SUs are not aware of each other and $\eta = 1\%$. If each SU causes 1% collision randomly (e.g., using the VX scheme [13] or the random access policy (15) described in Section IV-C,) the cumulative collision on the PU could be close to 2%. When there are more SUs, the impact of such cumulative intrusion can be devastating. Our objective is to protect the PU against the collective use of SUs without a centralized controller while still enabling spatial reuse. Our approach explicitly explores the structure of the optimal scheme presented in the previous section.

A. Perfect sensing case

According to Theorem 1, the optimal transmission strategy is threshold-based, where the threshold is obtained as a function of the idle time distribution. Assume SUs have perfect sensing and the knowledge of the idle time distribution, SUs have the same knowledge about the radio environment or the virtual channel. Basically, the decision maker at each SU (or SU sub-group) has the same input, and thus each SU can synchronize with the same virtual channel.



(a) ROC curve of spectrum sensor

(b) packet collision constraint: $\eta = 10^{-3}$

(c) packet collision constraint: $\eta = 8 \times 10^{-3}$

Fig. 5. Throughput of SU in threshold-based spectrum access policy with imperfect sensing

The remaining question is thus how SUs can obtain the knowledge of the PU activity. While there are schemes to estimate distribution functions, they usually require a large number of observations. This is challenging especially in the presence of multiple independent SUs. In addition, the PU traffic pattern is time varying, and so does the idle time distribution. Again, we explicitly explore the structure of the proposed optimal scheme. We propose the following adaptive approach for SUs to achieve the non-intrusive spectrum access. Each SU independently executes the following adaptive algorithm and makes transmission decisions.

Note that when the idle time distribution of the PU is such that $g(t)$ is monotonically decreasing, the optimal transmission strategy is a time-threshold-based approach. In this case, we use the result in Corollary 1 to develop an adaptive transmission strategy for SUs. Here, we assume that all SUs have knowledge about the average number of the PU's packets in an idle-busy period. Recall that the packet length of the SU is denoted by Δ. Each SU, indexed by m, maintains $N_c^m(j), j = 1, \cdots$, the number of collisions caused by SU m during the jth busy-idle period. Each SU begins transmitting its packets after sensing the idle channel, and stops when the internal timer t reaches the time threshold $T^m(i)$ (the ith estimate of T^*) or it collides with the PU. Each updating interval for the time threshold $T^m(i)$ consists W idle-busy periods of the PU. The SU estimates the collision probability in the ith updating interval as:

$$\tilde{\eta}^m(i+1) = \frac{\sum_{j=(i-1)W+1}^{iW} N_c^m(j)}{W \cdot n_p}. \quad (18)$$

Then the SU updates the time threshold T^m based on the observed collision probability as follows:

$$T^m(i+1) = T^m(i) + \mu(i) \cdot T^m(0) \cdot \frac{\eta - \tilde{\eta}^m(i)}{\eta}, \quad (19)$$

where $T^m(0)$ is the initial value, and $\mu(i)$ is the step-size.

A natural choice of $T^m(0)$ is the value of T^* when the idle time distribution is exponential, i.e., $T^m(0) = -v_p * \ln(1 - n_p\eta)/\Delta$. When $\tilde{\eta}^m > \eta$, the SU reduces its transmission time; otherwise the SU increases its transmission time. The step-size determines the trade-off between the convergence speed and the variance of the updated values. For example, one can show that $T^m(i)$ converges to T^* when $\mu(i) = 1/i$ because the estimation error has zero mean, using standard stochastic approximation techniques.

As an illustration, we use numerical simulations where the idle time initially follows the Weibull distribution with $\beta = 2$ and changes to uniform distribution later without informing SUs. Simulation parameters are set as: $v_p = 200ms$, $n_p = 100$, $\Delta = L = 1ms$, $\eta = 0.1\%$, $W = 50$, and $\mu(i) = 0.4$. Each SU has the knowledge of n_p and v_p (both are set to be constants), but not $f_{V_p}(t)$. Initially, there is only one SU in the network. Another SU joins the network after 50000 idle-busy periods (about 7500 seconds) and runs the proposed algorithm independently without observing the collisions caused by the first SU to the PU. The two SUs cannot sense each other. After

150000 idle-busy periods (about 45000 seconds), the PU's idle time distribution changes from Weibull to uniform distribution.

The performance of the proposed algorithm is shown in Fig. 6, where the value of P_p^c and C_s is evaluated every 5000 idle-busy periods, or 100 updating periods. For comparison purpose, we plot the optimal throughput and the optimal value of T^* in the figure. From Fig. 6, we can observe that the proposed algorithm has several desirable properties. First, it satisfies the protection requirement of the PU when there are multiple SUs opportunistically accessing the spectrum. Second, it converges very quickly to the optimal value of T^*, and obtains close-to optimal throughput performance derived for $\Delta \rightarrow 0$. Note that each SU independently achieves similar throughput C_s rather than shares the total available time, thus achieving spatial reuse. Third, it adapts to the dynamics of the PU's traffic pattern rapidly. Additionally, it is a distributed algorithm with no central controller. Each SU performs its algorithm and functions independently.

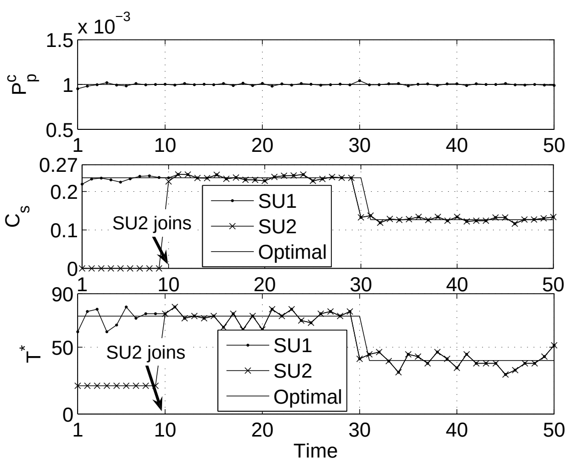


Fig. 6. Convergence of the Proposed Adaptive Algorithm (Unit of x-axis: 5000 idle-busy periods)

Compared to the multi-SU VX scheme proposed in [13], where perfect sensing of other SUs' signal is required to enable perfect cooperation among SUs, which reduces the spatial reuse gain. If SUs cannot hear each other perfectly, it is difficult to guarantee that the cumulative collisions observed by the PU is limited for the VX scheme. With our proposed algorithm, both the spatial reuse and the collective protection on the PU are possible.

This proposed adaptive algorithm works well for a wide class of idle time distribution with monotonically decreasing $g(\cdot)$. When this conditioin is not satisfied, T^* policy as in Corollary 1 may not be optimal, so neither is the adaptive scheme. However, the adaptive scheme can still garantee the collaborative protection of the PU in a distributed manner while enable spatial reuse.

B. Imperfect sensing case

In the presence of imperfect sensing, the SU cannot track accurately the status of the PU, neither the exact beginning of the idle time. We propose the following modifications to address the impact of the imperfect sensing in the above algorithm. We assume that a SU does not know the presence of other SUs outside its sensing range, and has no knowledge about other SUs' missed-detection probabilities or the information about other SUs' collisions with the PU.

First, each SU estimates the collision probability caused by missed detection, i.e., p_{pm}^c, using Eq. (16). It can estimate its own type-II collision probability and calculate the margin left for type-I collision as: $\hat{\eta} = \eta - p_{pm}^c(j)$. SUs update their time threshold independently using (19) where η is replaced by $\hat{\eta}$.

If the SU transmits until $T^m(i)$ without colliding with the PU, the SU will only confirm the return of the PU after detecting N_f consecutive busy slots, where N_f is a design parameter. This reduces the mistakes that the SU loses the track of the idle-busy period of the PU, and initializes its timer t after falsely detect an busy-idle transition when the PU is still idle.

Simulations are used to validate the performance of the proposed algorithm. The simulation setup is the same as in the previous section. We consider two SUs with different spectrum sensing performances. The missed detection probabilities of SU1 and SUs are $P_m(1) = 1.3203 \times 10^{-8}$, $P_m(2) = 10^{-6}$, respectively. The false alarm probability is $P_f(1) = 0.67$ and $P_f(2) = 0.2$, respectively. The sensing time is $\sigma_s = 50\mu s$. The number of packets in a busy period of the PU follows a uniform distribution with mean $n_p = 100$. The design parameter N_f is heuristically set as $N_f = \frac{n_p \cdot \Delta}{10\sigma_s}$. Each updating period for the SU consists $W = 50$ idle-busy periods.

The performance of the proposed algorithm is shown in Figure 7. The convergence to T^* is fast. Each SU achieves close-to optimal throughput performance. The proposed algorithm can approximate the PU protection requirement well. It is slightly above the threshold after the second SU joins the network. This is due to the fact that the type-II collisions accumulates at the PU because each SU is oblivious to other SUs type-II collisions. To address this issue, one can tune the spectrum sensors to reduce the missed detection probability. (We intently leave the missed detection probability of the second SU high to illustrate this accumulative effect.) In addition, SUs should be aware the potential existence of other SUs and thus leave a margin on the collision probability constraint. Because of the geographical area constraint, the total number of SUs that are unaware of each other is indeed limited.

VII. Conclusion

We studied the fundamental limit on the throughput of cognitive SU networks based on the primary packet collision constraint. Under the perfect sensing assumption, we derived an optimal threshold-based transmission strategy for SUs under generic idle time distributions of the PU. We showed that the idle time distribution and the packet collision probability constraint of the PU have significant impact on the throughput performance. Both lower and upper bounds on the throughput performance of the SU are given. We analyzed

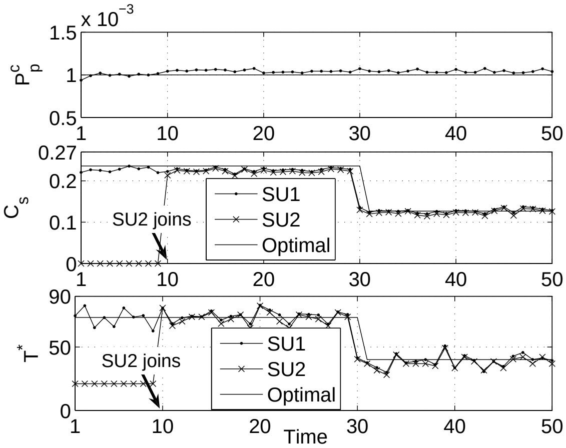


Fig. 7. Performance of the Proposed Adaptive Algorithm with Imperfect Sensing (Unit of x-axis: 5000 idle-busy periods)

the impact of imperfect sensing on the SU performance, and proposed a modified threshold-based spectrum access scheme that achieves close-to-optimal performance. We showed that it is critical to keep the missed detection probability (extremely) low and it is desirable to have a relatively long sensing time for good sensing performance. Moreover, we proposed a distributed scheme that adapts to the PU traffic pattern changes. The proposed scheme works well in the case of multiple secondary users, under both perfect sensing and imperfect sensing assumptions. Our results in this paper provide insights on the trade-off between the protection of the PU and throughput performance of the SU under various assumptions.

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