LETTER

Power Allocation for Amplify-and-Forward Opportunistic Relaying Systems*

Hui HUI[†,††a)], *Student Member*, Shihua ZHU[†], *Nonmember*, *and* Gangming LV[†], *Student Member*

SUMMARY In this letter, power allocation methods are devised for Amplify-and-Forward (AF) opportunistic relaying systems aiming at minimizing the outage probability. First, we extend the result on outage probability in [8] and develop an approximate expression to simplify the power allocation problem. A corresponding optimization problem is constructed and proved to be convex. Then an iterative numerical method is proposed to find the optimal power allocation factor. We also propose a near-optimal method which can directly calculate the power allocation factor to reduce computational complexity. Numerical results show that the proposed methods have a similar performance with the ideal one, and outperform equal power allocation significantly with little overhead.

key words: *cooperative communication, amplify-and-forward, power allocation, outage probability*

1. Introduction

Cooperative communication techniques, which have been proposed to exploit spatial diversity without the need for physical antenna arrays, are continuing to flourish due to their ability to combat fading in wireless environments [1]. Various cooperative protocols have been proposed and analyzed. In [2], several cooperative protocols were described such as amplify-and-forward (AF) and decode-and-forward (DF), which are widely used these days. Distributed space-time coded (DSTC) cooperative protocols have been proposed to improve the bandwidth efficiency for networks consisting of multiple relays [3], [4]. However, the techniques, which employ all potential relays to forward messages, may not be effective because the relays may not be helpful in the presence of deep fades. Besides, there are some extra issues need to be addressed, e.g., when the number of relays is large, the information exchange and the synchronization of the nodes become difficult.

Relay selection is one approach to solve the problem mentioned above. It was proved in [5] that with one "best" relay, AF protocol can still achieve full diversity order. Relay selection scenario studied in [6] obtains higher bandwidth efficiency for DF cooperative system with partial channel state information (CSI). Bletsas et al. proposed an opportunistic relaying system [7], [8], in which one "best" relay is selected and it was proved that opportunistic relaying can provide the same multiplexing and diversity tradeoff as DSTC.

In this paper, we address the power allocation problem for the AF opportunistic relaying system. We first extend the result on outage probability in [8] and develop an approximate expression. Then an optimal power allocation problem is constructed to minimize the outage probability under an aggregate power constrain with the knowledge of channel statistics. An iterative numerical method is proposed and we also propose a near-optimal method which can directly calculate the power allocation factor. Numerical results show that the two proposed methods significantly outperform the equal power allocation method and can achieve a similar outage performance with the ideal one.

The rest of this paper is organized as follows. Section 2 introduces the multi-node relay selection scenario. In Sect. 3, we derive an approximate expression of the outage performance and present two power allocation methods. Numerical results are discussed in Sect. 4. Finally, Sect. 5 concludes the paper.

2. System Model

We consider a wireless cooperative network in which a source communicates to a destination with the help of N potential relays employing AF. The channels between any two nodes are assumed to be random, independent, and remain constant during a transmission period. The channel coefficients from the source to the ith relay and from the ith relay to the destination are denoted by h_{si}, and h_{id} respectively for $i = 1, \cdots, N$, which are assumed to be zero-mean, circularly symmetric, complex Gaussian random variables with variances $E[|h_{si}|^2] = \Omega_{si}$ and $E[|h_{id}|^2] = \Omega_{id}$, respectively.

For any two nodes i and j, when i sends its message x with power P, the signal received at node j is

$$y_j = h_{ij}x + n_{ij}, \tag{1}$$

where $n_{ij} \sim \mathcal{CN}(0, N_0)$ is a zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension, which models the additive noise term, and $E[|x|^2] = P$.

We assume the system is subject to a total transmit power constraint P_t, and the source transmit power is $P_s = \zeta P_t$, where $\zeta \in (0, 1)$ is the power allocation factor. The aggregate relay power is $P_r = \sum_{i=1}^{N} P_i = (1-\zeta)P_t$, where P_i is the transmit power of relay i. Note that $P_i = 0$ if relay i

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[†]The authors are with the School of Electronics and Information Engineering, Xi'an Jiaotong University, Xi'an, China.
[††]The author is also with the School of Automation and Information Engineering, Xi'an University of Technology, Xi'an, China.

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a) E-mail: huihui@mail.xjtu.edu.cn

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does not forward the message. For opportunistic relaying, the "best" relay which minimizes outage probability is [8]

$$b^* = \arg\max_{i=1,\cdots,N} \frac{|h_{si}|^2\,|h_{id}|^2}{\frac{\zeta}{1-\zeta}\left(1+\frac{1}{\eta_{si}}\right)\Omega_{si} + |h_{id}|^2} \tag{2}$$

And the corresponding outage probability is [8]

$$P_{out} = \prod_{i=1}^{N}\left\{1 - \frac{1}{\Omega_{id}}\int_0^\infty \exp\left[-\frac{2^{2R}-1}{\eta_{si}}\left(1 + \frac{\zeta}{1-\zeta}\left(1+\frac{1}{\eta_{si}}\right)\frac{\Omega_{si}}{z}\right) - \frac{z}{\Omega_{id}}\right]dz\right\} \tag{3}$$

where R is the end-to-end spectral efficiency, and $\eta_{si} \triangleq \Omega_{si}P_s/N_0$ is the average received signal-to-noise ratio (SNR) of the ith relay.

3. Power Allocation Methods

In this section, power allocation methods are studied to minimize the outage probability under a total power constraint, which can be expressed as

$$\begin{aligned} &\min P_{out} \\ &s.t. P_s + P_r \le P_t. \end{aligned} \tag{4}$$

One can find from (3) that the expression of outage probability in [8] is too complicated to be used for the study of power allocation directly. Fortunately, it can be proved that the expression can be approximated by

$$P_{out} = \prod_{i=1}^{N}\left\{1 - \exp\left(-\frac{(2^{2R}-1)N_0}{\zeta P_t\Omega_{si}}\right) 2\sqrt{\frac{2^{2R}-1}{P_t\Omega_{id}}\frac{N_0}{1-\zeta}\left(1+\frac{N_0}{\zeta P_t\Omega_{si}}\right)} \cdot K_1\left(2\sqrt{\frac{2^{2R}-1}{P_t\Omega_{id}}\frac{N_0}{1-\zeta}\left(1+\frac{N_0}{\zeta P_t\Omega_{si}}\right)}\right)\right\} \tag{5}$$

where $K_\alpha(x)$ is the modified Bessel function of the second kind, and α denotes the order of the Bessel function. (See details of the proof in Appendix A.)

Furthermore, we have found an asymptotic function of $K_1(x)$, which is closer than the commonly used asymptotic function in small x ($K_\alpha(x) \approx (2/x)^\alpha\Gamma(\alpha)/2$ for $\alpha > 0, 0 < x \le \sqrt{\alpha+1}$ [9]).

$$K_1(x) \approx e^{-x^2}/x. \tag{6}$$

Let $SNR \triangleq P_t/N_0$, since $1 - e^{-x} \approx x$ for small x, when SNR is large and using (6), we can obtain

$$P_{out} \approx \left(\frac{2^{2R}-1}{SNR}\right)^N \prod_{i=1}^{N}\left\{\frac{1}{\zeta}\left(\frac{1}{\Omega_{si}} + \frac{4}{\Omega_{si}\Omega_{id}SNR}\right) + \frac{4}{(1-\zeta)}\left(\frac{1}{\Omega_{id}} + \frac{1}{\Omega_{si}\Omega_{id}SNR}\right)\right\}. \tag{7}$$

Letting

$$f(\zeta) = \prod_{i=1}^{N}\left\{\frac{1}{\zeta}\left(\frac{1}{\Omega_{si}} + \frac{4}{\Omega_{si}\Omega_{id}SNR}\right) + \frac{4}{1-\zeta}\left(\frac{1}{\Omega_{id}} + \frac{1}{\Omega_{si}\Omega_{id}SNR}\right)\right\}, \tag{8}$$

the optimization problem can be expressed as

$$\zeta^* = \arg\min_{\zeta\in(0,1)} f(\zeta). \tag{9}$$

Before investigating this problem, we have the following observations from above:

1) When $\Omega_{si} \ll \Omega_{id}$, which implies that the relay is much closer to the destination, $\zeta^* \to 1$. It is intuitionally reasonable since when the relay is much closer to the destination, the source should consume more power to reach the relay, and only a small amount of power is needed for the relay to reach the destination.

2) When $\Omega_{si} \gg \Omega_{id}$, $\zeta^* \to 0$, which implies that when the relay is much closer to the source, most of the available power should be allocate to the relay.

3) When SNR is large enough, (9) can be written as

$$\zeta^* \approx \arg\min_{\zeta\in(0,1)} \prod_{i=1}^{N}\left\{\frac{1}{\zeta\Omega_{si}} + \frac{4}{(1-\zeta)\Omega_{id}}\right\}, \tag{10}$$

from which one can find that in high SNR regime, SNR plays a minor role in power allocation and the optimal power allocation factor will only depend on Ω_{si} and Ω_{id} for $i = 1, 2, ..., N$, which are the average power gains of the source-relay and relay-destination channels, respectively.

3.1 Proposed Method I

In the following, we will focus on obtaining the optimal power allocation factor. The stationary point of $f(\zeta)$ must satisfy

$$f'(\zeta) = \sum_{i=1}^{N}\left\{\left[-\frac{1}{\zeta^2\Omega_{si}}\left(1+\frac{4}{\Omega_{id}SNR}\right) + \frac{4}{(1-\zeta)^2\Omega_{id}}\left(1+\frac{1}{\Omega_{si}SNR}\right)\right] \cdot \prod_{\substack{j=1\\j\neq i}}^{N}\left[\frac{1}{\zeta\Omega_{sj}}\left(1+\frac{4}{\Omega_{jd}SNR}\right) + \frac{4}{(1-\zeta)\Omega_{jd}}\left(1+\frac{1}{\Omega_{sj}SNR}\right)\right]\right\} = 0. \tag{11}$$

It is hard to get the close-form solution of (11), therefore in this subsection we propose a numerical method to find the optimal power allocation factor.

Theorem 1: The problem of (9) is a convex optimization problem and the objective function $f(\zeta)$ is a strictly convex function with the domain restriction $0 < \zeta < 1$. Proof: see in Appendix B.

Therefore, the convex optimization problem (9) has only one global optimum solution. To obtain the optimal power allocation factor ζ^*, we propose an iterative method by the following steps.

Let ζ_n denote the value of ζ in the nth iteration for $n = 0, 1, 2, \cdots$, and ε is the threshold to stop the iteration. Initialization: $\zeta_0 = 0.5$, $\varepsilon = 10^{-4}$.

Step 1: If $|f'(\zeta_0)| < \varepsilon$, stop the iteration process, and the result is $\zeta^* = \zeta_0$. Otherwise go to Step 2;

Step 2: Compute $\zeta_{n+1} = \zeta_n + \Delta\zeta_n$, where $\Delta\zeta_n = -f'(\zeta_n)\lambda_n$. Here $-f'(\zeta_n)$ is the descent direction and λ_n denotes the step size which guarantees $0 < \zeta_{n+1} < 1$ and $f(\zeta_{n+1}) - f(\zeta_n) < 0$;

Step 3: The iteration process is stopped if $|f(\zeta_{n+1}) - f(\zeta_n)| < \varepsilon$ or $|\Delta\zeta_n| < \varepsilon$ or $|f'(\zeta_{n+1})| < \varepsilon$, and the output is $\zeta^* = \zeta_{n+1}$. Otherwise, go to Step 2.

3.2 Proposed Method II

Considering that the none zero value of the consecutive multiplication item in (11) is similar for different relays, (11) can be approximated to

$$\sum_{i=1}^{N}\left[\frac{4}{(1-\zeta)^2\Omega_{id}}\left(1+\frac{1}{\Omega_{si}SNR}\right)-\frac{1}{\zeta^2\Omega_{si}}\left(1+\frac{4}{\Omega_{id}SNR}\right)\right]=0 \tag{12}$$

Thus we obtain a near optimal power allocation method, which can directly calculate the power allocation factor

$$\zeta^* = \frac{\sqrt{\sum_{i=1}^{N}\frac{1}{\Omega_{si}}\left(1+\frac{4}{\Omega_{id}SNR}\right)}}{\sqrt{\sum_{i=1}^{N}\frac{1}{\Omega_{si}}\left(1+\frac{4}{\Omega_{id}SNR}\right)}+\sqrt{\sum_{i=1}^{N}\frac{4}{\Omega_{id}}\left(1+\frac{1}{\Omega_{si}SNR}\right)}}. \tag{13}$$

4. Numerical Results

In this section, the outage performances of the proposed power allocation methods are presented. It is assumed that there are $N = 6$ relays distributed in a 1×1 square area, the source and the destination are located at (0, 0) and (1, 0), respectively. The average channel gain $\Omega_{ij} = cd_{ij}^{-\alpha}$, where d_{ij} is the distance between node i and j, c is a constant, and α is the path loss exponent. In the simulation, without loss of generality, we use $\alpha = 3$ and $c = d_{sd}^{\alpha}$ so that Ω_{sd} is normalized to 1. The noise variances on all communication links are assumed to be $N_0 = 1$, and $R = 1$ bps/Hz.

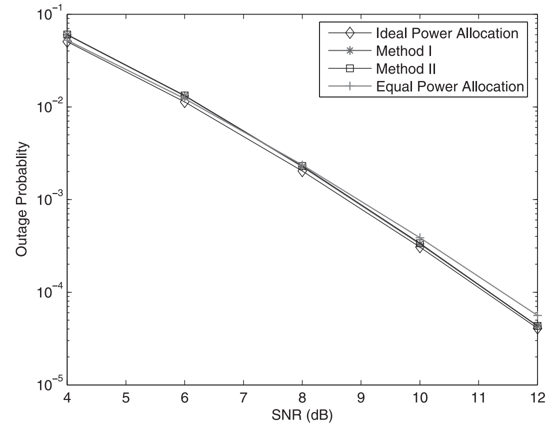

Fig. 1 Outage probability vs. SNR for different power allocation methods (Relays are distributed at the center area).

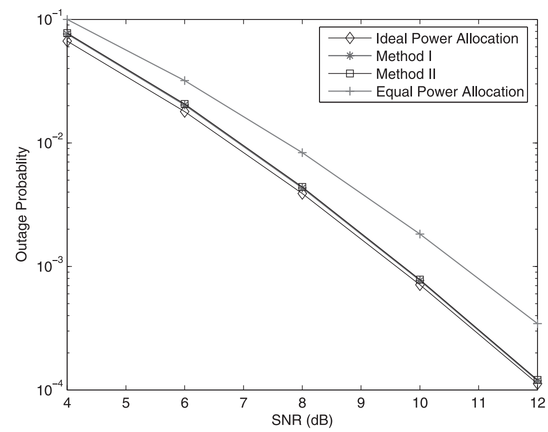

Fig. 2 Outage probability vs. SNR for different power allocation methods (Relays are distributed near the source).

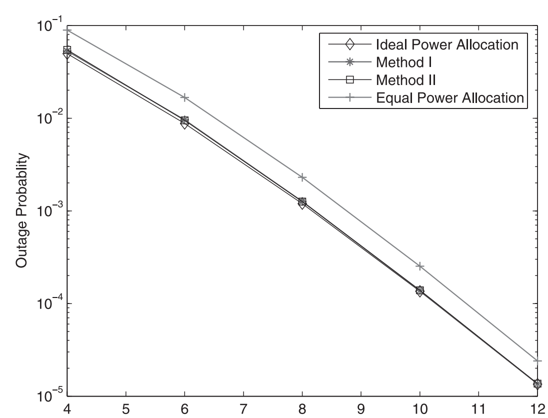

Fig. 3 Outage probability vs. SNR for different power allocation methods (Relays are distributed near the destination).

Figure 1 shows the simulation results when relays are randomly distributed at the center, which is $[0.4, 0.6] \times [-0.1, 0.1]$. We also provide the results of ideal and equal power allocation for comparison. The ideal one is derived by exhaustive search of ζ^* in (3), and the equal power allocation method evenly allocates the aggregate power to the source and the "best" relay as depicted in [8]. It can be found that these power allocation methods have a similar outage performance, which means that $\zeta^* = 0.5$ is a reasonable choice in this case.

Figures 2 and 3 compares the similar scenarios except that relays are randomly distributed near the source or destination, which is $[0.2, 0.4] \times [-0.1, 0.1]$ or $[0.6, 0.8] \times [-0.1, 0.1]$, respectively. It is revealed that in the two situations, both the proposed methods have a similar outage performance with the ideal one. When relays near the source, they outperform the equal power allocation method by about 1 dB, and the performance gain is about 0.5 dB when relays

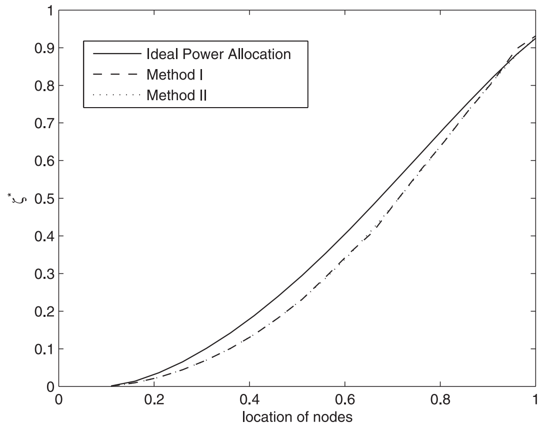

Fig. 4 Changing of power allocation factor.

are close to the destination.

To address the influence of the approximation we have made in developing our methods, Fig. 4 shows the changing of ζ^* with the location of nodes when $SNR = 10\,\mathrm{dB}$. It is assumed that the relays are located at the same place of the line connecting the source and destination. The abscissa denotes the distance between the source and the relays. One can see that ζ^* is changing from 0 to 1 as predicted in Sect. 3. The proposed two methods have almost the same value of ζ^*, and it is close to that of the ideal power allocation factor.

As one can see from the simulation results, when ε is set to 10^{-4}, the iterative and the simplified method have a similar outage performance, which is significantly better than that of the equal power allocation method. Since the accuracy of the iterative algorithm is determined by ε, in practice one can adjust ε so as to obtain a more accurate ζ^* than the simplified method at a price of computational complexity. However, in those scenarios where the reduction of computational complexity needs to be considered with high priority, the simplified method is more applicable.

5. Conclusions

In this paper, we studied the power allocation methods for AF opportunistic relaying systems. We first derived an approximate expression of the outage probability, based on which the power allocation problem is simplified into a convex optimization problem. Then we proposed an iterative numerical method to find the optimal power allocation factor. We also proposed a method which can directly calculate the power allocation factor thus reducing the complexity. Numerical results showed that the proposed methods can achieve a similar performance with the ideal one and outperform the equal power allocation method significantly.

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Appendix A: Proof of Eq. (5)

Substituting $a_i = \frac{2^{2R}-1}{\eta_{si}}$, $b_i = a_i\frac{\zeta}{1-\zeta}(1+\frac{1}{\eta_{si}})\Omega_{si}$, $c_i = \Omega_{id}$ into (3), and letting $z = e^t\sqrt{b_i c_i}$, we obtain

$$
\begin{aligned}
&\int_0^\infty e^{-\sqrt{b_i/c_i}(\sqrt{b_i c_i}/z + z/\sqrt{b_i c_i})}dz \\
&= \int_{-\infty}^{\infty} e^{-\sqrt{b_i/c_i}(e^{-t}+e^t)}(e^t\sqrt{b_i c_i})dt \\
&= \sqrt{b_i c_i}\left(\int_0^\infty e^{-2\sqrt{b_i/c_i}(\frac{e^{-t}+e^t}{2})}e^t dt + \int_{-\infty}^0 e^{-2\sqrt{b_i/c_i}(\frac{e^{-t}+e^t}{2})}e^t dt\right) \\
&= \sqrt{b_i c_i}\left(\int_0^\infty e^{-2\sqrt{b_i/c_i}(\frac{e^{-t}+e^t}{2})}e^t dt + \int_0^\infty e^{-2\sqrt{b_i/c_i}(\frac{e^{t}+e^{-t}}{2})}e^{-t} dt\right) \\
&= 2\sqrt{b_i c_i}\int_0^\infty e^{-2\sqrt{b_i/c_i}(\frac{e^{-t}+e^t}{2})}\frac{e^t+e^{-t}}{2}dt \\
&= 2\sqrt{b_i c_i}\int_0^\infty e^{-2\sqrt{b_i/c_i}\cosh(t)}\cosh(t)dt \\
&= 2\sqrt{b_i c_i}K_1(2\sqrt{b_i/c_i}) \qquad \text{(A· 1)}
\end{aligned}
$$

where the last step stems from [10, Eq. (9.6.24)]. Re-substituting a, b, c in (A· 1) will result in (5).

Appendix B: Proof of Theorem 1

It can be readily proved that the strict convexity of $\ln f(\zeta)$ is the sufficient condition for the strict convexity of $f(\zeta)$. Thus we let $g(\zeta) = \ln f(\zeta) = \sum_{i=1}^{N} g_i(\zeta)$, where

$$g_i(\zeta) = \ln\left[\frac{1}{\zeta\Omega_{si}}\left(1+\frac{4}{\Omega_{id}SNR}\right)+\frac{4}{(1-\zeta)\Omega_{id}}\left(1+\frac{1}{\Omega_{id}SNR}\right)\right] \tag{A$\cdot$ 2}$$

The second derivative of $g_i(\zeta)$ can be expressed by

$$g_i''(\zeta)=\left[\frac{1}{\zeta}\left(\frac{1}{\Omega_{si}}+\frac{4}{\Omega_{si}\Omega_{id}SNR}\right)+\frac{4}{1-\zeta}\left(\frac{1}{\Omega_{id}}+\frac{1}{\Omega_{si}\Omega_{id}SNR}\right)\right]^{-2} \left\{\left[-\frac{1}{\zeta^2}\left(\frac{1}{\Omega_{si}}+\frac{4}{\Omega_{si}\Omega_{id}SNR}\right)+\frac{4}{(1-\zeta)^2}\left(\frac{1}{\Omega_{id}}+\frac{1}{\Omega_{si}\Omega_{id}SNR}\right)\right]^2 + \frac{8}{\zeta(1-\zeta)}\left(\frac{1}{\Omega_{si}}+\frac{4}{\Omega_{si}\Omega_{id}SNR}\right)\left(\frac{1}{\Omega_{id}}+\frac{1}{\Omega_{si}\Omega_{id}SNR}\right)\left(\frac{1}{\zeta}+\frac{1}{1-\zeta}\right)^2\right\} \tag{A$\cdot$ 3}$$

It can be found that for any i and $\zeta \in (0,1)$, $g_i''(\zeta) > 0$. Therefore $g_i(\zeta)$ is a strictly convex function in $\zeta \in (0,1)$. Thus $g(\zeta)$ is also a strictly convex function in $\zeta \in (0,1)$, which implies $f(\zeta)$ is a strictly convex function.