Simple SNR Wall Calculation by Equating the Medians of the Detector's Test Statistic

Dayan Adionel Guimarães

***Abstract*— An apparently definitive conclusion that can be drawn from the literature is that the calculation of the signal-to-noise ratio wall (SNR_w) of detectors for spectrum sensing is not trivial. Conventionally, to make this calculation one has to find the expressions of the probabilities of detection and false alarm, and of the required number of samples to achieve target probabilities under worst-case noise uncertainty. However, a simple calculation of the SNR_w can be devised based on a theorem stating that the existence of an SNR_w requires that the test statistics have overlapping medians under the two test hypotheses. Grounded on this theorem, in this paper it is devised such a simple calculation method, which is applied to find the SNR_w of the absolute value cumulating (AVC) detector and the energy detector (ED) under Gaussian and Laplacian noise. Simulation results are presented to support and complement the analytical findings.**

***Keywords*— Absolute value cumulating detector, energy detector, Gaussian noise, Laplacian noise, signal-to-noise ratio wall.**

I. INTRODUCTION

The increased demand for wireless communication services nowadays has become the main driver of new technologies, as exemplified by the recent advances on the fifth generation (5G) of communication networks and the Internet of things (IoT), and by the research initiatives on the sixth generation (6G) of these networks [1], [2]. As a consequence of this demand, radio-frequency (RF) spectrum scarcity has arisen, owed to the fact that current fixed spectrum allocation polices grant to the incumbent (primary user, PU) network the exclusive right to use certain RF portions.

The potential solution to the RF spectrum scarcity is to adopt a dynamic spectrum access (DSA) policy, in which PU and secondary user (SU) networks are allowed to share frequency bands, as long as no harm are caused to the primary network operation.

The cognitive radio (CR) paradigm [3] arose in this DSA context. Among a multitude of cognition-related attributes of a CR, it is also capable of identifying unoccupied primary RF bands for opportunistic access by secondary terminals, applying the technique known as spectrum sensing [1], [4].

Spectrum sensing is a binary hypothesis test in which $\mathcal{H}_0$ denotes the absence of the PU signal in the sensed band, whereas $\mathcal{H}_1$ denotes the presence of the PU signal. The test is made by forming a test statistic T from the received signal samples and comparing it with a decision threshold γ to decide in favor of $\mathcal{H}_1$ if $T > \gamma$ or $\mathcal{H}_0$ if $T < \gamma$.

The performance of spectrum sensing is commonly measured by means of the probability of detection $P_\text{d} = \Pr\{T > \gamma|\mathcal{H}_1\}$, and the probability of false alarm $P_\text{fa} = \Pr\{T > \gamma|\mathcal{H}_0\}$. The former is the probability of declaring the PU signal present in the sensed band, when it is indeed present. The latter is the probability of declaring the PU signal present in the sensed band, given that it is in fact absent.

The SU terminals must be able to detect very weak PU signals, but there is a fundamental limit to the detection at low signal-to-noise ratio (SNR), meaning that accurate detection is impossible below a certain level of SNR called SNR wall [5]. Simply stating, the SNR wall, which is hereafter denoted by SNR_w, is the upper limit below or equal to which it is impossible to control P_d and P_fa to achieve target values.

The SNR_w is an important metric mainly when the detector is semi-blind, that is, when it does not need any knowledge about the PU signal, but it makes use of the noise variance information in the computation of the test statistic or the decision threshold. This is the case of the well-known energy detector (ED), and the recently proposed absolute value cumulating (AVC) detector [6], [7].

The SNR_w does not depend only on the detector type and on the noise uncertainty level, but also on the characteristics of the noise impairing the received signal.

Hence, the analysis of the SNR wall of semi-blind detectors under different types of sensing channel noise is of practical and theoretical interest.

A. Related work

In [7], the performances of the ED and the AVC are assessed when impaired by Laplacian noise. The SNR_w of both detectors is computed under the conventional way, making use of the Gaussian approximation of the test statistics in the large number of samples regime, and making use of an approximate computation of the mean and variance of the test statistics in the Laplacian noise case, assuming that the PU signal is a baseband binary phase-shift keying (BPSK) signal.

The performance of the AVC over Laplacian noise is also analyzed in [8], where are derived the optimal decision threshold for minimizing the total error rate, and the exact mean and variance of the test statistic, which led to accurate expressions for the probabilities of detection and false alarm under the Gaussian approximation grounded on the central limit theorem. A baseband BPSK primary signal has been also considered in [8]. The SNR_w is not addressed.

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This work was supported in part by RNP, with resources from MCTIC, Grant No. No 01245.010604/2020-14, under the 6G Mobile Communications Systems project of the Radiocommunication Reference Center (*Centro de Referência em Radiocomunicações*, CRR) of Inatel, Brazil, and in part by CNPq and FAPEMIG.

In [9], the generalized energy detector (GED) is analytically studied when impaired by generalized noise with McLeish distribution. Particular cases of the closed form expressions and the noise model given in [9] can also be used to address the performance of the AVC and the ED, which are especial cases of the GED, over Gaussian and Laplacian noise. The SNR_w is not addressed in [9].

As far as the calculation of the signal-to-noise ratio wall is concerned, the conventional approach corresponds to finding the expressions of the probabilities of detection and false alarm, subsequently finding the required number of samples n to achieve target probabilities under worst-case noise uncertainty. The SNR_w is then obtained from the expression for computing the number of samples in the limit of $n \to \infty$. Many works addressing the SNR_w in the conventional way can be found; see for instance [5], [10]–[16] and references therein. To the best of the author's knowledge, no explicit calculation of the SNR_w using the simple approach proposed in this paper has been put forward in the literature so far.

B. Contribution and organization of the article

This paper proposes a simple method for calculating the SNR_w of detectors under Gaussian and Laplacian noise, without the need of operating on the expressions of the probabilities of detection and false alarm, and the number of samples required to achieve target performances. The method is applied to find the SNR_w of the detectors AVC and ED, where the approximate theoretical mean of the test statistics given in [7] is shown to be sufficiently accurate and simple for the purpose of finding the SNR_w in the Laplacian noise case, as opposed to the intricate exact mean derived in [8], even when the PU signal is Gaussian distributed. Moreover, the novel noise uncertainty model proposed in [16] is used instead of the conventional model often adopted in the literature.

The remainder of this paper is organized as follows: Section II presents the signal and noise models, and the test statistics of the ED and the AVC. Section III describes the proposed method for SNR_w calculation. Numerical results and discussions are given in Section IV. The conclusions are drawn in Section V.

II. SIGNAL AND NOISE MODELS AND TEST STATISTICS

It is assumed that the SUs monitor the signal transmitted by a single PU. The SU collects n samples of the received signal during the sensing interval. The n-dimensional vector $\mathbf{y}$ of received samples can be written as

$$\mathbf{y} = \begin{cases} h\mathbf{x} + \mathbf{v}, & \text{under } \mathcal{H}_1 \\ \mathbf{v}, & \text{under } \mathcal{H}_0 \end{cases}, \tag{1}$$

where $\mathbf{x}$ e $\mathbf{v}$ are the vectors that contain the samples of the signal and the noise, respectively, and h is the channel gain between the PU and the SU.

The elements of $\mathbf{x}$ are assumed to be zero-mean Gaussian random variables with variance σ_x^2. Two noise distributions are considered, that is, $\mathbf{v}$ is a vector of independent and identically distributed Gaussian or Laplacian random variables with zero mean and variance σ^2. Thus, the average signal-to-noise ratio, in dB, at the SU receiver is $\text{SNR} = 10\log_{10}(\sigma_\text{x}^2/\sigma^2)$, whose value can be set to the values chosen for σ_x^2 and $/\sigma^2$. Hereafter, it is assumed that $h = 1$, which models an additive noise sensing channel without fading.

The Gaussian noise is the common assumption adopted to represent the receiver front-end additive thermal noise, whereas the Laplacian noise is adopted to model the thermal noise added to an impulsive noise component.

The Gaussian and Laplacian probability density functions (PDFs) of a sample $v \equiv v_i$ in $\mathbf{v}$ are respectively

$$f(v) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{v^2}{2\sigma^2}\right) \tag{2}$$

and

$$f(v) = \frac{1}{\sqrt{2\sigma^2}} \exp\left(-\sqrt{\frac{2}{\sigma^2}}|v|\right), \tag{3}$$

where $|\cdot|$ denotes the absolute value operation.

From a practical standpoint, the ED and the AVC test statistics can be respectively written as

$$T_\text{ED} = \frac{1}{n\hat{\sigma}^2}\sum_{i=1}^{n} y_i^2 \tag{4}$$

and

$$T_\text{AVC} = \frac{1}{n\sqrt{\hat{\sigma}^2}}\sum_{i=1}^{n} |y_i|, \tag{5}$$

where y_i is the i-th sample in $\mathbf{y}$, and $\hat{\sigma}^2$ is the estimated noise variance that takes into account any noise uncertainty that may arise. Here, it is considered that this uncertainty is owed to the fact that, when the noise variance is unknown, it is estimated on-the-run and, hence, carries the inherent estimation error. An alternative approach considers that, even if $\hat{\sigma}^2$ is accurately determined during the detector design phase, variations in the actual σ^2 may occur due to imperfect calibration of the receiver's front-end, or due to unwanted signals entering the receiver as if they were noise.

III. SIGNAL-TO-NOISE RATIO WALL

Formally stating, the SNR_w is the upper limit below or equal to which it is impossible to control the probability of detection, P_d, and the probability of false alarm, P_fa, to yield $P_\text{fa} \approx 0$ and $P_\text{d} \approx 1$, or at least to attain desired values of these probabilities within their useful limits, which are $0 \leq P_\text{fa} \leq 0.5$ and $0.5 \leq P_\text{d} \leq 1$. Given an SNR of operation, such control is performed through the number of samples, n, meaning that, if $\text{SNR} \leq \text{SNR}_\text{w}$, the increase of n does not bring improvement of the spectrum sensing performance [17].

A. Preliminaries

Fig. 1 illustrates the PDFs of a test statistic $T \equiv T_\text{ED} \equiv T_\text{AVC}$ under $\mathcal{H}_0$ and $\mathcal{H}_1$, for arbitrary high values of n and SNR. Notice that it is possible to set the decision threshold somewhere in-between the PDFs so as to result in $P_\text{fa} \approx 0$ and $P_\text{d} \approx 1$, because μ_0 is easily distinguishable from μ_1. The reduction of the SNR corresponds to the approximation of the means of the two PDFs. However, (4) and (5) incorporate samples means, meaning that the variances of both PDFs tend to zero in the limit of $n \to \infty$, still allowing to find a correct place for the threshold even if $\text{SNR}_\text{w} \to 0$, that is, $\sigma_\text{x}^2/\sigma^2 \to 0$.

The SNR_w arises when noise uncertainty takes place, making the PDFs to overlap no matter how large n is. In this situation, there is no threshold capable of allowing control of the probabilities of detection and false alarm.

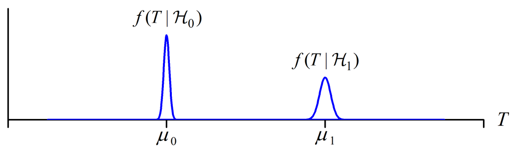

Fig. 1. Distributions of the test statistic for a large n and a large SNR.

The reasoning behind the method proposed herein is based on the above explanation, and is formally grounded on a theorem from [5], stating that the existence of an SNR wall below which every detector is not capable of meeting useful performance metrics requires the test statistics to have overlapping medians under the two hypotheses. In the cases of T_ED and T_AVC, the medians are equal to the corresponding means, due to the Gaussian approximation of the distributions of their test statistics for sufficiently large n.

Hence, the SNR_w calculation method works by equating the means of the test statistic under $\mathcal{H}_0$ and $\mathcal{H}_1$, for the worst-case noise uncertainty, subsequently determining the SNR $\sigma_\text{x}^2/\sigma^2$, which is the SNR_w. To exemplify the application of the method, the ED and the AVC test statistics are considered as case studies in the remainder of this section.

B. Means of T_{ED} and T_{AVC}

The mean of the ED test statistic T_ED given in (4) under $\mathcal{H}_0$, for Gaussian (G) and Laplacian (L) noise types, is $\mathbb{E}[T_\text{ED}|\mathcal{H}_0] = \mu_0 = (1/\hat{\sigma}^2)(1/n)n\mathbb{E}[y_i^2] = (1/\hat{\sigma}^2)\mathbb{E}[v_i^2]$, which yields

$$\mu_0^\text{ED,G,L} = \frac{\sigma^2}{\hat{\sigma}^2}. \tag{6}$$

Analogously, the mean of T_ED for Gaussian and Laplacian noise types under $\mathcal{H}_1$ is given by $\mu_1 = (1/\hat{\sigma}^2)\mathbb{E}[(x_i+v_i)^2] = (1/\hat{\sigma}^2)\mathbb{E}[x_i^2 + 2x_iv_i + v_i^2]$. Taking into account the independence between x_i and v_i, then it follows that

$$\mu_1^\text{ED,G,L} = \frac{\sigma_\text{x}^2 + \sigma^2}{\hat{\sigma}^2}. \tag{7}$$

The mean of the AVC test statistic T_AVC given in (5) under $\mathcal{H}_0$ and Gaussian noise is determined from the mean of a folded Gaussian distribution [18, Eqn. (7)]. Hence, $\mathbb{E}[T_\text{AVC}|\mathcal{H}_0] = \mu_0 = (1/\hat{\sigma})(1/n)n\mathbb{E}[|y_i|] = (1/\hat{\sigma})\mathbb{E}[|v_i|]$, which yields

$$\mu_0^\text{AVC,G} = \sqrt{\frac{2\sigma^2}{\pi\hat{\sigma}^2}}. \tag{8}$$

Analogously, the mean of T_AVC for Gaussian noise under $\mathcal{H}_1$ is $\mathbb{E}[T_\text{AVC}|\mathcal{H}_1] = \mu_1 = (1/\hat{\sigma})(1/n)n\mathbb{E}[|x_i+v_i|]$. Recalling that the sum of independent zero-mean Gaussian random variables is another zero-mean Gaussian random variable having variance equal to the sum of the variances of the added variables, then it immediately follows that

$$\mu_1^\text{AVC,G} = \sqrt{\frac{2(\sigma_\text{x}^2 + \sigma^2)}{\pi\hat{\sigma}^2}}. \tag{9}$$

In the case of Laplacian noise, the mean of T_AVC under $\mathcal{H}_0$ can be obtained based on a result in [7], yielding

$$\mu_0^\text{AVC,L} = \sqrt{\frac{\sigma^2}{2\hat{\sigma}^2}}. \tag{10}$$

Also based on [7], where an approximate mean of T_AVC under $\mathcal{H}_1$ and Laplacian noise is given, it follows that

$$\mu_1^\text{AVC,L} = \sqrt{\frac{\sigma_\text{x}^2 + \sigma^2/2}{\hat{\sigma}^2}}. \tag{11}$$

Fig. 2 shows approximate values of μ_1 obtained from (11), and the true μ_1, as functions of the SNR, assuming $\sigma_\text{x}^2 = 1$ and $\hat{\sigma}^2 = \sigma^2$ without loss of generality. It can be seen that (11) produces accurate results for SNRs below -5 dB, which suffices for the present analysis since the SNR_w is often below this value.

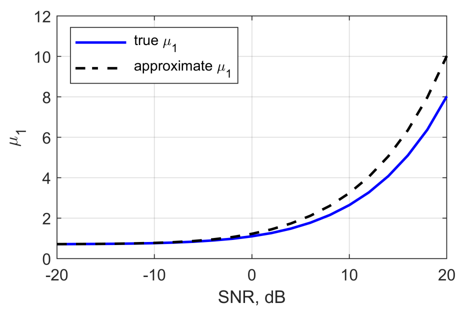

Fig. 2. True and approximate means of the absolute value of the sum of a Gaussian and a Laplacian random variable with zero means.

C. Noise uncertainty models

Two noise uncertainty models are considered in this paper. The conventional model assumes that $\hat{\sigma}^2$ lies in-between σ^2/ρ and $\rho\sigma^2$, that is,

$$\hat{\sigma}^2 \in [\sigma^2/\rho, \rho\sigma^2], \tag{12}$$

where $\rho \geq 1$ is the noise uncertainty parameter that governs the amount of uncertainty on σ^2.

A recently-proposed model [16] considers that $\hat{\sigma}^2$ lies in-between $(1-\rho)\sigma^2$ and $(1+\rho)\sigma^2$, that is,

$$\hat{\sigma}^2 \in [(1-\rho)\sigma^2, (1+\rho)\sigma^2], \tag{13}$$

where $0 \leq \rho < 1$ is the noise uncertainty parameter.

D. SNR_w computation

The calculations of the SNR_w are made by equating μ_0 and μ_1 for each test statistic, for each channel type, and for the worst cases in each of the above-described noise uncertainty models. The worst cases correspond to a reduction of $\hat{\sigma}^2$ under the hypothesis $\mathcal{H}_0$, and an increase of $\hat{\sigma}^2$ under $\mathcal{H}_1$, which makes the PDFs of the test statistic to get closer to each other.

Equating the means (6) and (7) and replacing $\hat{\sigma}^2$ by the worst-case uncertainty limits given in (12), one obtains $\sigma^2/(\sigma^2/\rho) = (\sigma_x^2 + \sigma^2)/(\rho\sigma^2)$. Using the fact that $\sigma_x^2/\sigma^2 =$ SNR, after some simple manipulations one obtains the SNR_w for the ED under Gaussian (G) or Laplacian (L) noise and uncertainty model (12), which is

$$\text{SNR}_\text{w}^{\text{ED,G,L,(12)}} = \rho^2 - 1. \tag{14}$$

Again equating (6) and (7), now replacing $\hat{\sigma}^2$ by the worst-case uncertainty limits given in (13), one obtains $\sigma^2/[(1-\rho)\sigma^2] = (\sigma_x^2 + \sigma^2)/[(1+\rho)\sigma^2]$. After some manipulations, it follows that the SNR_w for the ED under Gaussian or Laplacian noise and uncertainty model (13) is given by

$$\text{SNR}_\text{w}^{\text{ED,G,L,(13)}} = \frac{2\rho}{1-\rho}, \tag{15}$$

which matches [16, Eqn. (28)], a result that has been obtained using the conventional SNR_w calculation approach.

Equating (8) and (9) and replacing $\hat{\sigma}^2$ by the worst-case uncertainty limits given in (12), it is easy to find out that the SNR_w for the AVC detector under Gaussian noise is

$$\text{SNR}_\text{w}^{\text{AVC,G,(12)}} = \rho^2 - 1, \tag{16}$$

which is identical to the SNR_w given in (14), that is, the SNR wall of the ED and the AVC detectors when subjected to Gaussian noise are the same. This conclusion is consistent with the one stated in [11] regarding the immutability of the SNR wall for any particular case of the GED (recall that the ED and the AVC are particular cases of the GED).

Now equating (8) and (9) and replacing $\hat{\sigma}^2$ by the limits given in (13), it is also easy to conclude that the SNR_w for the AVC detector under Gaussian noise is

$$\text{SNR}_\text{w}^{\text{AVC,G,(13)}} = \frac{2\rho}{1-\rho}, \tag{17}$$

which is identical to (15), as expected according to the comments in the end of the previous paragraph.

When the AVC detector is impaired by Laplacian noise, equating (10) and (11) and replacing $\hat{\sigma}^2$ by the limits given in (12), it follows that

$$\text{SNR}_\text{w}^{\text{AVC,L,(12)}} = \frac{\rho^2 - 1}{2}, \tag{18}$$

which is half of the value determined from (16). This means that the AVC detector is more robust than the ED when subjected to Laplacian noise. From another perspective, it means that the AVC detector is capable of outperforming the ED when impaired by Laplacian noise [7].

Finally, if (10) and (11) are equated, with $\hat{\sigma}^2$ replaced by the limits given in (13), it is easily found that the SNR_w for the AVC detector under Laplacian noise becomes

$$\text{SNR}_\text{w}^{\text{AVC,L,(13)}} = \frac{\rho}{1-\rho}, \tag{19}$$

which is half of the value obtained from (17), as expected.

It is worth highlighting that the noise uncertainty determined by both models investigated herein is applied in the noise variance used in the test statistic (or, equivalently, in the decision threshold), aiming consistence with the uncertainty produced by noise variance estimation errors [16]. The conventional practice assumes prior knowledge of the nominal noise variance σ^2 (the decision threshold is set according to it), and noise uncertainty takes place by assuming that the actual noise variance $\hat{\sigma}^2$ deviates from σ^2 due to interfering signals treated as noise. Nonetheless, the simple method for calculating the SNR_w considered herein also applies to this conventional approach. For instance, if (4) is written as $T_\text{ED} = \frac{1}{n\sigma^2}\sum_{i=1}^{n} y_i^2$, with $\hat{\sigma}^2$ being the unknown noise variance impairing y_i, the means (6) and (7) would become $\mu_0 = \hat{\sigma}^2/\sigma^2$ and $\mu_1 = (\sigma_x^2 + \hat{\sigma}^2)/\sigma^2$, respectively. Equating these means under the limits given in (12), one easily obtains $\text{SNR}_\text{w}^{\text{ED,G,(12)}} = (\rho^2 - 1)/\rho$, which is a classical well-known outcome [5].

IV. Numerical results

This section presents Monte-Carlo simulation results showing P_fa versus P_d for the detectors ED and AVC, under Gaussian and Laplacian noise, with and without noise uncertainty, applying the worst-case noise uncertainty limits given in (13). All results were obtained by averaging 10^3 independent spectrum sensing trials.

Fig. 3 shows the results for SNR $= -10$ dB and $n = 1500$ samples, under different noise uncertainties. In the case of $\rho = 0.02$, from (15) and (17) one obtains $\text{SNR}_\text{w} \approx -13.89$ dB for the ED under Gaussian or Laplacian noise, and for the AVC under Gaussian noise, respectively. From (19) one obtains $\text{SNR}_\text{w} \approx -16.89$ dB for the AVC detector under Laplacian noise. Since SNR $> \text{SNR}_\text{w}$, full control of P_fa versus P_d can be made, in the case targeting $P_\text{d} > 0.9$ and $P_\text{fa} < 0.1$ for the best detector with $\rho = 0$.

From Fig. 3 it can be seen that the ED and the AVC have approximately the same performance under Gaussian noise, which is consistent with their equal SNR_w. A slight advantage of the ED is observed in this situation.

When impaired by Laplacian noise, it can be seen from Fig. 3 that the AVC outperforms the ED, which is also consistent with the smaller SNR_w of the AVC in this case. Comparing the two graphs in this figure, it can be seen the performance degradation of both detectors when noise uncertainty takes place, with a larger degradation of the ED under Laplacian noise.

Fig. 4 shows the performances of the ED and the AVC over Gaussian and Laplacian noise under different noise uncertainties, SNR $= -14$ dB, $n = 8000$ samples (left), $n = 50000$ samples (right). On the left-hand side graph the situation is similar to the corresponding graph in Fig. 3, highlighting the increase in n from 1500 to 8000 to maintain approximately the same performances even with the reduction in the SNR from -10 dB to -14 dB. Regarding the right-hand side graph of Fig. 4, where noise uncertainty takes place, the SNR $= -14$ dB is above the $\text{SNR}_\text{w} = -16.89$ of the AVC under Laplacian noise, which allowed the target performance by increasing n from 8000 to 50000 samples with respect to the left-hand side graph. However, the SNR $= -14$ dB is below the $\text{SNR}_\text{w} = -13.89$ dB of the AVC under Gaussian noise, and of the ED under Gaussian or Laplacian noise, which has causes a useless performance for both detectors.

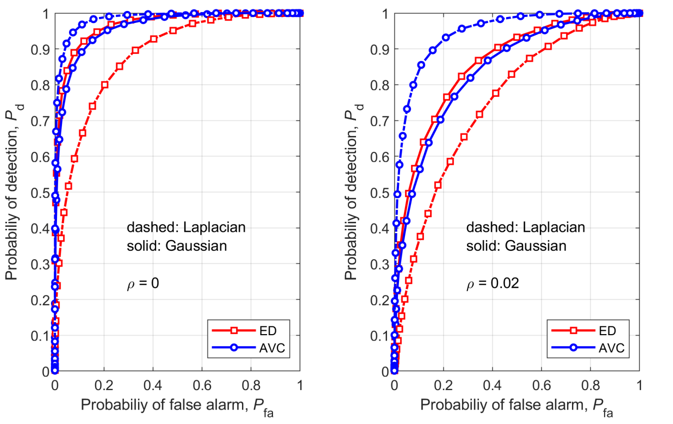

Fig. 3. Performances of ED and AVC over Gaussian and Laplacian noise under different noise uncertainties. SNR $= -10$ dB, $n = 1500$ samples.

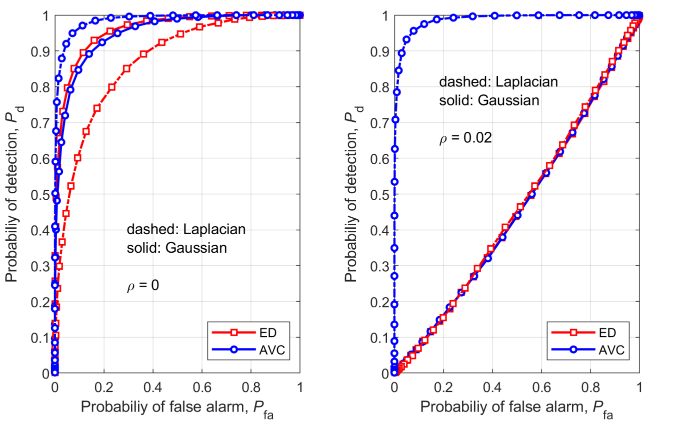

Fig. 4. Performances of ED and AVC over Gaussian and Laplacian noise under different noise uncertainties. SNR $= -14$ dB, $n = 8000$ (left), $n = 50000$ (right).

V. Conclusions

This paper proposed a simple method for calculating the SNR wall of detectors for spectrum sensing, removing the need of knowing the expressions for computing the probabilities of detection and false alarm, and the number of samples as a function of the SNR. The method was applied to find the SNR wall of the detectors AVC and ED when impaired by Gaussian or Laplacian noise.

The simple method proposed herein can be applied to any test statistic, as long as the expressions to compute the means (or medians) under both test hypotheses are known. Nonetheless, if they are unknown, one may resort to an empirical SNR wall computation using the same reasoning of equality in the means (or medians), which is exemplified by an algorithm recently proposed in [16].

A natural extension of this work is the application of the method considered herein to calculate the SNR wall of the ED and the AVC in the case of centralized cooperative spectrum sensing with soft-decision and hard-decision fusion, and to consider the conventional placement of the noise uncertainty in the received signal samples instead of the test statistics or the decision threshold.

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