Relay Selection and Power Allocation in Cooperative Cellular Networks

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Abstract

We consider a system with a single base station communicating with multiple users over orthogonal channels while being assisted by multiple relays. Several recent works have suggested that, in such a scenario, selection, i.e., a single relay helping the source, is the best relaying option in terms of the resulting complexity and overhead. However, in a multiuser setting, optimal relay assignment is a combinatorial problem. In this paper, we formulate a related convex optimization problem that provides an extremely tight upper bound on performance and show that selection is, almost always, inherent in the solution. We also provide a heuristic to find a close-to-optimal relay assignment and power allocation across users supported by a single relay. Simulation results using realistic channel models demonstrate the efficacy of the proposed schemes, but also raise the question as to whether the gains from relaying are worth the additional costs.

I. Introduction

In distributed wireless systems wherein each node possesses only a single antenna, relays can be used to provide spatial diversity and combat the impact of fading. Relaying has been an extremely active research area, especially since Sendonaris et al., in [1], proposed the idea of user cooperation wherein mobile users cooperate by relaying each others' data. Many cooperation schemes have now been studied, e.g., [1]–[4]. The work in [2] and [3] proposed repetition-based cooperation schemes including fixed amplify-and-forward (AF) and decode-and-forward (DF) using orthogonal channels (time/frequency slots). In networks with multiple relays, the traditional strategy has been to let all the relays forward their messages to the destination. However, having relays transmit on orthogonal bands is bandwidth inefficient. A proposed alternative is to use distributed space-time codes (DSTC) [3]; however, this requires symbol level synchronization, which is difficult to implement over a distributed network. It has recently been shown that most of the benefits of cooperative diversity can be achieved with minimum overhead if a single 'best' relay cooperates with the source. This scheme is referred to as selection cooperation [5], [6] and has now been investigated in various contexts [5]–[9].

In the case of a single source-destination pair, choosing the best relay is fairly straightforward and solved for both DF [5], [6] and AF [7] relaying. In both cases, the best relay is the one that contributes the most to the output signal-to-noise ratio (SNR). The selection gets significantly more complicated in the more practical case of multiple information flows [6]. Because a relay must now divide its available power between all flows it supports, a relay that is best for a single flow may not remain the best overall and relay selection becomes a combinatorial problem. In [6], the authors present ad hoc approaches to approximate the optimal solution with limited complexity, without addressing resource allocation.

In relay networks, an independent research theme is that of resource allocation, including power allocation, e.g., [10], [11] amongst many. Optimal allocation makes best use of the limited available power resources. However, to our knowledge there has not been any work jointly considering relay assignment and power allocation in the context of multiple source, multi-relay, especially cellular, networks.

Our system model comprises a single base station communicating to multiple users being assisted by a few dedicated relays. The users are to be assigned to the relays. The relays have limited power which must be divided among the users they support. Relaying in the context of a cellular wireless network has received limited attention [12]. In Section III, we develop an optimization problem for optimal relay assignment and power allocation at the relays. We try to answer the question, *what relay assignment and power allocation scheme maximizes the sum rate and what scheme maximizes the minimum rate to all the users*? Obtaining solutions to these requires exponential complexity.

The main theoretical contribution of this paper is in Section III, where we derive upper bounds to the rates and show that these bounds form a convex optimization problem for both figures of merit. We use the resulting Karush-Kuhn-Tucker (KKT) conditions to illustrate why the bound is tight and then derive a simplified, tight, lower bound. In Section IV, we simulate a cellular network, using the COST-231 model, to study the performance gains in a relay assisted network over a traditional single base station system. Interestingly, while the gains are significant, the results leave open the question of whether these gains adequately compensate for the additional infrastructure costs of a relay-assisted cellular system.

In terms of the available literature, our formulation is similar to tone assignment in orthogonal frequency division multiple access (OFDMA) systems. This is the problem of assigning users to tones to maximize a certain metric, subject to the constraint that no tone is assigned to two different users. In [13], Wong et al. solve the problem of minimizing the total transmission power in a multi-user OFDM system. The frequency spectrum is divided into discrete frequency bins and a convex relaxation technique is used to solve the discrete bin-assignment problem. Rhee and Cioffi [14] solve the problem of assigning users to tones on the downlink of a OFDMA based communication system with the objective of maximizing the minimum rate to users.

While our approach is similar to that taken in these papers; the roles of ‘user’ and ‘tone’ in these papers is played, respectively, by ‘relay’ and ‘user’ here. However, there are some significant differences, most importantly in the power allocation step. There a single power allocation is required across all tones, here each relay must meet a power constraint Furthermore, our problem formulation allows us to analyze the conditions under which the bounds are tight unlike the other works, wherein the authors state that the bound gets tight as the number of tones approaches infinity, but prove this for only the two user case.

This paper is organized as follows. In Section II, we describe the system model in some detail. In Section III, we then formulate the optimization problem and the upper bound to each of the two rates and illustrate why the bounds are tight. In Section IV, we illustrate this through simulations and use the bound to analyze the performance of relay assisted cellular networks. The paper wraps up with some conclusions in Section V.

II. System Model

Our system model consists of a cellular network with a single BS, communicating with K users, and assisted by J relays, as shown in Figure 1. Each of the users is assigned an orthogonal channel, over which the BS-to-user and the relay-to-user communications take place. The users are frequency division multiplexed, although the results here also apply to the case of time division multiplexing. The relays in the system are fixed wireless terminals, installed solely to aid the BS-user communication. The relays use the DF protocol with the same codebook as the transmitter.

The communication between the BS and a user happens over two time slots. In the first time slot the BS transmits, while the relays and the user try to decode the message. In the second time slot, one of the relays, chosen *a priori*, re-encodes and then transmits the information it has decoded in the first time slot. The user uses the messages received in the two time slots to decode the transmitted information.

Suppose that user k (denoted as d_k) is allotted to relay-j (r_j). For a system as described above, the maximum rate at which the BS can communicate with the receiver with the help of the relay is [6]

$$I_{d_k} = \min\left(I_{sr_j}, I_{sr_jd_k}\right), \tag{1}$$

$$I_{sr_j} = \frac{1}{2}\log_2\left(1+\mathsf{SNR}_s|h_{sr_j}|^2\right), \tag{2}$$

$$I_{sr_jd_k} = \frac{1}{2}\log_2\left(1+\mathsf{SNR}_s|h_{sd_k}|^2+\mathsf{SNR}_r\alpha_{jk}|h_{r_jd_k}|^2\right), \tag{3}$$

where, SNR_s and SNR_r are, respectively, the ratios of the transmit power at the BS (denoted as s) and relay to the noise power at the receiver. h_{sr_j} is the channel between the BS and relay j, denoted by r_j, similarly $h_{r_jd_k}$ is the channel between relay r_j and destination d_k. Finally, α_{jk} is the fraction of the total relay power used to communicate with user k. The factor of 1/2 accounts for the fact that the BS-user communication happens over two time slots. I_{sr_j} is the rate at which the source can communicate with relay-j while $I_{sr_jd_k}$ is the maximum rate at which the source can communicate to user k with the help of the relay. Equation (1) ensures that both the relay and the user can decode the message.

The channels between the BS, relays and users are modeled using the COST-231 model as recommended by the IEEE 802.16j working group [15]. The model includes the path loss, large-scale fading (a log-normal variable) and small-scale fading modeled as Rician random variable for line-of-sight (LoS) communication and Rayleigh random variable for non-LoS communication. When the BS and relays are both placed at some height above the ground, the fading has a LoS component. The existence of this component is crucial since it suggests that all relays will be able to decode a source codeword; hence the factor limiting the overall rate is the second term of Eqn. (1), $I_{sr_jd_k}$, significantly simplifying the problem at hand.

III. Problem formulation and solution

As described in the previous section, every user is assigned one of the J relays. This paper deals with optimizing this assignment to maximize two metrics of interest, the sum rate to all the users and the minimum of all the rates. In maximizing the sum rate (equivalently the average rate), the objective function is

$$\sum_{k=1}^{K} I_{d_k} = \sum_{k=1}^{K} \min\left(I_{sr(d_k)}, I_{sr(d_k)d_k}\right), \tag{4}$$

while in maximizing the minimum rate, the objective function is given by

$$\min_{k}\left\{I_{d_k}\right\} = \min_{k}\left\{\min\left(I_{sr(d_k)}, I_{sr(d_k)d_k}\right)\right\}, \quad k = 1, \ldots, K, \tag{5}$$

where, in both cases, $r(d_k)$ is the relay assigned to user k.

In practice, the number of users, K, will be much larger than the number of relays, J. Hence, a single relay will likely be required to support multiple users, and to meet its power constraint it must divide its power amongst these users. Thus, our objective is now two fold, one, finding the relay assignment scheme, and two, once the assignment is done, distributing powers at each of the relays amongst the users it supports.

To formulate a tractable problem, in this paper we investigate simplified versions of the above problems.

As mentioned earlier, in a cellular network, the data rate bottleneck is the compound source-relay-destination channel, the second term in Eqn. (1). We assume that

$$I_{sr_j} > I_{sr_j d_k} \quad \forall j, k, \tag{6}$$

and hence $\min\left(I_{sr(d_k)}, I_{sr(d_k)d_k}\right) = I_{sr(d_k)d_k}$.

In Section IV, we justify the validity of this assumption. Note that in spite of the assumption, the solution is not immediate. The fact that the relays divide their power amongst the users they support, makes the relay assignment an integer programming problem with the attendant exponential complexity.

A. Max sum rate

The sum rate measures the maximum throughput delivered by the base station. For the sake of brevity, let c_k represent $\mathsf{SNR}_s|h_{sd_k}|^2$ and p_{jk} represent $\mathsf{SNR}_r|h_{r_j d_k}|^2,\ j = 1, 2, \ldots, J$. Let α_{jk} be the fraction of the power of relay-j, used to communicate to user k. The optimization problem maximizes the sum rate to all the users subject to two constraints: only a single relay helps each user and each relay must meet a power constraint. The formal optimization problem is, therefore,

$$\max_{\{\alpha_{jk}\}} R \quad = \max_{\{\alpha_{jk}\}} \sum_{k=1}^{K} \frac{1}{2} \log_2\left(1 + c_k + \sum_{j=1}^{J} p_{jk}\alpha_{jk}\right), \tag{7}$$

$$\text{such that} \quad \forall k, \quad \alpha_{jk}\alpha_{lk} = 0, \qquad j \neq l, j, l \in \{1, 2, \ldots, J\}, \tag{8}$$

$$\sum_{k=1}^{K} \alpha_{jk} = 1 \qquad \forall \ j, \tag{9}$$

$$\alpha_{jk} \geq 0, \tag{10}$$

where the objective function assumes the relay uses the same codebook as the source. Equation (8) enforces the selection rule allowing only one α_{jk} term to be non-zero for all relays. The remaining two constraints force the power allocated to be positive and to meet a power constraint. The constraint in Eqn. (9) can also be written as an inequality constraint, $\sum_{k=1}^{K} \alpha_{jk} \leq 1, \forall j$. The solution to the optimization problem in either case is the same because the objective function is an increasing function of the powers, α_{jk}s. We cannot use the usual gradient based methods to maximize the objective function in Eqn. (7). Note that an inherent assumption is that the BS has knowledge of the parameters that define the problem. How this information is conveyed to the BS is beyond the scope of this paper.

The solution to the optimization problem in Eqns. (7)-(10) is complicated by the constraint in Eqn. (8).

An exhaustive search to find the solution would involve the following: for a given relay assignment, solving J water-filling problems corresponding to the power allocation at each of the relays. We need do this for every relay assignment and find the maximum of them. Each of the users can be assigned to any of the relays, hence, all J^K possible relay assignments must be tested. Doing so is impossible for realistic values of J and K. We therefore explore tractable approximate formulations.

The objective function of the optimization problem in Eqns. (7)-(10) is concave and the constraints, other than the one in (8), are affine. Our strategy to solve the optimization problem in hand is to ignore the constraints in Eqn. (8) and maximize the objective function subject to the power constraints alone:

$$\min_{\{\alpha_{jk}\}} -\sum_{k=1}^{K} \frac{1}{2} \log_2 \left(1 + c_k + \sum_{j=1}^{J} p_{jk}\alpha_{jk} \right), \tag{11}$$

$$\text{such that} \quad \sum_{k=1}^{K} \alpha_{jk} - 1 = 0 \qquad \forall \;\; j, \tag{12}$$

$$-\alpha_{jk} \leq 0. \tag{13}$$

Since we ignore a constraint, the solution so obtained will be an upper bound to the maximum sum rate achieved by selection. Note that since this minimization problem is now convex, solving this simplified problem is fairly straightforward, e.g., using interior point methods [16]. The computational complexity involved in solving the optimization problem is polynomial in K and J, and the problem is, hence, tractable for practical values of K and J. We use CVX, a package for specifying and solving convex programs [17], [18].

We now proceed to show that although we did not impose the selection rule explicitly, the solution to the optimization problem has the property that, for most k, $\alpha_{jk}\alpha_{lk} = 0, j \neq l, j, l \in \{1, 2, \ldots, J\}$. This means, when the power is optimally allocated, most users receive power from only *one* of the relays.

Tightness of Bound: The objective and the constraint functions are differentiable and the constraint conditions satisfy Slater's condition [16]. To show this, consider one possible choice for the power vectors, $\alpha_{jk} = 1/K$. This meets the sum power constraint with equality and the constraint on positive power with strict inequality. For a convex optimization problem with differentiable objective and constraint functions, which also satisfy Slater's condition, the solution to the optimization problem satisfies the KKT conditions [16].

Let us characterize the set of solutions to the optimization problem. For the sake of clarity, we start with the case with two relays. In such a case, the Lagrangian of the minimization problem is given by

$$\begin{aligned}\mathcal{L}(\{\alpha_{1k},\alpha_{2k}\};\{\lambda_k^1\},\{\lambda_k^2\},\nu_1,\nu_2) &= -R-\sum_{k=1}^{K}\lambda_k^1\alpha_{1k}-\sum_{k=1}^{K}\lambda_k^2\alpha_{2k}\\ &\quad +\nu_1\left(\sum_{k=1}^{K}\alpha_{1k}-1\right)+\nu_2\left(\sum_{k=1}^{K}\alpha_{2k}-1\right),\end{aligned}\tag{14}$$

where λ_k^1 and $\lambda_k^2, k=1,2,\ldots,K$ are the Lagrange multipliers associated with the constraint on positive power, and ν_1 and ν_2 are the Lagrange multipliers associated with the constraint on the total power at the two relays. Any solution to the optimization problem satisfies the KKT conditions, which are,

$$\frac{p_{1k}}{1+c_k+\sum_{i=1}^{2}p_{ik}\alpha_{ik}}+\lambda_k^1=\nu_1,\quad \lambda_k^1\alpha_{1k}=0,\quad \lambda_k^1\geq 0,\tag{15}$$

$$\frac{p_{2k}}{1+c_k+\sum_{i=1}^{2}p_{ik}\alpha_{ik}}+\lambda_k^2=\nu_2,\quad \lambda_k^2\alpha_{2k}=0,\quad \lambda_k^2\geq 0.\tag{16}$$

Now suppose for some $i\in\{1,2,\ldots,K\}$, α_{1i} and α_{2i} are both non-zero, then the conditions $\lambda_i^1\alpha_{1i}=0$ and $\lambda_i^2\alpha_{2i}=0$ dictate that λ_i^1 and λ_i^2 are both zero. From the KKT conditions it follows that

$$\frac{\nu_1}{p_{1i}}=\frac{\nu_2}{p_{2i}}.\tag{17}$$

Similarly if α_{1j} and α_{2j} are both non-zero for some $j\in\{1,2,\ldots,K\}$, then

$$\frac{\nu_1}{p_{1j}}=\frac{\nu_2}{p_{2j}}.\tag{18}$$

Unless $p_{1i}/p_{2i}=p_{1j}/p_{2j}$, Eqns. (17) and (18) cannot simultaneously be true. In the current problem, p_{jk} represent the powers of the channels between the relays and the users. If they are independent continuous random variables, as is the case with the wireless channels, then the probability that $p_{1i}/p_{2i}=p_{1j}/p_{2j}$ is zero. Hence, *when the power is optimally allocated,* at most *one of the K (α_{1k},α_{2k}) pairs has two non-zero entries* and $K-1$ of the pairs have at most one non-zero entry. This indicates that the selection rule, $(\alpha_{1k}\alpha_{2k}=0,\forall k)$, which we did not explicitly impose, is true for at least all but one of the K users. Hence, the solution obtained by ignoring Eqn. (8) comes quite close to the solution to the original optimization problem in Eqns. (7)-(10).

For the case of three relays, the KKT conditions are:

$$\frac{p_{1k}}{1+c_k+\sum_{i=1}^{3} p_{ik}\alpha_{ik}} + \lambda_k^1 = \nu_1, \quad \lambda_k^1 \alpha_{1k} = 0, \, \lambda_k^1 \geq 0, \tag{19}$$

$$\frac{p_{2k}}{1+c_k+\sum_{i=1}^{3} p_{ik}\alpha_{ik}} + \lambda_k^2 = \nu_2, \quad \lambda_k^2 \alpha_{2k} = 0, \, \lambda_k^2 \geq 0, \tag{20}$$

$$\frac{p_{3k}}{1+c_k+\sum_{i=1}^{3} p_{ik}\alpha_{ik}} + \lambda_k^3 = \nu_3, \quad \lambda_k^3 \alpha_{3k} = 0, \, \lambda_k^3 \geq 0, \tag{21}$$

where, for $k \in \{1, 2, \ldots, K\}$, λ_k^1, λ_k^2 and λ_k^3 are the Lagrangian multipliers associated with the constraint on positive power, and ν_1, ν_2 and ν_3 are the Lagrangian multipliers associated with the constraint on total power. In the solution to the optimization problem, we wish to find the maximum number of triplets (α_{1i}, α_{2i}, α_{3i}), in which more than one entry is non-zero. We do this by analyzing different possibilities for the solution. Suppose that in the solution, for some i, (α_{1i}, α_{2i}, α_{3i}) are all non-zero (user i receives power from all relays), then,

$$\frac{\nu_1}{p_{1i}} = \frac{\nu_2}{p_{2i}} = \frac{\nu_3}{p_{3i}}. \tag{22}$$

Now, for some j, if α_{1j} and α_{2j} are non-zero, then, along with Eqn. (22), this would imply that $p_{1i}/p_{2i} = p_{1j}/p_{2j}$, which occurs with probability zero. Hence, if the solution to the optimization problem has one triplet with all non-zero entries, then all other triplets can have only one non-zero entry, i.e., selection is imposed on all other users.

Now suppose that in the solution, for no i, (α_{1i}, α_{2i}, α_{3i}) are all non-zero. Without loss of generality, suppose for some j, α_{1j} and α_{2j} are non-zero, and for some k, α_{2k} and α_{3k} are non-zero, then,

$$\frac{\nu_1}{p_{1j}} = \frac{\nu_2}{p_{2j}}, \qquad \frac{\nu_2}{p_{2k}} = \frac{\nu_3}{p_{3k}}. \tag{23}$$

These two equations imply that in all other three-tuples $(\alpha_{1k}, \alpha_{2k}, \alpha_{3k})$, only one of the entries is non-zero. This is because, if for some l, α_{1l} and α_{3l} are non-zero, then, (23) would imply, $p_{1l}/p_{3l} = p_{2j}p_{1i}/p_{3j}p_{2i}$, which occurs with probability zero. Hence, for the case of three relays, at most two of triplets can have more than one non-zero entry. Like with the case of two relays, when the power is allocated optimally, the selection rule is followed in most of the triplets.

Generalizing this to J relays, when the power is allocated optimally, at most $J-1$ of the J-tuples $(\alpha_{1k}, \alpha_{2k}, \ldots, \alpha_{Jk})$ can have more than one non-zero entry. This indicates that if $K \gg J-1$, as expected in practice, a large fraction of the users are guaranteed to receive power from only one relay.

To summarize, we have shown that the power allocation matrix, $[\alpha_{jk}]_{J\times K}$ is *sparse*. Most of the rows of the matrix have only one non-zero entry. If all the rows of the matrix had at most a single non-zero entry, then we would have obtained the solution to the optimization problem given by Eqns. (7)-(10). A simple heuristic to find that solution, then, is to explicitly impose selection: assign users receiving power from multiple relays to the relays that allot the maximum power.

$$r(d_k) = r_m \quad \text{if} \quad m = \arg\max_j \{\alpha_{jk} p_{jk}\} \tag{24}$$

If there are multiple relays which allot the same maximum power, assign the user to any one of them arbitrarily. Once this relay assignment is done, J water-filling problems can be solved for the power distribution at each of the relays. However, we can also re-use the power allocation vector derived from the earlier step. Construct the matrix $[\alpha^{'}_{jk}]_{J\times K}$ as follows: for each $k \in \{1, 2, \ldots K\}$,

$$\alpha^{'}_{mk} = \alpha_{mk} \quad \alpha^{'}_{jk} = 0 \quad \forall j \neq m. \tag{25}$$

The matrix of the power allocation vectors $[\alpha^{'}_{jk}]_{J\times K}$ meet the constraints given by Eqns. (8) and (10) and satisfy $\sum_{k=1}^{K} \alpha^{'}_{jk} \leq 1, \forall j$. It is hence a lower bound to the solution to the optimization problem given by Eqns. (7)-(10). We avoid a second round of optimization because, as we shall see, the upper and lower bounds are already indistinguishable.

B. Max sum rate with a minimum rate constraint

Maximizing the sum rate does not ensure any fairness with respect to the distribution of power. In a cellular network, a more practical metric might be maximizing the sum rate while guaranteeing a minimum rate to each user. Formally, the optimization problem is,

$$\max_{\{\alpha_{jk}\}} R \;=\; \min_{\{\alpha_{jk}\}} -\sum_{k=1}^{K} \frac{1}{2}\log_2\left(1 + c_k + \sum_{j=1}^{J} p_{jk}\alpha_{jk}\right), \tag{26}$$

$$\text{such that} \quad \forall k, \quad \alpha_{jk}\alpha_{lk} = 0,\; j \neq l, j, l \in \{1, 2, \ldots, J\} \tag{27}$$

$$\sum_{k=1}^{K} \alpha_{jk} - 1 = 0 \qquad \forall \;\; j, \tag{28}$$

$$-\alpha_{jk} \leq 0, \tag{29}$$

$$R_k - \frac{1}{2}\log_2\left(1 + c_k + \sum_{j=1}^{J} p_{jk}\alpha_{jk}\right) \leq 0, \tag{30}$$

where R_k is the rate guaranteed to user k. Suppose we ignore the constraint given in Eqn. (27), the Lagrangian of the resulting optimization problem, for the case of $J = 2$ relays is given by :

$$\begin{aligned}\mathcal{L}(\{\alpha_{1k}, \alpha_{2k}\}\,; \{\lambda_k^1\}, \{\lambda_k^2\}, \nu_1, \nu_2, \{\gamma_k\}) \;=\; & -R - \sum_{k=1}^{K} \lambda_k^1 \alpha_{1k} - \sum_{k=1}^{K} \lambda_k^2 \alpha_{2k} \\ & + \nu_1 \left(\sum_{k=1}^{K} \alpha_{1k} - 1 \right) + \nu_2 \left(\sum_{k=1}^{K} \alpha_{2k} - 1 \right) \\ & - \sum_{k=1}^{K} \gamma_k \left(R_k - \frac{1}{2} \log_2 \left(1 + c_k + \sum_{j=1}^{2} p_{jk} \alpha_{jk} \right) \right), \quad (31)\end{aligned}$$

where λ_k^1 and $\lambda_k^2, k = 1, 2, \ldots, K$ are the Lagrange multipliers associated with the constraint on positive power. ν_1 and ν_2 are the Lagrange multipliers associated with the constraint on the total power at the two relays, and $\gamma_k, \; k = 1, 2, \ldots, K$ are the Lagrange multipliers associated with the constraint on the minimum rate. The solution, if it exists, satisfies the KKT conditions, which are,

$$\frac{p_{1k}(2 - \gamma_k)}{2 \left(1 + c_k + \sum_{i=1}^{2} p_{ik} \alpha_{ik} \right)} + \lambda_k^1 = \nu_1, \quad \lambda_k^1 \alpha_{1k} = 0, \quad \lambda_k^1 \geq 0, \tag{32}$$

$$\frac{p_{2k}(2 - \gamma_k)}{2 \left(1 + c_k + \sum_{i=1}^{2} p_{ik} \alpha_{ik} \right)} + \lambda_k^2 = \nu_2, \quad \lambda_k^2 \alpha_{2k} = 0, \quad \lambda_k^2 \geq 0, \tag{33}$$

$$\gamma_k \left(R_k - \frac{1}{2} \log_2 \left(1 + c_k + \sum_{j=1}^{2} p_{jk} \alpha_{jk} \right) \right) = 0, \quad \gamma_k \geq 0. \tag{34}$$

Suppose for some $i \in \{1, 2, \ldots, K\}$, α_{1i} and α_{2i} are both non-zero, then the conditions $\lambda_i^1 \alpha_{1i} = 0$ and $\lambda_i^2 \alpha_{2i} = 0$ dictate that λ_i^1 and λ_i^2 are both zero, and from the KKT conditions it follows that

$$\frac{\nu_1}{p_{1i}} = \frac{\nu_2}{p_{2i}}. \tag{35}$$

Similarly if α_{1j} and α_{2j} are both non-zero for some $j \in \{1, 2, \ldots, K\}$, then

$$\frac{\nu_1}{p_{1j}} = \frac{\nu_2}{p_{2j}}, \tag{36}$$

hence, like the case with the max sum rate metric, when the power is optimally allocated, at most one of the K $(\alpha_{1k}, \alpha_{2k})$ pairs can have more than one non-zero entry. To generalize this result, with J relays at most $J - 1$ of the J-tuples $(\alpha_{1k}, \alpha_{2k}, \ldots, \alpha_{Jk})$ have more than one non-zero entry. The additional constraint on minimum rate given in Eqn. (30) does not alter this property of the solution.

A lower bound to the solution of the optimization problem, can be formulated like before. Explicitly impose selection, by assigning users receiving power from multiple relays to the relays that allot the maximum power,

$$r(d_k) = r_m \quad \text{if} \quad m = \operatorname*{argmax}_j \{\alpha_{jk} p_{jk}\}\,. \tag{37}$$

This relay assignment has to be followed with solving J water-filling problems to meet the constraint of a minimum rate to each user. Note that it is possible that the simplified optimization problem is feasible where as the original optimization problem in Eqns. (26)-(30) is not. It is also possible that solving the J water-filling problems to compute the lower bound might be an infeasible optimization problem. In these cases, the bounds are not meaningful. However, these scenarios occur very rarely.

C. Max-min rate

We will now consider a third metric, the minimum rate to each user. The optimal power allocation ensures that each user receives the same data rate. The optimization problem maximizes the minimum rate to all the users subject to two constraints: only a single relay helps each user and each relay must meet a power constraint. The optimization problem is,

$$\max_{\{\alpha_{jk}\}} \min_k \left\{ \frac{1}{2} \log_2 \left(1 + c_k + \sum_{j=1}^{J} p_{jk}\alpha_{jk} \right) \right\} \tag{38}$$

$$\text{such that} \qquad \forall k,\ \alpha_{jk}\alpha_{lk} = 0, \qquad j \neq l, j, l \in \{1, 2, \ldots, J\} \tag{39}$$

$$\sum_{k=1}^{K} \alpha_{jk} = 1 \qquad \forall \ \ j, \tag{40}$$

$$\alpha_{jk} \geq 0 \qquad \forall \ \ k, j. \tag{41}$$

As before, ther than Eqn. (39), the optimization problem in Eqns. (38)-(41) is concave: the objective function is concave and the remaining constraints are affine [16]. As before, we ignore the constraints given in Eqn. (39) and maximize the objective function subject to the power constraints alone:

$$\max_{\{\alpha_{jk}\}} R \ = \ \max_{\{\alpha_{jk}\}} \min_k \left\{ \frac{1}{2} \log_2 \left(1 + c_k + \sum_{j=1}^{J} p_{jk}\alpha_{jk} \right) \right\}, \tag{42}$$

$$\text{such that} \qquad \sum_{k=1}^{K} \alpha_{jk} = 1 \qquad \forall \ \ j, \tag{43}$$

$$\alpha_{jk} \geq 0 \qquad \forall \ \ k, j. \tag{44}$$

Note that the objective function given by Eqn. (42) is not differentiable. To analyze this problem, we formulate an equivalent optimization problem with differentiable objective and constraint functions.

The logarithm function is a monotonically increasing function of its argument, and hence, maximizing the minimum of logarithm functions is same as maximizing the minimum of the arguments of the logarithm function. Also, for any positive real numbers $x_1, x_2, \ldots, x_n$,

$$\min_i \{x_i\} = \left(\max_i \left\{\frac{1}{x_i}\right\}\right)^{-1} \quad \text{and} \quad \max_i \{x_i\} = \lim_{l\to\infty} \left(\sum_{i=1}^{n} x_i^l\right)^{1/l}$$

Using these relations, the objective function can be reformulated as

$$\min_{\{\alpha_{jk}\}} \left(\frac{1}{R}\right) = \min_{\{\alpha_{jk}\}} \left[\lim_{l\to\infty} \left(\sum_{k=1}^{K} \frac{1}{\left(R_k^J\right)^l}\right)^{1/l}\right], \tag{45}$$

where $R_k^J = 1 + c_k + \sum_{j=1}^{J} p_{jk}\alpha_{jk}$.

Consider the optimization problem for some finite l,

$$\min_{\{\alpha_{jk}\}} \left(\frac{1}{R}\right) = \min_{\{\alpha_{jk}\}} \left(\sum_{k=1}^{K} \frac{1}{\left(R_k^J\right)^l}\right)^{1/l}, \tag{46}$$

$$\text{such that} \quad \sum_{k=1}^{K} \alpha_{jk} - 1 = 0 \qquad \forall\ j, \tag{47}$$

$$-\alpha_{jk} \leq 0 \qquad \forall\ k, j. \tag{48}$$

Again we show that the solution to the relaxed problem leads to selection in most cases. Note that while these conditions are the same as derived earlier for the max-sum rate, the approach to this getting here is very different. The optimization problem given by Eqns. (46)-(48) is also a convex optimization problem [16]. To show that when the power is optimally allocated, most users receive power only from one of the relays, let us characterize the set of solutions to the optimization problem. For the sake of clarity, we again start by looking at the case with $J = 2$ relays. In such a case, the Lagrangian of the minimization problem is given by

$$\begin{aligned}\mathcal{L}(\{\alpha_{1k}, \alpha_{2k}\}, \{\lambda_k^1\}, \{\lambda_k^2\}, \nu_1, \nu_2) = & \left(\sum_{k=1}^{K} \frac{1}{\left(R_k^2\right)^l}\right)^{1/l} + \sum_{k=1}^{K} \lambda_k^1(-\alpha_{1k}) + \sum_{k=1}^{K} \lambda_k^2(-\alpha_{2k}) \\ & + \nu_1 \left(\sum_{k=1}^{K} \alpha_{1k} - 1\right) + \nu_2 \left(\sum_{k=1}^{K} \alpha_{2k} - 1\right),\end{aligned} \tag{49}$$

where $R_k^2 = (1 + c_k + p_{1k}\alpha_{1k} + p_{2k}\alpha_{2k})$; λ_k^1 and λ_k^2, $k = 1, 2, \ldots, K$ are the Lagrange multipliers associated with the constraint of positive power given by Eqn. (48); and ν_1 and ν_2 are the Lagrange multipliers associated with the total power constraints given by Eqn. (47). The KKT conditions, which must be satisfied, are

$$\sum_{k=1}^{K} \alpha_{1k} = 1, \qquad \sum_{k=1}^{K} \alpha_{2k} = 1 \tag{50}$$

$$-\alpha_{jk} \leq 0 \qquad \forall j, k \tag{51}$$

$$-f_R p_{1k} - \lambda_k^1 + \nu_1 = 0, \quad \lambda_k^1 \alpha_{1k} = 0, \quad \lambda_k^1 \geq 0, \quad \forall k, \tag{52}$$

$$-f_R p_{2k} - \lambda_k^2 + \nu_2 = 0, \quad \lambda_k^2 \alpha_{2k} = 0, \quad \lambda_k^2 \geq 0, \quad \forall k, \tag{53}$$

where

$$f_R = \left(\sum_{k=1}^{K} \frac{1}{(R_k^2)^l} \right)^{\frac{1}{l}-1} (R_k^2)^{-l-1}.$$

Now suppose for some $i \in \{1, 2, \ldots, K\}$, α_{1i} and α_{2i} are both non-zero, then the conditions $\lambda_i^1 \alpha_{1i} = 0$ and $\lambda_i^2 \alpha_{2i} = 0$ dictate that λ_i^1 and λ_i^2 are both zero. ¿From the KKT conditions it follows that

$$\frac{\nu_1}{p_{1i}} = \frac{\nu_2}{p_{2i}}. \tag{54}$$

As discussed earlier, no other pair $(\alpha_{1k}, \alpha_{2k})$ can have two non-zero entries. Note that this property of the solution is true for all l. Therefore, the solution to the optimization problem with the objective function given by Eqn. (45), and constraints given by Eqns. (43)-(44), also has this property. The objective function given by Eqn. (45) is a reformulation of Eqn. (42), and, in turn, the solution to the optimization problem given by equations Eqns. (42)-(44) also has the aforementioned property for the case of two relays. Therefore, for the case of two relays, the solution obtained by ignoring Eqn. (39) comes quite close to the solution to the original optimization problem given by Eqns. (38)-(41).

To generalize this result, the solution to the max-min optimization problem given by Eqns. (42)-(44), for the case of J relays has at most $J-1$ of the J-tuples $(\alpha_{1k}, \alpha_{2k}, \ldots, \alpha_{Jk})$ with more than one non-zero entry.

The construction of a heuristic to the solution of the optimization problem given by Eqns. (38)-(41), is, as before: assign each user receiving power from multiple relays to that relay from which it receives maximum power. If there are multiple relays which allot the same maximum power, assign the user to any one of them arbitrarily. Once this relay assignment is done, if required, J max-min power allocation algorithms are solved for the power distribution at each of the relays.

D. Independent codebooks at the relays

In the previous section, relay selection and power allocation was done for the case when the transmitter and the relays use the same codebooks to encode the messages. The results can also be extended to the case when independent codebooks are employed at the source and the relays. Using independent codebooks results in higher rates [3], however, decoding of the source and relay messages is significantly more complex compared to the case of repetition coding [19]. When the source and the relays employ independent Gaussian codebooks, the optimization problem to maximize the sum rate to all users, similar to the ones given in equations (7)-(9), is given by:

$$\max_{\{\alpha_{jk}\}} R \quad = \quad \max_{\{\alpha_{jk}\}} \sum_{k=1}^{K} \left\{ \frac{1}{2} \log_2 (1 + c_k) + \frac{1}{2} \log_2 \left(1 + \sum_{j=1}^{J} p_{jk} \alpha_{jk} \right) \right\}, \tag{55}$$

$$\text{such that} \quad \forall k, \quad \alpha_{jk}\alpha_{lk} = 0, \qquad j \neq l, j, l \in \{1, 2, \ldots, J\} \tag{56}$$

$$\sum_{k=1}^{K} \alpha_{jk} = 1 \qquad \forall \;\; j, \tag{57}$$

$$\alpha_{jk} \geq 0. \tag{58}$$

It is not hard to show that other than the constraint given in Eqn. (56), the optimization problem is a concave maximization problem, and like before, solving it gives an upper bound to the sum rate. The heuristic which also serves as a lower bound can also be constructed from it. A max-min optimization problem can also be formulated in a similar manner.

IV. Numerical Results and Discussion

In this section we verify the validity of the assumption in Eqn. (6) and present the results of simulations to illustrate the tightness of the bounds developed in the previous section. We compare the performance of three cases: the baseline scenario uses a single-input single-output system (SISO) with a single antenna at the BS and user and relaying is not used. The alternative is a system with a single antenna at the BS and J relays with a single antenna each. The last system considered is a multiple-input single-output (MISO) system with $J + 1$ antennas at the BS and a single antenna at each user. In comparing these cases, all other system parameters, e.g., number of users, total power and bandwidth, remain constant.

TABLE I
PARAMETERS USED IN COST231 MODEL

Parameter	Value chosen	Parameter	Value chosen
BS height	50m	Rooftop height	30m
Relay height	50m	User height	1.5m
Frequency	1GHz	Road orientation	90 degrees
Building spacing	50m	Street width	12m
Transmit power	20dBm	Noise power spectral density	-174dBm/Hz

A. Channel Model

The simulations are implemented using the COST-231 channel model as described in [15]. The model assumes both the BS and relays are at some height off the ground and treats the BS-relay channel as Rician. The BS-destination and relay-destination channels are modeled as Rayleigh. The path loss in the BS-relay channel is made up of two components, free space loss and multi-screen loss. In addition to these two, the BS-user and the relay-user channels have a rooftop-to-street diffraction loss. For the values of the parameters that we consider, the COST-231 channel model suggets a distance attenuation in channel power of 20dB/km for the first 657 meters and 38dB/km for greater distances. The model therefore appears to be conservative in the sense that one would expect the LoS component in the Rician fading to attenuate slower than the other non-LoS components. In the MISO case, the large scale fading in all the channels between the transmit antennas and a particular user, is the same. *Each user is assigned an orthogonal channel of bandwidth of 200kHz*, resulting in a noise power of -120dBm. The chosen system parameters are given in Table I.

B. Decoding at the relays

To form a tractable problem, we had made the assumption that the relays always successfully decode the message transmitted by the BS in the first time slot, and the data rate is the limited by compound source-relay-destination channel capacity, as in Eqn. (6). To verify the assumption, we consider a circular cell, centered at a BS, of radius one kilometer with $J = 4$ relays positioned at ($\pm 200\sqrt{2}m, \pm 200\sqrt{2}m$), i.e., on a ring of radius 400m. 3×10^6 user locations in the cell are randomly generated. For each location, independent channels are generated using the channel model. As shown in Fig. 1, we divide the cell into annular rings of radius 100 meters. In Table II we list the percentage of number of locations where Eqn. (6) is valid. It is evident from the table that the assumption we make is valid whenever the user is farther than 300m from the BS. Essentially, for all user locations of interest, i.e., areas where users

TABLE II
PERCENTAGE OF LOCATIONS WHERE EQN. (6) IS SATISFIED

Distance from the BS (m)	% locations	Distance from the BS (m)	% locations
0-100	93.591	500-600	99.943
100-200	99.642	600-700	99.963
200-300	99.815	700-800	99.977
300-400	99.309	800-900	99.989
400-500	99.482	900-1000	99.992

have a relatively weak channel to the BS, the assumption is valid. It is worth emphasizing that these are conservative numbers.

C. Tightness of the bounds

Our next of simulations test the tightness of the upper bound as developed in this paper and the resulting heuristic which acts as a lower bound. Note that this heuristic is our final solution to the joint selection and power allocation problem. The relay assignment and the power allocation is done based on the instantaneous channel powers. For this simulation, the channels are generated as independent realizations of a unit-variance Rayleigh fading random variable. For a fair comparison, the power allocated to each relay is set to $1/J$, i.e., all curves use the same total power. The curves presented here are averages over one thousand random user locations.

Figure 2 plots the upper bound, and the sum rate achievable by the heuristic (that also acts as a lower bound on the achievable sum rate) for varying values of J and K. The average total transmit power to noise power ratio is set to 30dB. As is clear from the figure, the upper and lower bounds are indistinguishable. As explained in Section III, this is because it is quite rare for a user to be allocated power from multiple relays, i.e., selection is essentially inherent in the approximate solution. The heuristic, therefore, is an extremely effective solution to the joint selection and power allocation problem. By an exhaustive search, we also find the exact maximum sum rate for the case with $J = 2$ relays and K between one and eight. Note that since each exhaustive search requires solution of J^K water-filling problems, any larger value of J is infeasible.

Figure 3 plots the upper and lower bound to the max-min rate for varying values of J and K averaged over many channel realizations. In this simulation, the average total transmit power to noise power ratio is set to 20dB. Again, the bounds are extremely tight and the heuristic provides an effective solution. The slight difference is due to the rare case where a user is allocated power by two relays (see Section III).

In interests of brevity, we do not provide a similar plot for the max-sum rate with a rate constraint.

D. Results for a cellular network

In this section, we use the theory developed for solving the max-min and max sum rate problems, to estimate the performance gains, with respect to a SISO and MISO system in cellular network setting. We consider a cell of radius $\mathrm{r}_{\mathrm{cell}}$. Performance here is measured as the increase in cell-size made possible by relaying. Since we wish to study the improvement in the rates to the users with poor channels to the BS, we consider users in the outer annular ring, of inner radius $\mathrm{r}_{\mathrm{cell}}/2$ and outer radius $\mathrm{r}_{\mathrm{cell}}$, the area shaded in gray in Figure 1. Users are distributed uniformly in the region with a constant user density of $(30/\pi)$ per square kilometer. We consider the following three system models for comparison:

1) A cellular network with a single antenna BS, communicating to multiple users with single antenna receivers (multiuser SISO system).
2) A cellular network with a BS with five transmit antennas, communicating to multiple users with single antenna receivers (multiuser MISO system).
3) A cellular network with a BS with a single antenna and assisted by four relays positioned on a ring of radius $0.4\mathrm{r}_{\mathrm{cell}}$, communicating to multiple users with single antenna receivers.

For the simulation, we generate 50 random sets of locations for the users. We then use the COST-231 model to generate the BS-user and relay-user channels. For each set of locations, we generate one set of large-scale fading variables. To average over small-scale fading random variables, for every set of locations, we generate 500 small-scale fading random variables. As indicated in Table I, the total power used in communication is set to 20dBm.

In the first example, powers are allocated to maximize the sum rate to all the users. For a fair comparison, we use this to compute the data rate averaged over all users. In the SISO case, the system uses water-filling to allocate power to the multiple users. In the MISO case, the BS is assumed to know the channel vector to each user and can both match to the channel and allocate power via water-filling. Finally, in the case with relays, selection and power allocation uses the scheme developed in Section III.

In Figure 4, we plot the average user rate as a function of the radius of the cell. We compute the rates as given by the lower bound, assuming that the power allocation is done only in the second time slot. In the first time slot, the BS distributes the power equally among all the users. This is done to ensure that the relays are able to decode all the transmitted messages. In the second time slot, each of the relays uses one fourth of the available power to communicate with the users it assists. This ensures that the total power used is the same in all the three system models. Interestingly, Figure 4 shows that a MISO system provides higher average data rates (and hence the sum rate) compared to the system with relays. We explain these graphs in the following section.

Next, we repeat the simulations by allocating power using the max-min algorithm, and then computing the outage rates for each of the system models. For the SISO and MISO cases, computing this power allocation is fairly straightforward. The optimal power allocation is the one such that all the users have the same data rate. When relays are employed, we use the methodology developed in this paper to solve the max-min optimization problem.

We plot the outage rates for 10% and 1% outage, as a function of $\mathrm{r_{cell}}$ in Figure 5. Here we see a reversal in performances, with the system with relays providing higher outage rates compared to the MISO system. As expected, the BS-user communication in these systems is more susceptible to channel fluctuations. This plot is discussed further below.

E. Discussion

A user in a heavily shadowed region has a weak channel to the base station. Having multiple antennas at the base station does not help much. Relays aid such users by providing alternate paths to the base station. This is consistent with the data in Figure 5 where a relay system provides higher outage rates. This is because the outage rates depend on the data rates to the users with weakest channels. On the other hand, the max-sum rate algorithm, allocates more power to the users with the strongest channels. The MISO system provides higher data rates compared to a relay system. As shown in Fig. 4, the loss in half the bandwidth incurred in switching from direct transmission to co-operative transmission outweighs the benefits brought by the additional diversity.

In a network setting where a user has the same average channel to all the J relays, selection cooperation achieves order $J+1$ diversity [6]. However, because of the geometry of a cellular network and because of the rapid deterioration of the channel powers with distance, most users have good channels to only a small set of relays. The *effective* diversity order is, therefore, limited.

Figures 4 and 5 also lead to a cautionary result. These results indicate that, compared to a system with SISO communication, deploying relays does offer substantial improvements. The area serviced effectively by a single BS, helped by relays, can significantly expand. However, these improvements need to be commensurate with the infrastructure costs involved in the deployment of these relays including both the antenna system cost and ‘non-technical’ costs such as the required real-estate. If the cost of a relay were on the same order of magnitude as a base station, the improvements in the cell radius, as shown by the simulations do not justify the additional cost. Also, depending on the performance metric, a MISO system may perform better or almost as good as the relay system, but with significantly lower costs.

Clearly, a complete financial cost/benefit analysis is beyond the scope of this paper. Furthermore, the examples presented here are limited and do not explore every potential parameter. However, do note that the our results are optimistic in assuming the relays can always decode and that the transmitters have all the information they need to make optimal decisions. Our goal here is to indicate that significant gains are possible, but are context and scenario dependent. These results also indicate the need for exploring alternate ways of exploiting cooperative diversity. We also need to explore alternate hybrid schemes wherein the relays help only those users who need it.

V. Conclusion

This paper deals with the use of cooperation in a cellular network wherein a base station is assisted by a few dedicated relays. Previous work largely for mesh networks has shown the importance of *selection*, i.e., each user using only one relay, since this minimizes the overhead due to orthogonal channels. However, in a scenario with multiple data flows, selection has been either brute force or ad-hoc. Previous work has also largely ignored the problem of power allocation once the selection is achieved. In this paper we developed an optimization framework to solve the problem of joint selection and power allocation.

The optimization problem uses the achievable sum rate and max-min user-rate as two figures of merit. Given that the selection problem has exponential complexity, in this paper we formulate alternative convex optimization problems whose solution provides upper bounds on the two metrics. However, for practical values of number of users, the bound is indistinguishable from the true solution. Since this solution can violate the selection condition, a related heuristic is derived that assigns users to the relay which allocates it the maximum power. The resulting lower bound is also extremely tight and we have an efficient solution to the problem at hand. The numerical examples, using realistic channel models, illustrate the benefits achievable due to relaying.

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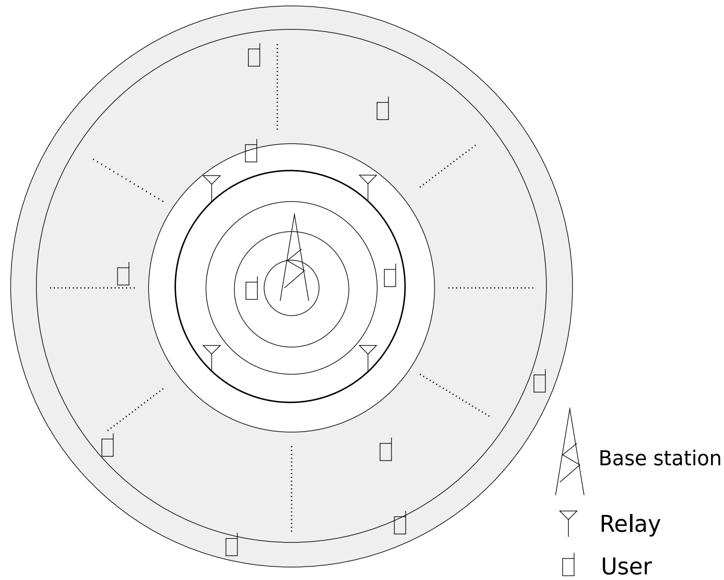

Fig. 1. A relay aided cellular network

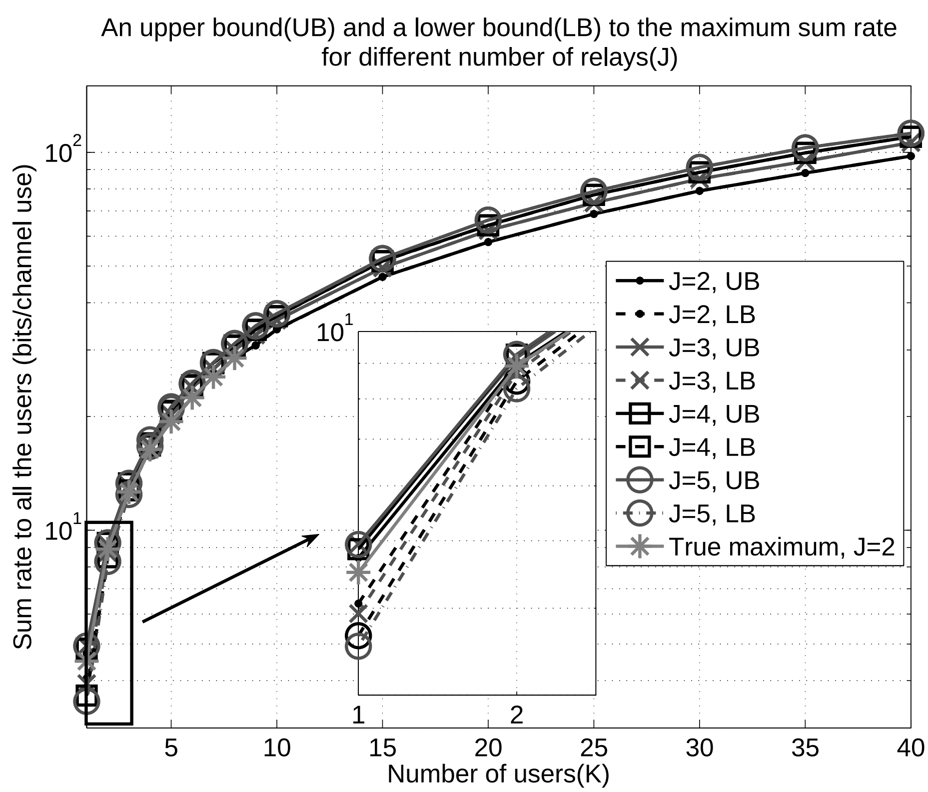

Fig. 2. The proposed upper bound to the maximum sum rate and the heuristic (a lower bound) as a function of the number of users. Note that both the bounds are extremely tight.

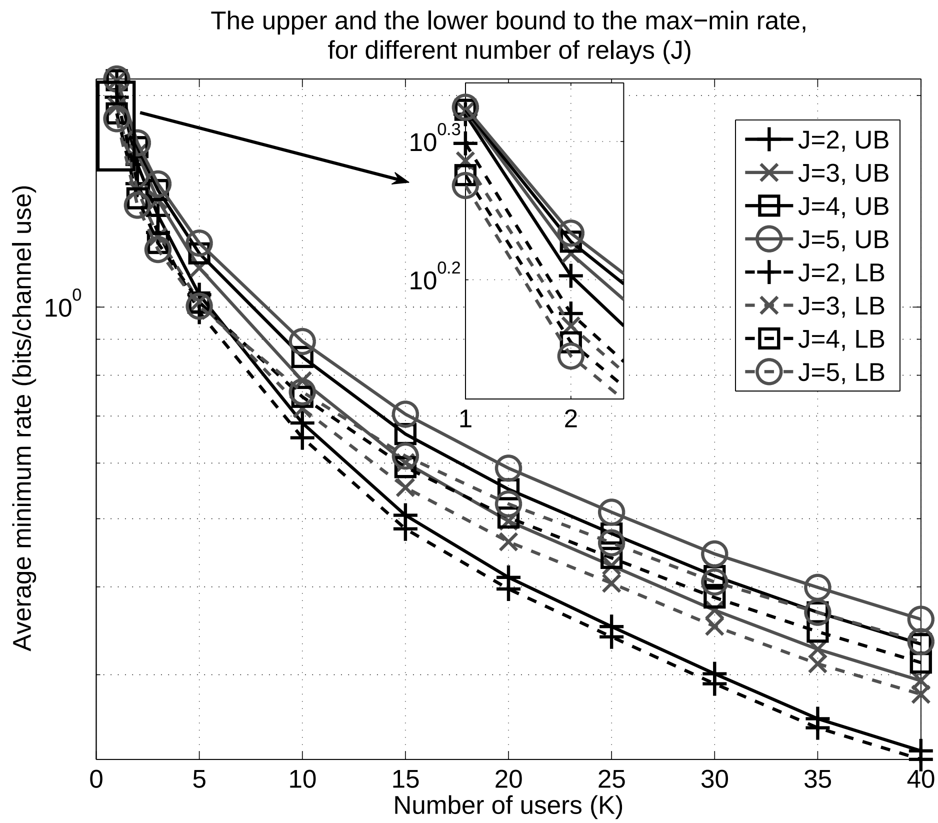

Fig. 3. The proposed upper and the lowerbound to the max-min rate.

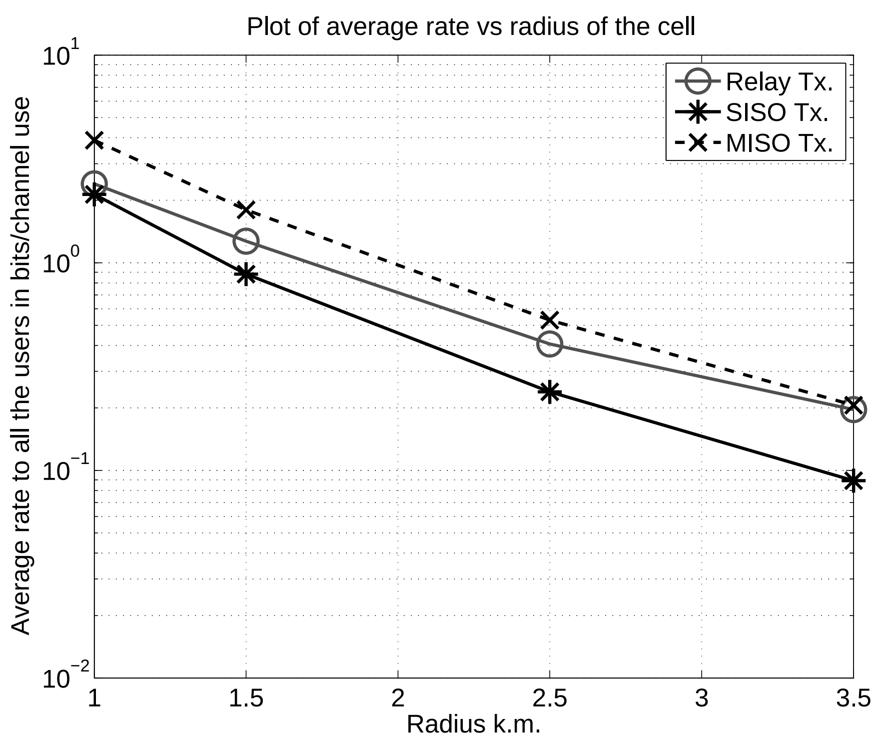

Fig. 4. Average data rate as a function of the radius of the cell.

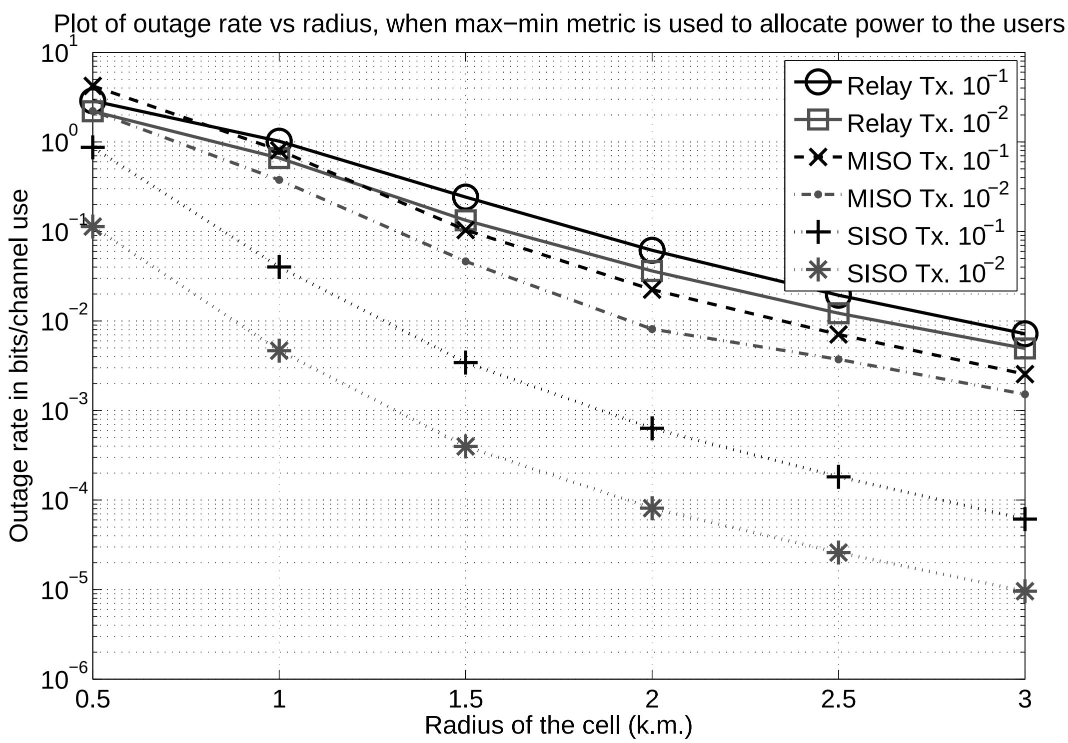

Fig. 5. Outage rate as a function of the radius of the cell.