Weighted Sum Rate Optimization of Multicell Cognitive Radio Networks

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***Abstract*— In this paper, we study the weighted sum rate maximization of multicell cellular cognitive radio networks (CRNs) which are overlaid with multicell primary radio networks (PRNs). We assume each CRN cell is collocated with a PRN cell and has a cellular structure with access point (AP) and multiple secondary users (SUs). We propose a unified framework to determine the operation parameters of the CRNs in the multicell environment. First, to avoid unacceptable interference to primary users (PUs), we propose methods to determine the power spectral masks (PSMs) of SUs and APs in uplink and downlink transmissions at each subchannel based on the target signal-to-interference-plus-noise ratio (SINR) outage probability of PRN base station (BS) receivers. Second, we utilize the duality optimization tool and design weighted sum rate maximization schemes which include the PSM optimally. Third, we accurately model the intercell interferences between CRNs and mutual interferences between the PRNs and CRNs, as a function of multiple system parameters. Our model and approaches provide powerful design tools and deep insights into achievable performance for overlaid CRNs and PRNs.**

I. Introduction

While the spectral resource becomes scarce today and yet many available spectrum bandwidths are not efficiently used [1], the cognitive radio (CR) technique has been proposed as a viable solution [2], [3]. In many cellular primary radio networks (PRNs), the channel resources are not utilized efficiently, for example, due to frequency reuse or low traffic duration when primary users (PUs) are idle. We can envision that the system is well under-utilized and more revenue can be obtained by deploying cognitive radio networks (CRNs), which opportunistically utilize the unused frequency bands available at PRNs and provide service to secondary users (SUs). Note that the PRNs that we considered include many existing systems, such as global system for mobile communication (GSM) and WiMax, which typically exploit multicell frequency reuse.

The orthogonal frequency division multiple access (OFDMA) is a promising candidate modulation and access scheme for 4G communication systems. This method can also be used for dynamic channel assignment technique in CRNs and will be exploited in this paper.

To successfully deploy CRNs, the transmission of SUs must not cause unacceptable interferences to rudimentary PRN receivers. A major technique to control CR interference is to impose power spectral mask (PSM) at CR transmitters. However, setting optimal PSM at each SU and AP for each subchannel is a difficult task, because of the multicell structure and user mobility (for PUs and SUs). Clearly, the PSMs for different CR transmitters are location and frequency-dependent, and this calls for a dynamic spectrum allocation design.

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The majority of past results for imposing PSM on SUs using OFDMA channel assignment have assumed the single CRN cell model [4]–[6]. In contrast to the above results, we will model the mutual interferences between CRNs and PRNs and design co-existence and Lagrangian duality-based optimization schemes for multicell overlaid PRN/CRN systems.

We assume that CRN uplink and downlink transmissions use time division duplex (TDD) mode, and they both take place in the PRN uplink transmission phase, and the following co-existence approach is taken: The PRN uplink target signal-to-interference-plus-noise ratio (SINR) outage probability is maintained by imposing PSM constraints at CRN transmitters.

Once the available channels and PSMs are determined, each CRN cell operates by maximizing its utility objective, which is defined as the weighted sum rate of SUs within each CRN cell. We propose several spectral access methods for cellular CRN channels based on the Lagrangian duality optimization framework. Different from existing approaches for OFDMA optimization, in our method, the PSM constraint is included by using the Lagrangian multipliers, and efficient primal-dual update approaches are designed for both downlink and uplink channels, which we term the *optimal duality-PSM algorithm*. For comparison purposes, we also design a duality update algorithm where PSM is ignored in the Lagrangian multipliers but is directly included in the power allocation, termed the *direct-PSM duality algorithm*.

Simulation results show that the proposed optimal duality scheme can provide a higher rate than the direct-PSM duality scheme which includes the PSM constraint in a suboptimal manner. Furthermore, the proposed PSM fitting Method 2 that utilizes the PUs' activity (i.e., on-off traffic) gives a significant larger sum rate than Method 1, and can efficiently mitigate the rate loss caused by the PSM constraints. These results provide guidelines for the co-existence and optimization of OFDMA-based CRN systems.

II. System Model

Consider the system model given in Fig. 1. Each PRN cell has one BS and multiple PUs, while each CRN cell has one AP and multiple SUs, and each CRN cell is collocated with a PRN cell. Assume cell 0 is the cell of interest. We consider the orthogonal PRN/CRN channel-sharing within each cell (spectral overlay) but non-orthogonal sharing among the cells (spectral underlay). We assume that in a CRN cell (say, at cell 0) there are K SUs who are communicating with the AP, and they compete for a set of N available frequency bands which are not in use by PRN cell 0.

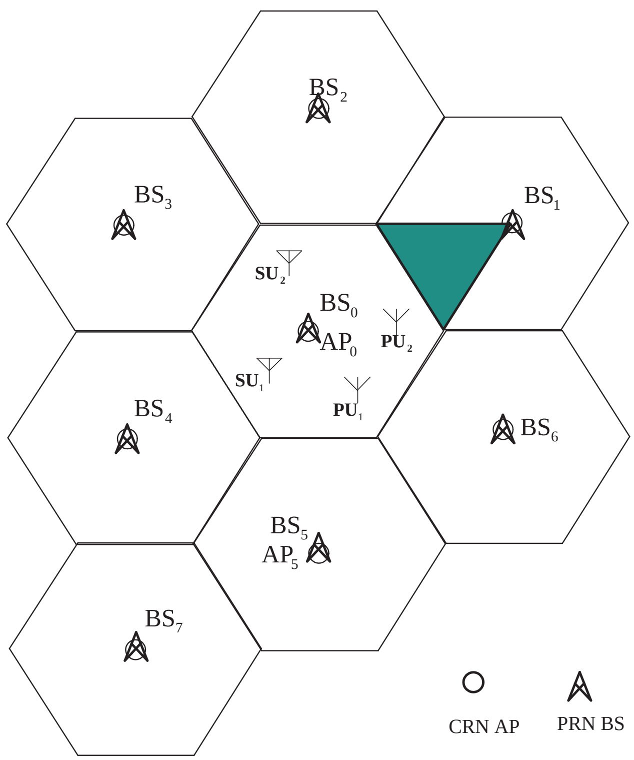

Fig. 1. System model of multicell PRNs and multicell cellular CRNs.

Due to the frequency reuse, other PRN cells (cells 1-6) may be using part of these N channels and therefore impose spectral mask constraints on the K SUs at the CRN cell 0. Furthermore, PRN communication causes interference to CRN cells.

On the other hand, though BS 0 (the BS at PRN cell 0) does not receive CRN interference from its own cell, it receives CRN interferences from surrounding cells. It is important that the sum interference power from all CRN transmitters at each BS receiver is limited by a certain bound, which will be calculated based on the SINR outage probability. We will consider the optimization of cellular CRNs when PRNs are at the uplink transmission phase. This is because it is relatively easier to measure the CRN-generated interference to BSs (for PRN uplink) than to multiple PUs (for PRN downlink).

A. Uplink Model

Consider the CRN cell 0 without loss of generality. We assume that every SU has a single transmit antenna and single receive antenna, and exclusive channel assignment (ECA) is implemented inside each CRN cell. Assume user k is assigned channel n. The received signal at AP 0 on carrier n, $y_{k,n}$, consists of the desired signal from SU k, external interference from surrounding PRN and CRN cells, and background noise, and is given by

$$y_{k,n}^{\text{UL}} = h_{k,n} x_{k,n}^{\text{UL}} + \sum_{m=1}^{M} \left[I_{n,m}^{\text{SU,UL}} + I_{n,m}^{\text{PU,UL}} \right] + z_n \tag{1}$$

where $h_{k,n}$ is the frequency domain channel gain from SU transmitter k to AP 0, $x_{k,n}^{\text{UL}}$ is the transmitted signal, and $I_{n,m}^{\text{SU}}$ and $I_{n,m}^{\text{PU}}$ are the interferences generated from SUs and PUs at neighboring cell m, $m = 1, \ldots, M$, all on carrier n. z_n is the background Gaussian noise at AP 0 on carrier n, which has zero mean and power spectral density (PSD) $\mathcal{N}_n$. The transmit power of user k on carrier n is given by $P_{k,n}^{\text{UL}} = |x_{k,n}^{\text{UL}}|^2$. Note that $\sum_{n=1}^{N} P_{k,n}^{\text{UL}} \le P_{T,k}$ holds, where $P_{T,k}$ is the available transmit power of SU k.

For convenience, we assume $M = 6$, since only the interferences from the first tier of cells are considered. Nonetheless, this model can be easily extended to include more tiers of interferences.

We assume that the N available channels experience independent but not necessarily identically distributed (i.n.d.) fading/shadowing channel gains. The channel SNR of user k on carrier n is given by $\gamma_{k,n} = |h_{k,n}|^2/\mathcal{N}_n$. The SINR of SU k on carrier n may be defined as

$$\gamma_{k,n}^{\text{SINR}} = |h_{k,n}|^2 / \{ \sum_{m=1}^{M} [E[|I_{n,m}^{\text{SU,UL}}|^2] + E[|I_{n,m}^{\text{PU,UL}}|^2]] + \mathcal{N}_n \} \tag{2}$$

where $E[x]$ is the expectation with respect to the Rayleigh fading part of channel gain. We have $E[|I_{n,m}^{\text{SU,UL}}|^2] = L_{k^*,m}^{\text{SU}-\text{AP}} P_{k^*,n,m}^{\text{SU}}$, where k^* is the SU who occupies channel n based on ECA, $P_{k^*,n,m}^{\text{SU}}$ is the transmit power of SU k^* at cell m on carrier n, and $L_{k^*,m}^{\text{SU}-\text{AP}}$ is the intercell interference channel gain (including path loss and shadowing part) between SU k^* and AP 0.

As a conservative measure, we assume at each of the neighboring cells, all the channels are occupied by either PUs or SUs. Similarly, we have $E[|I_{n,m}^{\text{PU,UL}}|^2] = L_{k^*,m}^{\text{PU}-\text{AP}} \cdot P_{k^*,n,m}^{\text{PU}}$, where k^* is the interfering PU, $P_{k^*,n,m}^{\text{PU}}$ is the transmit power of PU k^* at cell m on carrier n, and $L_{k^*,m}^{\text{PU}-\text{AP}}$ is the PU to AP 0 interference channel gain.

The SU uplink channel gain inside cell 0, $h_{k,n}$, can be expressed as

$$h_{k,n} = \sqrt{L_k^{\text{SU}}} \tilde{h}_{k,n} \tag{3}$$

where $\tilde{h}_{k,n} \sim \text{CN}(0,1)$ models the normalized Rayleigh fading part, and L_k models the distance-dependent channel loss and shadowing effect. Since we assume that the total channel bandwidth is much less than carrier frequency f_c, it follows that L_k is approximately independent of carrier index

n.

B. Downlink Model

The received signal at SU k on carrier n from AP 0 is given by

$$y_{k,n}^{\text{DL}} = h_{k,n}x_{k,n}^{\text{DL}} + \sum_{m=1}^{M}\left[I_{n,m}^{\text{SU,DL}} + I_{n,m}^{\text{PU,DL}}\right] + z_{k,n} \quad (4)$$

where $h_{k,n}$ is the channel gain from AP 0 to SU receiver k, $x_{k,n}^{\text{DL}}$ is the transmitted signal, and $I_{n,m}^{\text{SU,DL}}$ and $I_{n,m}^{\text{PU,DL}}$ are the interferences generated from the AP and the PUs at neighboring cell m, $m = 1, \ldots, M$, all on carrier n. $z_{k,n}$ is the background Gaussian noise at receiver k on carrier n, and it has zero mean and PSD $\mathcal{N}_{k,n}$.

III. Spectral Mask Fitting Approaches

We assume that the maximum allowed total interference power caused by all the SUs (from the surrounding cells) to BS m at each subchannel n is given by $P_{n,m}^{\text{BS,lim}}$. Without loss of generality, we consider PRN cell 0, and the received sum CRN interference power from cells 1-6 to BS 0 is upper-bounded by $P_{n,0}^{\text{BS,lim}}$.

A. Spectral-Mask Fitting Methods for CRNs

1) Interference-Channel-Tracking Based PSM Fitting: To determine the PSM, each CRN transmitter must be able to track the interference channel gain between itself and the BSs at neighboring PRNs. We assume the shadowing and distance-related average channel gain between SU transmitter k at cell m to BS 0, $L_{k,m}^{\text{SU}-\text{BS}}$, can be tracked accurately. To implement the interference channel tracking, we assume that the BS at each PRN periodically broadcasts pilot signals with transmission powers known to CRNs. Each SU (and AP) can use the reciprocity between the forward and reverse channels to estimate the interference channel gain between itself and the interfered BS.

For the uplink CRN, the maximum transmission power at SU k at cell m on carrier n is given by (Method 1)

$$P_{k,n}^{\text{SU,PSM}} = P_{n,0}^{\text{BS,lim}}/L_{k,m}^{\text{SU}-\text{BS}} \quad (5)$$

For the downlink CRN, the maximum transmission power at AP m $(m = 1, \ldots, 6)$ on carrier n is

$$P_{n}^{\text{AP,PSM}} = P_{n,0}^{\text{BS,lim}}/L_{k,m}^{\text{AP}-\text{BS}}$$

where $L_{k,m}^{\text{AP}-\text{BS}}$ is the interference channel gain between AP m and BS 0.

2) Joint PRN-Status and Interference-Channel-Tracking Based PSM Fitting: Since some of PRN subchannels can be idle, interferences generated to unused channels at these PRNs can thus be excluded in the PSM calculation. In Method 2, besides interference channel tracking, we assume that the CRN APs can also track the activity of PRN BSs at every subchannels, and and include such information in the PSM fitting for uplink and downlink scheduling decisions.

For uplink CRN, the maximum transmit power of SU k on channel n imposed by neighboring BS m' is given by

$$P_{k,n,m'}^{\text{SU}} \leq \begin{cases} P_{n,m'}^{\text{BS,lim}}/L_{k,m'}^{\text{SU}-\text{BS}} & \text{with prob. } \Pr(m',n) \\ P_{T,k} & \text{with prob. } 1-\Pr(m',n) \end{cases} \quad (6)$$

where $P_{m',n}^{\text{BS,lim}}$ is the interference power limit at BS m' on carrier n, and $\Pr(m', n)$ is the carrier occupation probability (COP) of PRN cell m' is using channel n in its uplink phase. The downlink CRN maximum AP transmit power on carrier n, $P_{n,m'}^{\text{AP}}$, can be obtained similarly, and is omitted for brevity.

Since the power masks imposed by all the neighboring PRN BSs must be met, for both PSM fitting Method 1 and Method 2 the actual PSM at SU k on carrier n at CRN cell 0 is bounded by (for uplink CRN)

$$P_{k,n}^{\text{SU,PSM}} = \min\{P_{k,n,1}^{\text{SU}}, \ldots, P_{k,n,6}^{\text{SU}}\}. \quad (7)$$

and $P_{n}^{\text{AP,PSM}} = \min\{P_{n,1}^{\text{AP}}, \ldots, P_{n,6}^{\text{AP}}\}$ holds for downlink CRN.

Though (5) – (7) show the PSM constraints, we still need to determine $P_{n,m}^{\text{BS,lim}}$. This procedure is given next.

B. Acceptable Interference Power Limit at PRN BS

We show how to determine $P_{n,0}^{\text{BS,lim}}$ so that the PRN SINR outage probability $P_{n,\text{out}}^{\text{BS}}$ at BS 0 is not violated. Here, the SINR outage event does not necessarily cause the BS receiver to break down, but rather brings a rate loss. For example, with adaptive modulation and coding (AMC), a lower transmission rate for the affected PUs can be employed in the case of SINR outage event.

Let P_{n}^{BS} be the received signal power from PU k on carrier n. We have $P_{n}^{\text{BS}} = P_{k,n}^{\text{PU}}L_{k,n}^{\text{PU}-\text{BS}}$, where $P_{k,n}^{\text{PU}}$ is the transmit power of PU who is assigned channel n, and $L_{k,n}^{\text{PU}-\text{BS}}$ is the channel gain between PU k and BS 0. In our design, we assume a constant transmit power $P_{k,n}^{\text{PU}}$, and it uses AMC to adapt to the channel fading and shadowing effects. Due to the shadowing, both $L_{k,n}^{\text{PU}-\text{BS}}$ and P_{n}^{BS} have the Lognormal distribution with variance $\sigma_{\psi,\text{PU}-\text{BS}}^{2}$,

The SINR at BS at carrier n is given by (suppressing indices k and n)

$$\text{SINR} = P^{\text{BS}}/(P_{0}^{\text{BS,lim}} + N_0 B_w) \quad (8)$$

where N_0 and B_w are the PSD of the noise and the bandwidth per subchannel, respectively.

To guarantee that the specified outage probability $P_{\text{out}}^{\text{BS}}$ is not violated, we need

$$\Pr(\text{SINR} < \text{SINR}_{\text{tar}}^{\text{PU}}) \leq P_{\text{out}}^{\text{BS}} \quad (9)$$

where $\text{SINR}_{\text{tar}}^{\text{PU}}$ is the target SINR at PRN BS 0 for the uplink transmitter on carrier n. For convenience, we assume $P_{0}^{\text{BS,lim}} \gg N_0 B_w$ holds which is a reasonable assumption in cellular communications. It is convenient to rewrite (9) as

$$\Pr\{P_{\text{dB}}^{\text{BS}} - P_{0,\text{dB}}^{\text{BS,lim}} - \text{SINR}_{\text{tar,dB}}^{\text{PU}} < 0\} \leq P_{\text{out}}^{\text{BS}} \quad (10)$$

where $\Pr\{A\}$ denotes the probability of event A, $\text{SINR}_{\text{tar,dB}}^{\text{PU}} = 10\log_{10}\text{SINR}_{\text{tar}}^{\text{PU}}$, and $P_{\text{dB}}^{\text{BS}} = 10\log_{10}P^{\text{BS}}$.

We determine the time-varying acceptable maximum interference power $P^{\text{BS,lim}}_{0,\text{dB}}$. For this case (10) leads to

$$Q\left(\frac{\overline{P}^{\text{BS}}_{\text{dB}} - P^{\text{BS,lim}}_{0,\text{dB}} - \text{SINR}^{\text{PU-BS}}_{\text{tar,dB}}}{\sigma_{\psi,\text{PU-BS}}}\right) \leq P^{\text{BS}}_{\text{out}} \qquad (11)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-t^2/2)dt$ is the Gaussian-Q function. The dB-valued interference limit at the BS is calculated as

$$P^{\text{BS,lim}}_{0,\text{dB}} \leq \bar{P}^{\text{BS}}_{\text{dB}} - \text{SINR}^{\text{PU}}_{\text{tar,dB}} - \sigma_{\psi,\text{PU-BS}}Q^{-1}(P^{\text{BS}}_{\text{out}}) \qquad (12)$$

where $Q^{-1}(\cdot)$ is the inverse-Q function. Based on the interference limit at BS given by (12), one can calculate the PSM at each SU and AP.

IV. Weighted Sum-Rate Optimization for CRNs

The weighted sum rate optimization can capture the effect of different utilities (or revenues) of different classes of services, and will be studied below. Here, the problem is different from the general OFDMA optimization task, because the PSM constraints for K SUs (in uplink) and the AP (in downlink) at all N carriers have to be included. The optimization problem for uplink CRN can be posed as:

$$\max_{\{P_{k,n}\},\{\mathcal{S}_k\}} \sum_{k=1}^{K} w_k \sum_{n\in\mathcal{S}_k} R_{k,n} \quad \text{s.t.} \quad \sum_{n\in\mathcal{S}_k} P_{k,n} \leq P_{T,k}, \quad \text{and} \quad P_{k,n} \leq P^{\text{PSM}}_{k,n} \qquad (13)$$

where w_k is the utility weight factor for CRN user k, $\mathcal{S}_k$ is the set of carriers assigned to user k, and $\mathcal{S}_1,\ldots,\mathcal{S}_K$ are non-overlapping sets. The throughput rate of user k on carrier n is given by

$$R_{k,n} = B_n \log_2\left(1 + \Gamma_k P_{k,n}\gamma^{\text{SINR}}_{k,n}\right) \qquad (14)$$

where $\gamma^{\text{SINR}}_{k,n}$ was given by (2), Γ_k is an SNR gap due to modulation format and target bit-error-rate (BER) requirement. When $\Gamma_k = 1$, (14) is the Shannon capacity, and when $\Gamma_k = -1.5/\log(5P_{e,k})$, where $P_{e,k}$ is the target BER for user k assuming continuous-rate quadrature amplitude modulation (CR-QAM) [7], (14) gives the throughput which satisfies the target BER $P_{e,k}$.

Define the Lagrangian

$$\begin{aligned}\mathcal{L}(\{P_{k,n}\},\{\mathcal{S}_k\}_{k=1}^K) &= \sum_{k=1}^{K}\sum_{n\in\mathcal{S}_k} w_k R_{k,n} \\ &- \sum_{k=1}^{K}\lambda_k\left(\sum_{n\in\mathcal{S}_k} P_{k,n} - P_{T,k}\right) \\ &- \sum_{k=1}^{K}\sum_{n\in\mathcal{S}_k}\mu_{k,n}\left(P_{k,n} - P^{\text{PSM}}_{k,n}\right), \qquad (15)\end{aligned}$$

where $\{\lambda_k\}$ and $\{\mu_{k,n}\}$ are non-negative Lagrangian multipliers.

Define the duality function as

$$g(\{\lambda_k\},\{\mu_{k,n}\}) = \max_{\{\mathcal{S}_k\},\{P_{k,n}\}} \mathcal{L}(\{\mathcal{S}_k\},\{P_{k,n}\}). \qquad (16)$$

The dual problem can be expressed as $\{\lambda_k\},\{\mu_{k,n}\} = \text{argmin}_{\{\lambda_k\},\{\mu_{k,n}\}} g(\{\lambda_k\},\{\mu_{k,n}\})$. The dual function $g(\{\lambda_k\},\{\mu_{k,n}\})$ can be re-written as

$$\begin{aligned}g(\{\lambda_k\},\{\mu_{k,n}\}) &= \sum_{n=1}^{N} g_n(\{\lambda_k\},\{\mu_{k,n}\}) \\ &+ \sum_{k=1}^{K}\{\lambda_k P_{T,k} + \sum_{n=1}^{N}\mu_{k,n}P^{\text{PSM}}_{k,n}\} \qquad (17)\end{aligned}$$

where

$$g_n(\{\lambda_k\},\{\mu_{k,n}\}) = \max_{\{\mathcal{S}_k\},\{P_{k,n}\}}\{w_k\log_2(1 + P_{k,n}\Gamma_k\gamma^{\text{SINR}}_{k,n}) - (\lambda_k + \mu_{k,n})P_{k,n}\} \qquad (18)$$

By deriving the Karush-Kuhn-Tucker (KKT) conditions, we obtain the following results.

Carrier n is allocated to user k^* if

$$k^* = \text{argmax}_k\{w_k\log(1 + P_{k,n}\Gamma_k\gamma^{\text{SINR}}_{k,n}) - (\lambda_k + \mu_{k,n})P_{k,n}\} \qquad (19)$$

The transmit power allocated to carrier n for user k (if $k = k^*$) is given by

$$P_{k,n} = \left(w_k/(\lambda_k + \mu_{k,n}) - 1/[\Gamma_k\gamma^{\text{SINR}}_{k,n}]\right)^+ \qquad (20)$$

where $(x)^+ = \max(0,x)$, and $P_{k,n} = 0$ if $k \neq k^*$.

Generally, to account for the transmit power and the PSM constraints, the following iterations based on sub-gradient search may be implemented:

$$\mu^{s+1}_{k,n} = \mu^s_{k,n} - \beta_k(P^{\text{PSM}}_{k,n} - P_{k,n}) \qquad (21)$$

$$\lambda^{s+1}_k = \lambda^s_k - \beta_k(P_{T,k} - \sum_{n\in\mathcal{S}_k} P_{k,n}) \qquad (22)$$

where $0 < \beta_k < 1$ is a gradient search step size, and λ^s_k and $\mu^s_{k,n}$ denote the values of λ_k and $\mu_{k,n}$ at stage s, respectively. Note in (21) and (22) the stopping conditions $P^{\text{PSM}}_{k,n} - P_{k,n} = 0$ (for all n) and $P_{T,k} - \sum_{n\in\mathcal{S}_k} P_{k,n}$ (for all k) may not be both fulfilled, and the dual iteration should stop when either of these two conditions is attained.

Gradient-based search generally has a low speed of convergence, and modified bisection search can be exploited to improve the convergence speed of the dual update.

We design the duality-based weighted sum rate maximization algorithm below (**Optimal Duality-PSM Scheme (UL)**):

(i) Initialize. Assume stage $s = 0$ and the weighted sum rate $R^{(s)}_{\text{tot}} = 0$.

(ii) For $n = 1,\ldots,N$, do the following:
   a) For $k = 1,\ldots,K$, assume carrier n is allocated to user k, and find the allocated power $P_{k,n}$ using (20).
   b) Carrier n is assigned to user k^* according to (19). Update the carrier set $\mathcal{S}^*_k$ and remove carrier n from the user who originally possessed this carrier.

(iii) Check if transmit power and the PSM constraints for every user and carrier has been fulfilled. First, check if $P^{\text{PSM}}_{k,n} - P_{k,n} \geq 0$. If no, update $\mu_{k,n}$. Next, check if $P_{T,k} - \sum_{n\in\mathcal{S}_k} P_{k,n} = 0$ holds. If not, update λ_k using a bisection search.

(iv) Find the sum rate $R_{\text{tot}}^{(s)}$ at stage s according to the carrier sets $\mathcal{S}_k$ and $P_{T,k}$ for all k. Compute the sum rate difference $\Delta R_{\text{tot}}^{(s)} = R_{\text{tot}}^{(s)} - R_{\text{tot}}^{(s-1)}$. Stop if the difference is negligible. Otherwise, go to step (ii) and repeat.

■

For comparison purposes, we also derive a duality approach by using direct PSM fitting, as given below. Define the Lagrangian

$$\mathcal{L}(\{P_{k,n}\},\{\mathcal{S}_k\}_{k=1}^K) = \sum_{k=1}^{K}\sum_{n\in\mathcal{S}_k} w_k R_{k,n} - \sum_{k=1}^{K}\lambda_k\left(\sum_{n\in\mathcal{S}_k} P_{k,n} - P_{T,k}\right) \quad (23)$$

where $\{\lambda_k\}$ are Lagrangian multipliers.

Based on the KKT conditions, we obtain the following results. Carrier n is allocated to user k^* if

$$k^* = \text{argmax}_k\{w_k \log(1 + P_{k,n}\Gamma_k\gamma_{k,n}^{\text{SINR}}) - \lambda_k P_{k,n}\} \quad (24)$$

The transmit power allocated to carrier n for user k (if $k = k^*$) is given by

$$P_{k,n} = \min\{\left(w_k/\lambda_k - 1/(\Gamma_k\gamma_{k,n}^{\text{SINR}})\right)^+, P_{k,n}^{\text{PSM}}\} \quad (25)$$

and $P_{k,n} = 0$ if $k \neq k^*$. Note that in (25) a direct PSM truncation in power allocation is implemented, and thus we call it direct-PSM duality algorithm.

Finally, λ_k is updated based on gradient or bisection search, or using the following closed-form formula,

$$\lambda_k = \frac{w_k N_k}{P_{T,k} + \sum_{n\in\mathcal{S}_k}(1/\Gamma_k\gamma_{k,n}^{\text{SINR}})} \quad \text{s.t.} \quad \lambda_k \leq \Gamma_k\gamma_{k,n}^{\text{SINR}} \quad (26)$$

Based on eqns. (24) – (26) we design the duality-based rate maximization algorithm below (**Direct-PSM Duality Scheme (UL)**):

(i) Initialize. Assume stage $s = 0$ and the weighted sum rate $R_{\text{tot}}^{(s)} = 0$.

(ii) For $n = 1, \ldots, N$, do the following:

a) For $k = 1, \ldots, K$, assume carrier n is allocated to user k, and find the allocated power $P_{k,n}$ using (25) for all k.

b) Carrier n is assigned to user k^* according to (24). Update the carrier set $\mathcal{S}_k^*$ and remove carrier n from the user who originally possessed this carrier.

(iii) Update the WF level λ_k using (26), where N_k is replaced by $N_{k,\text{eff}}$, which is the actual number of carriers in $\mathcal{S}_k$ which has non-zero power allocation.

(iv) Check if the weighted sum rate converges. If yes, stop; otherwise, increase stage s by one and go to step (ii).

■

This algorithm based on duality optimization fits the PSM constraint directly into the power allocation procedure in (25), while the WF level λ_k is obtained by ignoring such constraint. Though this method is simpler, its performance is inferior to the proposed optimal duality-PSM method, as will be shown by simulation results.

The downlink optimization algorithms can be designed by changing the K SU transmit power constraints to the AP transmit power constraint. For the *downlink optimal PSM-duality scheme*, we need to replace λ_k by λ in (19) and (20), where λ is the Lagrangian multiplier which includes the effect of AP transmit power constraint. Furthermore, (22) in the uplink optimization is replaced by

$$\lambda^{s+1} = \lambda^s - \beta(P_T - \sum_{k=1}^{K}\sum_{n\in\mathcal{S}_k} P_{k,n}) \quad (27)$$

where β ($0 < \beta < 1$) is the gradient-search step size, λ^s is the sth stage value for λ, and P_T is the available power at the AP. The details of the downlink optimal PSM-duality scheme and the *direct-PSM duality scheme* are omitted due to the space limitation, but their performances will be compared by simulation results.

V. Numerical results

We provide simulation results for the proposed PSM determination and rate optimization schemes in multicell overlaid CRNs and PRNs. For the K active SUs at each CRN cell, N, the number of available channels, is a function of N_c and the COP, and is thus a random integer for each scheduling slot. We assume 10 PUs at each PRN cell.

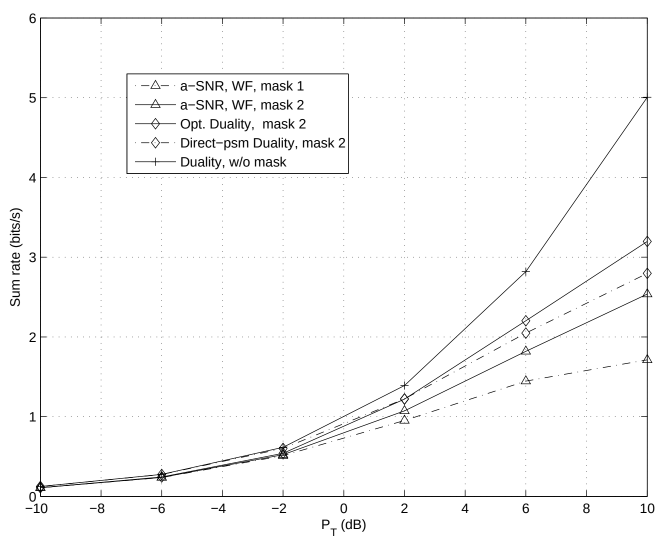

Fig. 2. Sum rate vs. total transmit power P_T for the duality and the *a*-SNR schemes in CRN uplink transmission. $N_c = 16$, $K = 6$, and $P_{\text{COP}} = 0.5$.

For CRN uplink transmissions, an equal available transmission power case for all K SUs is considered, that is $P_{T,k} = P_T/K$ for all k, and P_T is the sum transmit power of all K SUs. For a fair comparison between CRN downlink and uplink, we assume the total available power at each AP is equal to P_T. All CRN cells are in a quasi-synchronous mode, and they are either all in uplink transmission or all in downlink transmission phases. However, they have no coordination in multi-channel assignment to reduce mutual

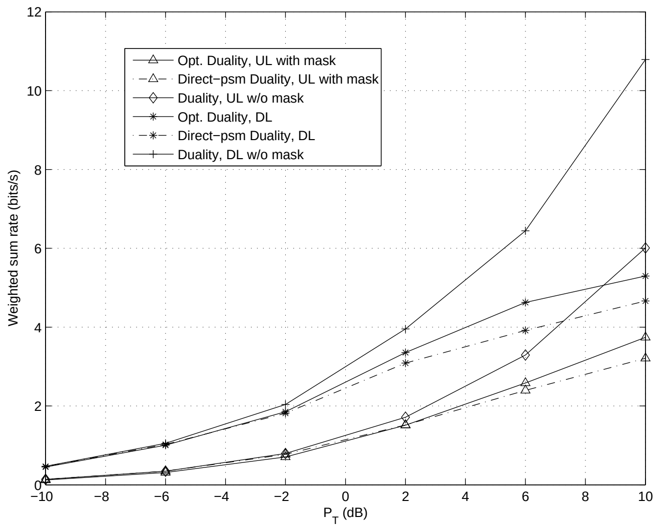

Fig. 3. Weighted sum rate vs. total transmit power P_T for the duality schemes in CRN uplink and downlink transmissions. $N_c = 16$, $K = 6$, and $P_{\text{COP}} = 0.5$.

interferences. This assumption is exploited to calculate the CRN intercell interferences.

The PUs at all cells are using each of the N_c subchannels independently with the same probability $\Pr(m,n) = P_{\text{COP}}$, for $m = 0, 1, \ldots, 6$ and $n = 1, \ldots, N_c$. Furthermore, to calculate the interference from PUs to SUs, we take a pessimistic assumption that all the PUs at all cells transmit using the maximum power $P_T^{\text{PU}} = 10$ dB on each carrier assigned, and the AMC is used based on SINR at each subchannel.

In the rate formula the SNR gap factor is calculated based on target BER of 10^{-3} for all K SUs, and the carrier bandwidth is $B_n = B$ for all n. The noise power at each subchannel experienced by the SUs and the AP is assumed to be -120 dBm. In all the figures, the rate is normalized by B.

To calculate the PSM for each SU on every subchannel, we assume a 1% SINR outage probability at the BS with target SINR of 12 dB, for a PU transmitter located $3R/4$ distance away from the BS, where R is the cell radius for both PRN and CRN cells.

We consider the IEEE 802.16 SUI model [8], [9], with $R = 1000$ m, reference distance $d_0 = 100$ m, carrier frequency $f_c = 2.5$ GHz, and antenna height 1.5 m. We also assume that for all the SUs and PUs the path loss exponents are $\alpha = 4$ and the shadowing standard deviation (STD) values are $\sigma_{\psi,\text{SU}-\text{BS}} = \sigma_{\psi,\text{PU}-\text{BS}} = 6$ dB.

To calculate the weighted sum rate, the weight vector $\mathbf{w} = [w_1, \ldots, w_K]^T$ is a normalized version of $[K, K-1, \ldots, 1]$ such that $\sum_{k=1}^{K} w_k = K$ holds. For unweighted sum rate, $\mathbf{w} = \mathbf{1}_{K\times 1}$ holds.

Fig. 2 presents the unweighted sum rate of the duality schemes vs. P_T for the uplink CRN assuming $K = 6$. The performance of the absolute channel SNR (*a*-SNR) based selective multiuser diversity (SMuD) [10], [11] scheme is shown for comparison. The result shows that the PSM fitting Method 2 gives better rate than Method 1. Also, when $P_T \geq 0$ dB, the PSM constraint causes some performance loss, which increases with P_T. The optimal duality scheme performs substantially better than the direct-PSM duality scheme, as expected, and both give high rates than the *a*-SNR SMuD scheme.

Fig. 3 shows the weighted sum rate vs. P_T for the duality schemes for both downlink and uplink CRN channels. The gaps between the optimal PSM duality schemes and the direct-PSM duality schemes are shown to be significant for both downlink and uplink. The performance gap between the cases of optimal duality schemes with and without PSM constraints is very small for P_T values between -10 to 0 dB for uplink but becomes significant when P_T increases. This shows that the negative effect of PSM is closely related to the operating CRN transmit powers.

VI. Conclusions

We have studied the optimization of multicell CRNs overlaid with multicell PRNs and proposed efficient mask fitting and weighted sum rate maximization approaches which included the optimal duality-PSM and the direct-PSM duality schemes. Simulation results have shown that the optimal duality-PSM performs substantially better than the direct-PSM duality scheme. While the PSM constraint can bring a significant performance loss to the CRN, the proposed PSM fitting methods can efficiently mitigate such losses. Based on our results, one can check the effects of other system and channel parameters on the weighted sum rates of SUs and optimize the performance of overlaid CRNs.

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