Influence of Noise Uncertainty Source and Model on the SNR Wall of Energy Detection

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***Abstract*—Recently, we conducted a comprehensive analysis of energy detection (ED) signal-to-noise ratio wall (SNRw) due to noise uncertainty (NU) in cognitive-radio (CR)-based non-cooperative spectrum sensing (nCSS) and cooperative spectrum sensing (CSS) with soft-decision (SD) and hard-decision (HD) fusion under the k-out-of-M rule. It derived the SNRw for a novel NU source and model adopting a truncated Gaussian NU distribution at the CRs and proposed empirical algorithms for SNRw estimation. Based on it, this article conducts another extensive ED study by deriving new closed-form SNRw expressions combining novel and traditional NU sources and models in nCSS and CSS with SD and HD k-out-of-M rule. Besides the conventional test statistic in CSS with SD, it also considers a more general one that, to our best knowledge, was never studied under NU. This new ED computation improves detection performance when CRs are under unequal noise powers and leads to a more conservative (higher) SNRw when CRs are under unequal NU levels in the novel NU source and model combination. Yet, this article maps new and previous derivations for easier comparisons involving any NU source and model combination, more easily highlighting its advantages. Simulations validate the theoretical findings.**

***Index Terms*—Cognitive radio; energy detection; noise uncertainty; signal-to-noise ratio wall; spectrum sensing.**

I. Introduction

RECENT studies unveil that free radio-frequency (RF) spectrum are currently scarce and that those already allocated are underutilized [1]. Spectrum scarceness is a consequence of the growth in demand for new telecommunication services in the last decades, as one can notice, e.g., from the Internet of things (IoT) and the fifth-generation (5G) communication networks [2]. Spectrum underutilization is owed mainly to the current fixed allocation policy, which gives spectrum access rights only to incumbents, or primary users (PUs), and to the low activity of incumbents in accessing their bands during the contracted time in a given geographical area [1], [3]. Spectrum scarceness and underutilization have been motivating studies on dynamic spectrum access (DSA) policies, in which non-incumbents, or secondary users (SUs), can share RF bands with PUs simultaneously in a non-interfering basis, or opportunistically when bands vacated. The cognitive radio (CR) concept has arisen in this context as a promising DSA solution to the RF spectrum shortage by using spectrum sensing (SS) [4] to detect unused frequency portions.

SS can be non-cooperative or cooperative. In non-cooperative spectrum sensing (nCSS), each CR/SU is in charge of sensing a band and deciding whether it is vacant or occupied, independently of other SUs in the secondary network. In cooperative spectrum sensing (CSS), the secondary network makes this occupation decision under the collaboration of a group of SUs, conferring more accuracy to the detection process. CSS can be centralized, distributed, or relay-assisted [4]. In centralized CSS, the SUs sense a given band and share the sensing information with a fusion center (FC), which combines all received information and infers about its occupation state. The sensing information can be local SUs decisions under the hard-decision (HD) fusion or the received signal samples under the soft-decision (SD) fusion. In the HD fusion, the k-out-of-M rule is generally adopted at the FC to combine the decisions received from the SUs and make the final global decision, where k is the number of received decisions favoring the hypothesis $\mathcal{H}_1$ that the PU signal is present in the sensed band, and M is the number of SUs in cooperation. The FC decides in favor of $\mathcal{H}_1$ if k or more decisions favor $\mathcal{H}_1$ or in favor of $\mathcal{H}_0$ otherwise, in which $\mathcal{H}_0$ denotes the PU signal absence hypothesis. When $k = 1$, $k = M$, or $k = \lfloor M/2 + 1 \rfloor$, the k-out-of-M rule becomes the well-known OR, AND, or majority (MAJ) voting rule [4], respectively, where $\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x.

Secondary networks achieve SU decisions in nCSS and CSS with HD fusion or FC decisions in CSS with SD fusion by comparing a test statistic computed from the received signal samples with a decision threshold. There are test statistics for different detection strategies [5], but energy detection (ED) is the most widely used due to its low complexity. However, high sensitivity to inaccuracy in the additive white Gaussian noise (AWGN) variance in the receiver input is its main drawback, and the literature refers to this inaccuracy as noise uncertainty (NU) [6]. NU can severely degrade the detection performance of ED and also impose a limit on the signal-to-noise ratio

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(SNR) called signal-to-noise ratio wall (SNRw). The SNRw is the SNR below or equal to which a detector cannot detect signals accurately, no matter the sensing time spent.

Motivated by the relevance of the SNRw for the design and analysis of ED-based SS, this article addresses the ED detection performance under NU and the ED SNRw due to NU, in nCSS, CSS with SD, and CSS with HD under the k-out-of-M rule.

A. Related research

There are several studies on deriving expressions for the ED SNRw due to NU, on analyzing the detection performance of ED under NU, or on proposals for decreasing the effect of NU over the ED in different SS circumstances, scenarios, NU models, and NU distributions [7]–[23].

The works in [7] and [8], e.g., proved the existence of an SNRw due to NU in generalized energy detection (GED), in which ED is a particular case, and derived the ED SNRw due to NU in general signal detection applications in low SNR regimes, respectively. On the other hand, the works in [9]–[12] and [13] derived the SNRw of GED or ED due to NU in CR-based SS applications. More specifically, [9] derived the GED SNRw in nCSS and showed that GED SNRw is identical to the SNRw of ED. Besides, it adopted uniform distribution in decibels (dB) for the NU at the SU and calculated the SNRw of GED under lognormally distributed NU numerically. [10] derived a more general ED SNRw expression considering heterogeneous SUs, in which SUs may experience different NU levels and nominal PU signal and noise powers in CSS with SD. Results showed that the traditional expression derived for the ED SNRw assuming homogeneous SUs is a particular case of the generalized ED SNRw for heterogeneous SUs. [11] adopted the same NU distribution used in [9] but derived the GED SNRw in CSS with SD and HD under the k-out-of-M rule and showed that, as in nCSS, the SNRw of GED and ED are identical in CSS. One can find more results related to [11] in [12], which considered SS analyses in different circumstances and scenarios and derived an expression for numerical computation of GED SNRw under Nakagami fading. Finally, [13] presented a comprehensive study on ED under NU with a novel (truncated Gaussian) NU distribution at the SUs. It derived the SNRw of ED in nCSS and CSS with SD and HD under the k-out-of-M rule for the more general expressions for the probabilities of false alarm and detection at the FC compared to [12], also considering a novel NU source and model. It also proposed two empirical algorithms for SNRw estimation in nCSS and CSS that one can apply to ED or other related detectors.

Regarding detection performance analyses and proposals for decreasing the NU effect over the ED, [14] presents a design of ED in CR-based nCSS under NU. It proposed a discrete and a continuous NU model and showed that assuming a prior knowledge of NU distribution, choosing a proper decision threshold can improve performance as much as possible under NU. [15] studied the effects of noise power estimation on ED in nCSS and established the SNR penalty in dB imposed on practical ED compared to ideal ED, which assumes the exact knowledge of the noise variance at the SU. [23] studies the optimal ED sensing duration under NU to attain the desired detection performance at a given SNR. [17] presented a performance analysis of ED under NU in CSS and showed that CSS, or a suitable decision threshold setting, is hopeful for decreasing the limitation imposed by the SNRw. [18] verified the unusability of the traditional ED in CSS under NU and proposed a dual threshold-based optimum power ED algorithm that, even under NU, can outperform the conventional ED without NU. [19] and [20] also proposed double threshold-based algorithms for the ED under NU in CSS with SD and HD fusion that enhance detection performances compared to conventional ED with SD and HD, respectively. Finally, [21] and [22] present performance analyses of ED in CSS, nCSS, or both, under NU over generalized fading channels modeled by Nakagami-m/Gamma and $\kappa - \mu$ distribution, respectively.

As shown in the ED-related research in [7]–[23], the most used NU modeling considers the noise variance estimate, $\hat{\sigma}_{\text{v}}^2$, as being a random variable uniformly distributed in dB in the linear range of $[a = (1/\rho)\sigma_{\text{v}}^2, b = \rho\sigma_{\text{v}}^2]$, *i.e.*, $\hat{\sigma}_{\text{v}}^2 \in [a = (1/\rho)\sigma_{\text{v}}^2, b = \rho\sigma_{\text{v}}^2]$, in which $\rho \geq 1$ is an NU parameter that defines the amount of uncertainty on the knowledge of the exact noise variance value σ_{v}^2. Thus, see that most of the works employ a biased NU model since $(a + b)/2 \neq \sigma_{\text{v}}^2$ for $\rho > 1$. Besides, these works commonly assume *a priori* knowledge of σ_{v}^2 in the decision threshold computation, meaning that they consider unwanted interfering signals arriving at the SUs during SS as NU sources. In these cases, the decision threshold does not change due to NU since it uses σ_{v}^2, but the mean and variance of the ED test statistic change according to the existence of interference signals. Due to the assumption on the threshold computation or interfering signals, one can not mitigate the effect of NU or model NU mitigation. Hereafter, we refer to the model associated with the assumptions regarding the above NU interfering signals and NU interval as the traditional NU source and model, respectivley.

Differently from the assumptions above, [13] adopted a novel NU distribution at the SUs, proposed a new NU source and model, and derived new closed-form expressions for the SNRw of ED due to NU in nCSS and CSS with SD and HD fusion under the k-out-of-M rule using the more general expressions for the probabilities of false alarm and detection at the FC. More specifically, it considered the noise variance estimate as a truncated Gaussian random variable in the linear range of $[a = (1 - \rho)\sigma_{\text{v}}^2, b = (1 + \rho)\sigma_{\text{v}}^2]$, *i.e.*, $\hat{\sigma}_{\text{v}}^2 \in [a = (1 - \rho)\sigma_{\text{v}}^2, b = (1 + \rho)\sigma_{\text{v}}^2]$, in which $0 \leq \rho < 1$ is the NU parameter in this case. Thus, see that [13] considered an unbiased NU modeling since now $(a + b)/2 = \sigma_{\text{v}}^2$ for $0 \leq \rho < 1$. Yet, [13] adopted a different perspective on the source of NU, where the received signal at each SU is affected by a possibly variable thermal noise (with a distinct expected value, meaning unequal average noise powers at the SUs) plus interfering signals having unknown variance σ_{v}^2. This means that the decision threshold uses the noise variance estimate $\hat{\sigma}_{\text{v}}^2$ instead of its exact value σ_{v}^2 in its computation. So, it is clear that the decision threshold now changes due to NU according to the level of accuracy in estimating the noise variance at each SU, but the mean and variance of the test statistic do not change since they follow σ_{v}^2. Because of the assumption on the threshold computation and interfering signals, one can now mitigate the effect of NU (by improving the estimation accuracy) and also model NU mitigation since they track the variances of noise and interference signals in the estimation process. Hereafter, we refer to the model associated with the above interfering signals and NU interval assumptions as the novel NU source and model, respectively.

B. Contributions and structure of the article

Based on [13] and motivated by the possibility of defining different combinations involving the traditional and novel NU sources and models, we present a comprehensive analysis of ED detection performances under NU and ED SNRw due to NU in CR-based nCSS and CSS with SD and HD fusion under the k-out-of-M rule in this article. We adopt SUs under unequal noise powers and the same NU distribution adopted in [13], *i.e.*, the truncated Gaussian random variable for modeling SU noise variance estimates. However, we assume possibly unequal PU signal powers at the SUs in CSS, differently from [13]. Specifically, about the PU signal power at the ith SU, we consider $\sigma_{\mathrm{s}_i}^2 \neq \sigma_{\mathrm{s}_j}^2$ for $i, j = 1, \ldots, M$, $i \neq j$, while [13] considers $\sigma_{\mathrm{s}_i}^2 = \sigma_{\mathrm{s}}^2$ for all i. Assuming these, we derive closed-form SNRw expressions for the combinations between i) the traditional NU source and the traditional NU model, ii) the traditional NU source and the novel NU model, iii) the novel NU source and the novel NU model, and iv) the novel NU source and the traditional NU model. For the CSS with SD, we use a novel and more general ED test statistic computation at the FC that improves detection performances when the SUs are under unequal noise powers (compared to the conventional ED) and leads to a more conservative (higher) SNRw when they are under different NU parameters when using this new computation joined with the combination iii). To the best of our knowledge, there are no analyses of detection performance under NU or SNRw due to NU considering it in the literature yet. We also map our derivations into previous ones in the literature so one can easily make comparisons and highlight the advantages and disadvantages of each NU combination. Finally, in Appendix A, we also use the proposed mapping scheme to derive the ED SNRw in CSS with SD under the conventional ED for the NU combinations i), ii), iii), and iv) for allowing comparisons involving this novel ED computation. Empirical and Monte Carlo simulation results validate the theoretical results.

One may notice at this point that NU can arise from the dynamic nature of SS environments and that the above-described approach does not explicitly employ any modeling for node mobility. In other words, one may notice that our approach does not expressly consider the effects caused by mobility on the interfering signals. However, we assume that our NU modeling encompasses these effects implicitly. We treat mobility by adopting unequal average PU signal powers, σ_{s}^2, and unequal average and time-varying noise powers, $\hat{\sigma}_{\mathrm{v}}^2$, at the SUs, mimicking different average SNRs and, thus, different distances between nodes in each SS round. We assume that these considerations mimic [24], in a simple way, the dynamism of SS areas, and we treat interfering signals plus noise just as noise via the estimates in $\hat{\sigma}_{\mathrm{v}}^2$. See from [7]–[23] that modeling NU simply by varying $\hat{\sigma}_{\mathrm{v}}^2$ in $[a, b]$ with a given distribution is widely accepted in the literature.

Considering these, the following summarizes the main contributions of this article.

1) It assumes heterogeneous non-incumbents compared to [13] by adopting unequal PU signal powers in addition to unequal noise powers and NU levels at the SUs in all CSS analyses;
2) It proposes an NU mapping scheme that facilitates the SNRw derivations for the combinations i), ii), iii), and iv) and that favors comparisons among the expressions derived;
3) It provides performance analyses and SNRw derivations for the combinations i), ii), iii), and iv) in nCSS and CSS with SD and HD, all via the proposed NU mapping scheme;
4) It proposes the use of a novel ED test statistic computation, in addition to the conventional one, for performance analyses and SNRw derivations in CSS with SD under NU.

Lastly, we emphasize that [7]–[23] consider only the traditional NU source and model, and [13] only the novel NU source and model, both NU combinations respectively discussed in the last two paragraphs of Subsection I-A and ordered as i) and iii) in the first paragraph of this subsection, in detection analyses; yet with only the conventional ED test statistic computation in CSS with SD. Besides, all derivations here are new, but those based on the combination i) lead to the well-known literature SNRw expression, and those based on the combination iii) to two SNRw expressions in [13], in which [13] served as a basis for this article. To our best knowledge, none of the existing literature SNRw derivations (including those in [13]) proposed or used any mapping scheme to convert any NU modeling into any other available as in this article.

Subsequent sections organize the remaining parts of this article as follows. Section II presents the system models and ED performance metrics in nCSS and CSS with HD and SD fusion for the traditional and novel test statistic computations at the FC in the absence of NU and devises the NU modeling. Section III addresses the performance metrics under NU and derives the SNRw in nCSS and CSS with HD and SD fusion with the novel test statistic computation under all four NU source and model combinations. Section IV presents all numerical results, and Section V concludes the article. Appendix A gives the ED SNRw derivations in CSS with SD fusion using the traditional test statistic computation for all four NU source and model combinations.

II. Models and Performance Metrics

A. Signal model

SS is a binary hypothesis test given by $\mathcal{H}_0$ and $\mathcal{H}_1$ in which $\mathcal{H}_0$ or $\mathcal{H}_1$ represents the hypothesis of the absence or presence of a PU signal in the sensed band, respectively. For a primary network with a single PU transmitter and a secondary network with M SUs, the nth sample received at the ith SU in a given sensing interval, for $n = 1, \ldots, N$ and $i = 1, \ldots, M$, can be represented by

$$y_i(n) = \begin{cases} v_i(n) & \text{, under } \mathcal{H}_0, \\ s_i(n) + v_i(n) & \text{, under } \mathcal{H}_1, \end{cases} \quad (1)$$

where $s_i(n)$ and $v_i(n)$ are zero-mean circularly symmetric complex Gaussian-distributed [6] random variables with variances $\sigma^2_{\mathrm{s}_i}$ and $\sigma^2_{\mathrm{v}_i}$, that is, $s_i(n) \sim \mathcal{CN}(0, \sigma^2_{\mathrm{s}_i})$ and $v_i(n) \sim \mathcal{CN}(0, \sigma^2_{\mathrm{v}_i})$, which represent the nth PU signal and AWGN samples received at the ith SU, respectively. The secondary network decides between $\mathcal{H}_0$ or $\mathcal{H}_1$ by computing a test statistic T and comparing it with a predefined decision threshold τ. The decision is made in favor of $\mathcal{H}_1$ if $T > \tau$ or $\mathcal{H}_0$ otherwise. Performance analyses of these decisions commonly are made through the probabilities of false alarm, P_{fa}, and detection, P_{d}, respectively defined as [4]

$$\begin{cases} P_{\mathrm{fa}} = \Pr(\text{decision} = \mathcal{H}_1 | \mathcal{H}_0) = \Pr(T > \tau | \mathcal{H}_0) \text{ and} & (2a) \\ P_{\mathrm{d}} = \Pr(\text{decision} = \mathcal{H}_1 | \mathcal{H}_1) = \Pr(T > \tau | \mathcal{H}_1). & (2b) \end{cases}$$

P_{fa} and P_{d} are respectively the probability of having a decision favoring hypothesis $\mathcal{H}_1$ when the sensed PU band is under hypothesis $\mathcal{H}_0$ or $\mathcal{H}_1$, or, in other words, the probability of having $T > \tau$ when the PU signal is absent or present in the sensed PU band.

B. Local performance metrics without NU

The ED test statistic computation at the ith SU can be given by [11], [13], [16]

$$T_i = \tfrac{1}{N} \sum\nolimits_{n=1}^{N} |y_i(n)|^2, \quad (3)$$

with $|x|$ being the absolute value of x. According to the central limit theorem, T_i closely follows a Gaussian distribution if N is sufficiently large. Thus, the mean and variance of T_i are [11]

$$\begin{cases} \mu_{0_i} = \sigma^2_{\mathrm{v}_i} \text{ and } \sigma^2_{0_i} = \sigma^4_{\mathrm{v}_i}/N \text{ under } \mathcal{H}_0, \text{ and} & (4a) \\ \mu_{1_i} = \sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i} \text{ and } \sigma^2_{1_i} = (\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})^2/N \text{ under } \mathcal{H}_1, & (4b) \end{cases}$$

respectively. Hence, the local performance metrics are

$$\begin{cases} P_{\mathrm{fa}_i} = Q\left[(\tau_i - \mu_{0_i})/\sigma_{0_i}\right] = Q\left[(\tau_i - \sigma^2_{\mathrm{v}_i})\sqrt{N}\sigma^{-2}_{\mathrm{v}_i}\right] \text{ and} & (5a) \\ P_{\mathrm{d}_i} = Q\left(\dfrac{\tau_i - \mu_{1_i}}{\sigma_{1_i}}\right) = Q\left(\dfrac{\tau_i - (\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})}{(\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})/\sqrt{N}}\right), & (5b) \end{cases}$$

where $Q(t) = \int_t^{\infty} \exp\left(-\frac{x^2}{2}\right)/\sqrt{2\pi} dx$ is the standard Gaussian Q-function, $\tau_i = \lambda \sigma^2_{\mathrm{v}_i}$ is the decision threshold at the ith SU, and $\lambda > 0$ is a constant set to meet the target P_{fa_i}, P_{d_i}, or both.

C. Global performance metrics without NU

1) CSS with HD fusion under the k-out-of-M rule: One can write the more general expressions for the global performance metrics Q_{fa} and Q_{d} at the FC as [13]

$$Q_\chi = \sum\nolimits_{\ell=k}^{M} \sum\nolimits_{\mathbf{d}: w_{\mathrm{H}}(\mathbf{d})=\ell} \prod\nolimits_{i=1}^{M} P^{d_i}_{\chi_i} (1 - P_{\chi_i})^{1-d_i}, \quad (6)$$

where P_{χ_i} comes from the local metric in (5a) or (5b), with χ = 'fa' or χ = 'd' accordingly. The vector $\mathbf{d} = (d_1, \ldots, d_M)$ represents the binary M-tuple whose ith entry is the local occupation decision made by the ith SU in favor of $\mathcal{H}_0$, given by $d_i = 0$, or $\mathcal{H}_1$, by $d_i = 1$, and $w_{\mathrm{H}}(\cdot)$ denotes the Hamming weight, $w_{\mathrm{H}}(\mathbf{d}) = \sum_{i=1}^{M} d_i$.

2) CSS with SD fusion:

a) Traditional test statistic computation: The traditional ED test statistic is [11]

$$T = \tfrac{1}{M} \sum\nolimits_{i=1}^{M} T_i, \quad (7)$$

where T_i comes from (3). If N is large enough, T closely follows a Gaussian distribution, with mean and variance under $\mathcal{H}_0$ and $\mathcal{H}_1$ given respectively by

$$\begin{cases} \mu_0 = \frac{1}{M} \sum_{i=1}^{M} \sigma^2_{\mathrm{v}_i} \text{ and } \sigma^2_0 = \frac{1}{M^2 N} \sum_{i=1}^{M} \sigma^4_{\mathrm{v}_i}, \text{ and} & (8a) \\ \mu_1 = \frac{1}{M} \sum_{i=1}^{M} (\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i}) \text{ and } \sigma^2_1 = \frac{1}{M^2 N} \sum_{i=1}^{M} (\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})^2. & (8b) \end{cases}$$

Therefore, one can write Q_{fa} and Q_{d} as

$$\begin{cases} Q_{\mathrm{fa}} = Q\left(\dfrac{\tau - \mu_0}{\sigma_0}\right) = Q\left(\dfrac{M\tau - \sum_{i=1}^{M} \sigma^2_{\mathrm{v}_i}}{\frac{1}{\sqrt{N}} \sqrt{\sum_{i=1}^{M} \sigma^4_{\mathrm{v}_i}}}\right) \text{ and} & (9a) \\ Q_{\mathrm{d}} = Q\left(\dfrac{\tau - \mu_1}{\sigma_1}\right) = Q\left(\dfrac{M\tau - \sum_{i=1}^{M} (\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})}{\frac{1}{\sqrt{N}} \sqrt{\sum_{i=1}^{M} (\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})^2}}\right), & (9b) \end{cases}$$

where $\tau = \frac{\lambda}{M} \sum_{i=1}^{M} \sigma^2_{\mathrm{v}_i}$ is the global decision threshold computed at the FC. Since the FC achieves global decisions by comparing T against τ, one can rewrite the test statistic T in (7) as

$$T = \left(\sum_{i=1}^{M} \sigma^2_{\mathrm{v}_i}\right)^{-1} \sum_{i=1}^{M} T_i = \left(\sum_{i=1}^{M} \sigma^2_{\mathrm{v}_i}\right)^{-1} \sum_{i=1}^{M} \frac{1}{N} \sum_{n=1}^{N} |y_i(n)|^2, \quad (10)$$

where the new threshold $\tau = \lambda$ applies. In this case, it follows that $\mu_0 = 1$, $\sigma^2_0 = \frac{1}{K} \sum_{i=1}^{M} \sigma^4_{\mathrm{v}_i}$, $\mu_1 = (1/\sum_{i=1}^{M} \sigma^2_{\mathrm{v}_i}) \sum_{i=1}^{M} (\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})$ and $\sigma^2_1 = \frac{1}{K} \sum_{i=1}^{M} (\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})^2$, where $K = (\sqrt{N} \sum_{i=1}^{M} \sigma^2_{\mathrm{v}_i})^2$. If one considers $\sigma^2_{\mathrm{v}_i} = \sigma^2_{\mathrm{v}}$ for all i, the test statistic T in (10) becomes

$$T = \tfrac{1}{MN\sigma^2_{\mathrm{v}}} \sum\nolimits_{i=1}^{M} \sum\nolimits_{n=1}^{N} |y_i(n)|^2, \quad (11)$$

in which (11) is the most used form of computing T in the literature [11], [17], [25] and leads to $\mu_0 = 1$, $\sigma^2_0 = (MN)^{-1}$, $\mu_1 = \frac{1}{M} \sum_{i=1}^{M} (1 + \gamma_i)$ and $\sigma^2_1 = (M^2 N)^{-1} \sum_{i=1}^{M} (1 + \gamma_i)^2$, where $\gamma_i = \sigma^2_{\mathrm{s}_i}/\sigma^2_{\mathrm{v}_i}$ denotes the SNR of the ith SU.

b) Novel test statistic computation: Differently from (7) or (10), an alternative form of computing the ED test statistic is by applying a weighted sum of the partial energies $\{T_i\}$, with the ith weight being inversely proportional to the noise variance of the ith SU. That is,

$$T = \tfrac{1}{M} \sum\nolimits_{i=1}^{M} \tfrac{T_i}{\sigma^2_{\mathrm{v}_i}} = \tfrac{1}{M} \sum\nolimits_{i=1}^{M} \tfrac{1}{N\sigma^2_{\mathrm{v}_i}} \sum\nolimits_{n=1}^{N} |y_i(n)|^2. \quad (12)$$

Since (12) weights the energy of the ith set of samples by $1/\sigma^2_{\mathrm{v}_i}$, it attains more detection power than (10) under unequal noise powers because computing T as in (10) is like having NU on the knowledge of the noise power at the ith SU. Yet, verify that (12) specializes to (11) if $\sigma^2_{\mathrm{v}_i} = \sigma^2_{\mathrm{v}}$ for all i is assumed. For sufficiently large N, (12) has mean and variance identical to those of (11), independently of having equal or unequal noise powers. More specifically, (12) has

$$\begin{cases}\mu_0 = 1 \text{ and } \sigma_0^2 = (MN)^{-1}, \text{ and} & \text{(13a)}\\ \mu_1 = \frac{1}{M}\sum_{i=1}^{M}(1+\gamma_i) \text{ and } \sigma_1^2 = \frac{1}{M^2N}\sum_{i=1}^{M}(1+\gamma_i)^2. & \text{(13b)}\end{cases}$$

Thus, the global performance metrics associated with the use of (12) are (with $\tau = \lambda$),

$$\begin{cases} Q_{\text{fa}} = Q\left[(\tau-\mu_0)/\sigma_0\right] = Q\left[(\tau-1)\sqrt{MN}\right] \text{ and} & \text{(14a)}\\ Q_{\text{d}} = Q\left(\dfrac{\tau-\mu_1}{\sigma_1}\right) = Q\left(\dfrac{M\tau - \sum_{i=1}^{M}(1+\gamma_i)}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M}(1+\gamma_i)^2}}\right). & \text{(14b)}\end{cases}$$

Comparing the global metrics achieved with T in (7) and (12) for a given λ shows that the Q_{fa} in (9a) is greater than or equal to the Q_{fa} in (14a), a condition we express as "Eq. (9a) ≥ Eq. (14a)", and that the Q_{d} in (9b) is smaller than or equal to the Q_{d} in (14b), *i.e.*, "Eq. (9b) ≤ Eq. (14b)", with the equality holding if $\sigma_{\text{v}_i}^2 = \sigma_{\text{v}}^2$ for all i. Besides, under unequal noise powers, *i.e.*, $\sigma_{\text{v}_i}^2 \neq \sigma_{\text{v}_j}^2$ for $i, j = 1, \ldots, M$, $i \neq j$, the condition "Eq. (9b) > Eq. (14b)" is achieved if, and only if, $\gamma_i = \gamma$ for all i and sufficiently low SNR, *i.e.*, $\gamma < \lambda - 1$, since $\sum_{i=1}^{M}\sigma_{\text{v}_i}^2/\sqrt{\sum_{i=1}^{M}\sigma_{\text{v}_i}^4} \leq \sqrt{M}$, with the equality achieved when $\sigma_{\text{v}_i}^2 = \sigma_{\text{v}}^2$ for all i. Thus, (12) can indeed achieve better detection performance than (7) or (10) in practice, where one should expect heterogeneous SUs.

D. NU modeling

The traditional and novel NU intervals/models described in the last two paragraphs of Subsection I-A can be related by redefining the range $[a_i, b_i]$ as

$$[a_i, b_i] = \left[(1-\rho_{1_i})\sigma_{\text{v}_i}^2, (1+\rho_{2_i})\sigma_{\text{v}_i}^2\right]. \quad (15)$$

This relation is the referred NU mapping scheme proposed to facilitate SNRw derivations, allow easy comparisons involving traditional and novel SNRw expressions, and highlight their advantages and disadvantages. Specifically, one can analyze the ED SNRw under the NU sources and models combinations simply substituting ρ_{1_i} by $(\rho_i - 1)/\rho_i$ and ρ_{2_i} by $\rho_i - 1$ in (15) (to consider the traditional NU model), with $\rho_i \geq 1$, or $\rho_{1_i} = \rho_{2_i}$ by ρ_i (to the novel NU model), with $0 \leq \rho_i < 1$, or directly using ρ_{1_i} and ρ_{2_i} in what we will call 'generalized SNRw' expressions.

As in [13], we model the noise variance estimate $\hat{\sigma}_{\text{v}_i}^2$ at the ith SU as a truncated Gaussian-distributed random variable in $[a_i, b_i]$, with probability density function (PDF) given by

$$f_{\hat{\sigma}_{\text{v}_i}^2}(x_i) = \begin{cases} 0 & , x_i \leq a_i,\\ \frac{\phi(\mu_i,\sigma_i^2;x_i)}{\Phi(\mu_i,\sigma_i^2;b_i)-\Phi(\mu_i,\sigma_i^2;a_i)} & , a_i < x_i < b_i,\\ 0 & , x_i \geq b_i,\end{cases} \quad (16)$$

where $\phi(\mu_i, \sigma_i^2; x_i)$ denotes the Gaussian function in the variable $x_i = \hat{\sigma}_{\text{v}_i}^2$, with mean $\mu_i = \sigma_{\text{v}_i}^2$ and variance σ_i^2, such that six standard deviations about $\sigma_{\text{v}_i}^2$, which is approximately 99.73% of its variation, lies in $[a_i, b_i]$. Specifically, $6\sigma_i = b_i - a_i = (1+\rho_{2_i})\sigma_{\text{v}_i}^2 - (1-\rho_{1_i})\sigma_{\text{v}_i}^2 = (\rho_{1_i}+\rho_{2_i})\sigma_{\text{v}_i}^2$, yielding $\sigma_i^2 = [(\rho_{1_i}+\rho_{2_i})\sigma_{\text{v}_i}^2/6]^2$. The constants $\Phi(\mu_i, \sigma_i^2; a_i)$ and $\Phi(\mu_i, \sigma_i^2; b_i)$ are the values of the associated Gaussian cumulative distribution function (CDF) at $x_i = a_i$ and $x_i = b_i$, respectively. Notice that this truncated distribution guarantees unbiased estimations for the novel NU model given by $[a_i, b_i] = \left[(1-\rho_i)\sigma_{\text{v}_i}^2, (1+\rho_i)\sigma_{\text{v}_i}^2\right]$, $0 \leq \rho_i < 1$, which we view as the expected behavior for any noise variance estimator in practice (remember that since the noise variance is a positive quantity, estimates below the expected value, $\sigma_{\text{v}_i}^2$, must be truncated to avoid $\hat{\sigma}_{\text{v}_i}^2 < 0$, thus, estimates above $\sigma_{\text{v}_i}^2$ should also be truncated to guarantee a symmetric/unbiased estimator). In addition, we found it more reasonable to consider that estimation errors further apart from the expected value should occur with less probability than closer apart errors. We view this as the behavior expected for any noise variance estimator in practice, and that justifies our choice to adopt the Gaussian distribution instead of the uniform distribution to model $\hat{\sigma}_{\text{v}_i}^2$, especially when considering the novel NU source in this article. Moreover, this article adopts the NU distribution given in (16) even when considering the concept of the traditional NU source for simplicity.

III. SNR Wall in Non-Cooperative and Cooperative Spectrum Sensing

From (4a), (4b), (13a), and (13b), see that the variance of the ED test statistic under $\mathcal{H}_0$ and $\mathcal{H}_1$ tends to zero as $N \to \infty$ in the absence of NU, that is, if $\rho_i = 0$ for all i. Hence, it is possible to distinguish between noise only and noise plus PU signal for any SNR level. In other words, in the absence of NU, the ED in nCSS and CSS can reach the ideal performance metrics independently of the SNR if N is sufficiently large. In this case, the following hold

$$\begin{cases}\lim_{N\to\infty} P_{\text{fa}_i} = 0 \text{ and } \lim_{N\to\infty} P_{\text{d}_i} = 1, \text{ and} & \text{(17a)}\\ \lim_{N\to\infty} Q_{\text{fa}} = 0 \text{ and } \lim_{N\to\infty} Q_{\text{d}} = 1. & \text{(17b)}\end{cases}$$

To derive the SNRw of the ED, we use (17a) and (17b) and the fact that when $N \to \infty$, the Q-function value becomes [11]

$$\lim_{N\to\infty} Q(g\sqrt{N}) = 0, \text{if } g>0, \text{ or } \lim_{N\to\infty} Q(g\sqrt{N}) = 1, \text{if } g<0. \quad (18)$$

We highlight that the literature has already derived the SNRw of the ED under the traditional NU source and model and the novel NU source and model, as shown in [16], [11], and [13] for nCSS and CSS with SD and HD under the k-out-of-M rule, for example. However, in the following three subsections, we make the SNRw derivations combining the NU sources and models via the relation in (15). To the best of our knowledge, there is no derivation of the SNRw of the ED combining the traditional or novel NU source with the novel or traditional NU model. Moreover, there is no SNRw derivation for CSS with SD under the novel test statistic computation proposed in (12), nor under the traditional test statistic computation in (7) via (15).

A. Performance metrics and SNRw in nCSS with NU

1) Traditional NU source combined with traditional or novel NU model: In the traditional NU source, the decision threshold, means, and variances of T_i in (3) are respectively

$\tau_i = \lambda\sigma^2_{\mathrm{v}_i}$, $\hat{\mu}_{0_i} = \hat{\sigma}^2_{\mathrm{v}_i}$, $\hat{\mu}_{1_i} = \hat{\sigma}^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i}$, $\hat{\sigma}^2_{0_i} = \hat{\sigma}^4_{\mathrm{v}_i}/N$ and $\hat{\sigma}^2_{1_i} = (\hat{\sigma}^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})^2/N$. Thus, for a given $\hat{\sigma}^2_{\mathrm{v}_i}$ value,

$$\begin{cases} P_{\mathrm{fa}_i} = Q\left[(\tau_i - \hat{\mu}_{0_i})/\hat{\sigma}_{0_i}\right] = Q\left[(\tau_i - \hat{\sigma}^2_{\mathrm{v}_i})\sqrt{N}\hat{\sigma}^{-2}_{\mathrm{v}_i}\right] \text{ and} & (19a) \\ P_{\mathrm{d}_i} = Q\left(\dfrac{\tau_i - \hat{\mu}_{1_i}}{\hat{\sigma}_{1_i}}\right) = Q\left(\dfrac{\tau_i - (\hat{\sigma}^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})}{(\hat{\sigma}^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})/\sqrt{N}}\right). & (19b) \end{cases}$$

Averaging (19a) and (19b) over all possible values of $\hat{\sigma}^2_{\mathrm{v}_i}$ according to the PDF in (16), yields

$$\begin{cases} \bar{P}_{\mathrm{fa}_i} = \displaystyle\int_{a_i}^{b_i} Q\left[(\tau_i - x_i)\sqrt{N}x_i^{-1}\right] f_{\hat{\sigma}^2_{\mathrm{v}_i}}(x_i)dx_i \text{ and} & (20a) \\ \bar{P}_{\mathrm{d}_i} = \displaystyle\int_{a_i}^{b_i} Q\left(\dfrac{\tau_i - (x_i + \sigma^2_{\mathrm{s}_i})}{(x_i + \sigma^2_{\mathrm{s}_i})/\sqrt{N}}\right) f_{\hat{\sigma}^2_{\mathrm{v}_i}}(x_i)dx_i, & (20b) \end{cases}$$

respectively, where we apply $\hat{\sigma}^2_{\mathrm{v}_i} = x_i$ for notation simplicity.

Applying (18) in (20a), it follows that the inequality $\lambda\sigma^2_{\mathrm{v}_i} - x_i > 0$ must be satisfied to yield $\bar{P}_{\mathrm{fa}_i} = 0$. Thus, $\lambda > x_i/\sigma^2_{\mathrm{v}_i}$, where, taking into account (15), $x_i = b_i = (1 + \rho_{2_i})\sigma^2_{\mathrm{v}_i}$ to guarantee $\bar{P}_{\mathrm{fa}_i} = 0$ in the entire NU range (recall that $b_i > a_i$). Likewise, applying (18) in (20b), it follows that $\lambda\sigma^2_{\mathrm{v}_i} - (x_i + \sigma^2_{\mathrm{s}_i}) < 0$ must be satisfied to yield $\bar{P}_{\mathrm{d}_i} = 1$, that is, $\lambda < (x_i + \sigma^2_{\mathrm{s}_i})/\sigma^2_{\mathrm{v}_i}$, such that $x_i = a_i = (1 - \rho_{1_i})\sigma^2_{\mathrm{v}_i}$ to guarantee $\bar{P}_{\mathrm{d}_i} = 1$ in the whole NU range. Hence, to guarantee $\bar{P}_{\mathrm{fa}_i} = 0$ and $\bar{P}_{\mathrm{d}_i} = 1$, it is imperative to guarantee

$$\begin{cases} \lambda > 1 + \rho_{2_i} \text{ and} & (21a) \\ \lambda < 1 - \rho_{1_i} + \gamma_i, & (21b) \end{cases}$$

which leads to $\gamma_i > \rho_{1_i} + \rho_{2_i}$.

Hereafter, the term 'generalized SNRw' refers to SNRw expressions that can be mapped into the traditional or novel NU interval/model according to the relations expressed in (15). Thus, from (21a) and (21b), the generalized SNRw of the ith SU is

$$\gamma_{\mathrm{w}_i} = \rho_{1_i} + \rho_{2_i}. \quad (22)$$

When combining the traditional NU source with the novel NU model, the SNRw of the ith SU in nCSS is obtained by applying $\rho_{1_i} = \rho_{2_i} = \rho_i$ in (22), for $0 \leq \rho_i < 1$, yielding

$$\gamma_{\mathrm{w}_i} = 2\rho_i. \quad (23)$$

It is worth highlighting that (23) is a novel SNRw expression obtained from the NU source and model combination ii) proposed in the first paragraph of Subsection I-B. Alternatively, when considering the traditional NU source and model, which is the NU combination i) described in Subsection I-B and widely used in the literature, one can derive the SNRw of the ith SU in nCSS by applying $\rho_{1_i} = (\rho_i - 1)/\rho_i$ and $\rho_{2_i} = \rho_i - 1$ in (22), with $\rho_i \geq 1$, resulting in

$$\gamma_{\mathrm{w}_i} = (\rho_i^2 - 1)/\rho_i. \quad (24)$$

It is also worth highlighting that (24) is the well-known SNRw expression reported, for example, in [16]. Achieving (23) and (24) via (22) proves the utility of the mapping scheme proposed in (15) since it allows one to derive the SNRw expression belonging to the traditional NU source combined with the novel or traditional NU model simply by substituting the proper NU parameters in the generalized SNRw expression in (22).

The number of samples N_i required to achieve given target performance metrics at the ith SU under NU can be derived by eliminating τ_i from (19a) and (19b) [10] assuming their worst NU cases: using $\hat{\sigma}^2_{\mathrm{v}_i} = (1 + \rho_{2_i})\sigma^2_{\mathrm{v}_i}$ in (19a) and $\hat{\sigma}^2_{\mathrm{v}_i} = (1 - \rho_{1_i})\sigma^2_{\mathrm{v}_i}$ in (19b). This derivation leads to

$$N_i = \frac{\left[Q^{-1}(P_{\mathrm{fa}_i})(1+\rho_{2_i}) - Q^{-1}(P_{\mathrm{d}_i})\,(1-\rho_{1_i}+\gamma_i)\right]^2}{\left[\gamma_i - (\rho_{1_i} + \rho_{2_i})\right]^2}, \quad (25)$$

where it is clear that $N_i \to \infty$ as $\gamma_i \to \rho_{1_i} + \rho_{2_i}$, a result that is consistent with the SNRw given in (22). Thus, it is necessary to have $\gamma_i > \rho_{1_i} + \rho_{2_i}$ in nCSS under the traditional NU source.

2) Novel NU source combined with traditional or novel NU model: In the novel NU source, $\hat{\sigma}^2_{\mathrm{v}_i}$ is in the local decision threshold given by $\hat{\tau}_i = \lambda\hat{\sigma}^2_{\mathrm{v}_i}$. Thus, the mean and variance in (4a), under $\mathcal{H}_0$, and in (4b), under $\mathcal{H}_1$, hold for P_{fa_i} and P_{d_i}, respectively. Therefore, for a given $\hat{\sigma}^2_{\mathrm{v}_i}$,

$$\begin{cases} P_{\mathrm{fa}_i} = Q\left[(\hat{\tau}_i - \mu_{0_i})/\sigma_{0_i}\right] = Q\left[(\hat{\tau}_i - \sigma^2_{\mathrm{v}_i})\sqrt{N}\sigma^{-2}_{\mathrm{v}_i}\right] \text{ and} & (26a) \\ P_{\mathrm{d}_i} = Q\left(\dfrac{\hat{\tau}_i - \mu_{1_i}}{\sigma_{1_i}}\right) = Q\left(\dfrac{\hat{\tau}_i - (\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})}{(\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})/\sqrt{N}}\right). & (26b) \end{cases}$$

Averaging (26a) and (26b) over all possible values of $\hat{\sigma}^2_{\mathrm{v}_i}$ according to the PDF in (16) yields

$$\begin{cases} \bar{P}_{\mathrm{fa}_i} = \displaystyle\int_{a_i}^{b_i} Q\left[(\lambda x_i - \sigma^2_{\mathrm{v}_i})\sqrt{N}\sigma^{-2}_{\mathrm{v}_i}\right] f_{\hat{\sigma}^2_{\mathrm{v}_i}}(x_i)dx_i \text{ and} & (27a) \\ \bar{P}_{\mathrm{d}_i} = \displaystyle\int_{a_i}^{b_i} Q\left(\dfrac{\lambda x_i - (\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})}{(\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})/\sqrt{N}}\right) f_{\hat{\sigma}^2_{\mathrm{v}_i}}(x_i)dx_i. & (27b) \end{cases}$$

Applying (18) in (27a), one must guarantee $\lambda x_i - \sigma^2_{\mathrm{v}_i} > 0$ to yield $\bar{P}_{\mathrm{fa}_i} = 0$. Thus, $\lambda > \sigma^2_{\mathrm{v}_i}/x_i$, where, considering (15), $x_i = a_i = (1 + \rho_{1_i})\sigma^2_{\mathrm{v}_i}$ to guarantee $\bar{P}_{\mathrm{fa}_i} = 0$ in the whole NU range. Similarly, applying (18) in (27b), it can be concluded that $\lambda x_i - (\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i}) < 0$ to yield $\bar{P}_{\mathrm{d}_i} = 1$. As a consequence, $\lambda < (\sigma^2_{\mathrm{v}_i} + \sigma^2_{\mathrm{s}_i})/x_i$, where $x_i = b_i = (1 - \rho_{2_i})\sigma^2_{\mathrm{v}_i}$ to guarantee $\bar{P}_{\mathrm{d}_i} = 1$ in the whole NU range. Hence, to guarantee $\bar{P}_{\mathrm{fa}_i} = 0$ and $\bar{P}_{\mathrm{d}_i} = 1$, it is imperative to guarantee

$$\begin{cases} \lambda > (1 - \rho_{1_i})^{-1} \text{ and} & (28a) \\ \lambda < (1 + \gamma_i)(1 + \rho_{2_i})^{-1}, & (28b) \end{cases}$$

which leads to $\gamma_i > (\rho_{2_i} + \rho_{1_i})(1 - \rho_{1_i})^{-1}$. Thus, the generalized SNRw of the ith SU is

$$\gamma_{\mathrm{w}_i} = (\rho_{2_i} + \rho_{1_i})(1 - \rho_{1_i})^{-1}. \quad (29)$$

Applying $\rho_{1_i} = (\rho_i - 1)/\rho_i$ and $\rho_{2_i} = \rho_i - 1$ in (29), with $\rho_i \geq 1$, the SNRw of the ith SU in nCSS when using the new NU combination iv) proposed in the first paragraph of Subsection I-B, *i.e.*, when combining the novel NU source with the traditional NU model, becomes

$$\gamma_{\mathrm{w}_i} = \rho_i^2 - 1. \quad (30)$$

We highlight that (30) is a novel SNRw expression since it follows the NU combination iv). Similarly, considering the novel NU source and model, *i.e.*, the NU combination iii), described in Subsection I-B and used in [13], one can derive the SNRw of the ith SU in nCSS by plugging $\rho_{1_i} = \rho_{2_i} = \rho_i$ into (29), with $0 \le \rho_i < 1$, yielding

$$\gamma_{\mathrm{w}_i} = 2\rho_i(1-\rho_i)^{-1}, \tag{31}$$

a result that agrees with [13, Eq. (28)] as expected.

The required number of samples, N_i, is found by eliminating λ from (26a) and (26b) assuming their NU worst-cases, *i.e.*, $\hat{\sigma}^2_{\mathrm{v}_i}=(1-\rho_{1_i})\sigma^2_{\mathrm{v}_i}$ in (26a) and $\hat{\sigma}^2_{\mathrm{v}_i}=(1+\rho_{2_i})\sigma^2_{\mathrm{v}_i}$ in (26b), yielding

$$N_i = \frac{\left[Q^{-1}(P_{\mathrm{fa}_i})\frac{1+\rho_{2_i}}{1-\rho_{1_i}} - Q^{-1}(P_{\mathrm{d}_i})\,(1+\gamma_i)\right]^2}{[\gamma_i - (\rho_{1_i}+\rho_{2_i})(1-\rho_{1_i})^{-1}]^2}, \tag{32}$$

where $N_i \to \infty$ as $\gamma_i \to (\rho_{1_i}+\rho_{2_i})(1-\rho_{1_i})^{-1}$, consistently with (29). Because of that, $\gamma_i > (\rho_{1_i}+\rho_{2_i})(1-\rho_{1_i})^{-1}$ must be satisfied in nCSS when the novel NU source is adopted.

Behold, the novel NU source is more conservative than the traditional one since "Eq. (31) ≥ Eq. (23)" for $0 \le \rho_i < 1$, and "Eq. (30) ≥ Eq. (24)" for $\rho_i \ge 1$ (recall that the novel NU model also has more practical significance since it is unbiased, which is not valid for the traditional NU model). Yet, adopting the novel NU model when using the traditional NU source is also more conservative than the traditional one since "Eq. (23) ≥ Eq. (24)" for $0 \le \rho_i < 1$ and $\rho_i \ge 1$, respectively. Likewise, notice that "Eq. (30) ≥ Eq. (23)" for $\rho_i \ge 1$ and $0 \le \rho_i < 1$, respectively.

B. Performance metrics and SNRw in CSS with HD fusion and NU

When NU is present, one can compute the global performance metrics in CSS with HD fusion under the k-out-of-M rule by plugging the average local metrics given in (20a) and (20b) or in (27a) and (27b) into (6) according to the traditional or novel NU source, respectively. That is,

$$\bar{Q}_{\mathcal{X}} = \sum\nolimits_{\ell=k}^{M} \sum\nolimits_{\mathbf{d}:w_{\mathrm{H}}(\mathbf{d})=\ell} \prod\nolimits_{i=1}^{M} \bar{P}_{\mathcal{X}i}^{d_i}(1-\bar{P}_{\mathcal{X}i})^{1-d_i}. \tag{33}$$

Following [13], a combinatorial analysis applied to (33) shows that $\bar{Q}_{\mathrm{fa}} = 0$ and $\bar{Q}_{\mathrm{d}} = 1$ if and only if $\bar{P}_{\mathrm{fa}_i} = 0$ and $\bar{P}_{\mathrm{d}_i} = 1$ for at least $M-k+1$ and k out of the M factors of the product over the index i, respectively. Thus, there are $\binom{M}{M-k+1}$ combinations of $M-k+1$ SUs that must attain $\bar{P}_{\mathrm{fa}} = 0$ to yield $\bar{Q}_{\mathrm{fa}} = 0$, and $\binom{M}{k}$ combinations of k SUs that must achieve $\bar{P}_{\mathrm{d}} = 1$ to achieve $\bar{Q}_{\mathrm{d}} = 1$. These statements are applied to derive the SNRw in the sequel.

1) Traditional NU source combined with traditional or novel NU model: To achieve $\bar{Q}_{\mathrm{fa}} = 0$ in (33), notice with the help of (21a) that, for $i = 1, \ldots, M$, the following must be satisfied

$$\lambda > \begin{cases} 1+\rho_{2_1}, \text{and}, \ldots, \text{and } 1+\rho_{2_{M-k+1}}, \text{ or} \\ \vdots \\ 1+\rho_{2_k}, \text{and}, \ldots, \text{and } 1+\rho_{2_M}. \end{cases} \tag{34}$$

Since $\bar{Q}_{\mathrm{fa}} = 0$ in (33) if and only if at least one group having $M-k+1$ conditions in at least one of the $\binom{M}{M-k+1}$ lines of (34) is satisfied, one can write λ as the minimum among the maximum of each line of (34). Specifically, one can write λ as

$$\lambda > \min\begin{pmatrix} \max\,(1+\rho_{2_1}, \ldots, 1+\rho_{2_{M-k+1}}), \\ \vdots \\ \max\,(1+\rho_{2_k}, \ldots, 1+\rho_{2_M}) \end{pmatrix}. \tag{35}$$

Similarly, to achieve $\bar{Q}_{\mathrm{d}} = 1$ in (33), notice with the help of (21b) that, for $i = 1, \ldots, M$,

$$\lambda < \begin{cases} 1-\rho_{1_1}+\gamma_1, \text{and}, \ldots, \text{and } 1-\rho_{1_k}+\gamma_k &, \text{or} \\ \vdots &, \text{or} \\ 1-\rho_{1_{M-k+1}}+\gamma_{M-k+1}, \text{and}, \ldots, \text{and}, 1-\rho_{1_M}+\gamma_M. \end{cases} \tag{36}$$

Since $\bar{Q}_{\mathrm{d}} = 1$ in (33) if and only if at least one group having the k conditions in at least one of the $\binom{M}{k}$ lines of (36) is satisfied, one can rewrite λ in (36) as

$$\lambda < \begin{cases} \min\,(1-\rho_{1_1}+\gamma_1, \ldots, 1-\rho_{1_k}+\gamma_k) &, \text{or} \\ \vdots &, \text{or} \\ \min\,(1-\rho_{1_{M-k+1}}+\gamma_{M-k+1}, \ldots, 1-\rho_{1_M}+\gamma_M). \end{cases} \tag{37}$$

Assuming descending ordered NU parameters (we will treat the equality of them later), that is, $\rho_j > \rho_{j+1}$ for $j = 1, \ldots, M-1$, it follows from (35) and (37) that

$$1+\rho_{2_k} < \begin{cases} \min(1-\rho_{1_1}+\gamma_1, \ldots, 1-\rho_{1_k}+\gamma_k) &, \text{or} \\ \vdots &, \text{or} \\ \min(1-\rho_{1_{M-k+1}}+\gamma_{M-k+1}, \ldots, 1-\rho_{1_M}+\gamma_M). \end{cases} \tag{38}$$

Since determining the minimum in each line of (38) requires the knowledge of γ_i if $k \ne 1$, it follows, in general, that

$$\begin{matrix} \gamma_1 > \rho_{2_k}+\rho_{1_1}, \ldots, \gamma_k > \rho_{2_k}+\rho_{1_k} &, \text{or} \\ \vdots &, \text{or} \\ \gamma_{M-k+1} > \rho_{2_k}+\rho_{1_{M-k+1}}, \ldots, \gamma_M > \rho_{2_k}+\rho_{1_M}. \end{matrix} \tag{39}$$

The conditions to achieve ideal values of the global performance metrics given in (33) show that at least one group having $M-k+1$ conditions in at least one of the $\binom{M}{M-k+1}$ lines of (34) must be met to yield $\bar{Q}_{\mathrm{fa}} = 0$, and at least one group having k conditions in at least one of the $\binom{M}{k}$ lines of (36) must be met to yield $\bar{Q}_{\mathrm{d}} = 1$. However, meeting these two sets of conditions is equivalent to meet at least one group of k conditions in at least one of the lines of (39) [13]. Therefore, the equality in each condition of (39) gives the generalized SNRw at the ith SU under the traditional NU source as

$$\gamma_{\mathrm{w}_i} = \rho_{2_k} + \rho_{1_i}. \tag{40}$$

If all SUs have equal SNRs, that is, $\gamma_i = \gamma$ for all i, for $k = 1$, the conditions in (38) become

$$\begin{matrix} \gamma > \rho_{2_1}+\rho_{1_1}, \text{or } \gamma > \rho_{2_1}+\rho_{1_2}, \text{or}, \ldots, \text{or} \\ \gamma > \rho_{2_1}+\rho_{1_{M-1}}, \text{or } \gamma > \rho_{2_1}+\rho_{1_M}. \end{matrix} \tag{41}$$

Since meeting any condition in (41) satisfies the SNR constraint, one can simplify (41) to $\gamma > \rho_{2_1}+\rho_{1_M}$ to yield the smallest generalized SNRw. Thus, for $k = 1$,

$$\gamma_{\mathrm{w}} = \rho_{2_1} + \rho_{1_M}. \tag{42}$$

Similarly, for $1 < k < M$, the conditions in (38) become

$$\gamma > \rho_{2_k}+\rho_{1_1}, \text{or } \gamma > \rho_{2_k}+\rho_{1_2}, \text{or}, \ldots, \text{or } \gamma > \rho_{2_k}+\rho_{1_{M-k+1}}, \tag{43}$$

which can be simplified to $\gamma > \rho_{2_k} + \rho_{1_{M-k+1}}$, if $k < M-k+1$, or to $\gamma > \rho_{2_k} + \rho_{1_k}$, if $k \geq M-k+1$, to yield the smallest generalized SNRw. Thus, for $1 < k < M$, one obtains

$$\gamma_{\mathrm{w}} = \begin{cases} \rho_{2_k} + \rho_{1_{M-k+1}}, & \text{if } k < M-k+1 \quad (44a) \\ \rho_{2_k} + \rho_{1_k}, & \text{if } k \geq M-k+1. \quad (44b) \end{cases}$$

Likewise, for $k = M$, (38) becomes $\gamma > \rho_{2_M} + \rho_{1_1}$, which leads to

$$\gamma_{\mathrm{w}} = \rho_{2_M} + \rho_{1_1}. \tag{45}$$

When adopting the MAJ rule, that is, $k = \lfloor M/2+1 \rfloor$, verify that $M-k+1 = k-1 = M/2$ if M is even, and $M-k+1 = \lfloor M/2+1 \rfloor$ if M is odd. Thus, one can simplify (43) to $\gamma > \rho_{2_{\lfloor M/2+1 \rfloor}} + \rho_{1_{M/2}}$, when M is even, or to $\gamma > \rho_{2_{\lfloor M/2+1 \rfloor}} + \rho_{1_{\lfloor M/2+1 \rfloor}}$ when M is odd. Then,

$$\gamma_{\mathrm{w}} = \begin{cases} \rho_{2_{M/2+1}} + \rho_{1_{M/2}}, & \text{if } M \text{ is even} \quad (46a) \\ \rho_{2_{\lfloor M/2+1 \rfloor}} + \rho_{1_{\lfloor M/2+1 \rfloor}}, & \text{if } M \text{ is odd.} \quad (46b) \end{cases}$$

Considering identical NU parameters, *i.e.*, $\rho_i = \rho$ for all i, all the above generalized SNRw expressions become

$$\gamma_{\mathrm{w}} = \rho_2 + \rho_1. \tag{47}$$

As before, one can derive particular SNRw expressions from the generalized ones. For example, substituting ρ_{2_k} by ρ_k and ρ_{1_i} by ρ_i in (40), with $0 \leq \rho_k, \rho_i < 1$, yields

$$\gamma_{\mathrm{w}_i} = \rho_k + \rho_i, \tag{48}$$

which is the SNRw at the ith SU when combining the traditional NU source with the novel NU model. If $\rho_i = \rho$ for all i, this SNRw becomes $\gamma_{\mathrm{w}_i} = 2\rho_i$, which is consistent with (23). On the other hand, substituting ρ_{2_k} by $\rho_k - 1$ and ρ_{1_i} by $(\rho_i - 1)/\rho_i$ in (40), with $\rho_k, \rho_i \geq 1$, yields

$$\gamma_{\mathrm{w}_i} = (\rho_k \rho_i - 1)/\rho_i, \tag{49}$$

which is the SNRw of the ith SU under the traditional NU source and model, and converts to $\gamma_{\mathrm{w}_i} = (\rho_i^2 - 1)/\rho_i$ with $\rho_i = \rho$ for all i, consistently with (24).

2) Novel NU source combined with traditional or novel NU model: To attain $\bar{Q}_{\mathrm{fa}} = 0$ in (33), notice with the help of (28a) that, for $i = 1, \ldots, M$,

$$\lambda > \begin{cases} (1-\rho_{1_i})^{-1}, \text{and}, \ldots, \text{and } (1-\rho_{1_{M-k+1}})^{-1}, \text{ or} \\ \vdots \qquad\qquad\qquad , \text{ or} \\ (1-\rho_{1_k})^{-1}, \text{and}, \ldots, \text{and } (1-\rho_{1_M})^{-1}. \end{cases} \tag{50}$$

Since $\bar{Q}_{\mathrm{fa}} = 0$ is attained in (33) if and only if at least one group having the $M-k+1$ conditions in at least one of the $\binom{M}{M-k+1}$ lines of (50) is satisfied, one can write λ as the minimum among the maximum of each line of (50), that is

$$\lambda > \min \begin{pmatrix} \max\left[(1-\rho_{1_i})^{-1}, \ldots, (1-\rho_{1_{M-k+1}})^{-1}\right], \\ \vdots \\ \max\left[(1-\rho_{1_k})^{-1}, \ldots, (1-\rho_{1_M})^{-1}\right] \end{pmatrix}. \tag{51}$$

Similarly, to attain $\bar{Q}_{\mathrm{d}} = 1$ in (33), notice with the help of (28b) that, for $i = 1, \ldots, M$,

$$\lambda < \begin{cases} \frac{1+\gamma_1}{1+\rho_{2_1}}, \text{and}, \ldots, \text{and } \frac{1+\gamma_k}{1+\rho_{2_k}} & \text{, or} \\ \vdots & \text{, or} \\ \frac{1+\gamma_{M-k+1}}{1+\rho_{2_{M-k+1}}}, \text{and}, \ldots, \text{and } \frac{1+\gamma_M}{1+\rho_{2_M}}. \end{cases} \tag{52}$$

Because $\bar{Q}_{\mathrm{d}} = 1$ is achieved in (33) if and only if at least one group having the k conditions in at least one of the $\binom{M}{k}$ lines of (52) are satisfied, one can rewrite λ in (52) as

$$\lambda < \begin{cases} \min\left(\frac{1+\gamma_1}{1+\rho_{2_1}}, \ldots, \frac{1+\gamma_k}{1+\rho_{2_k}}\right) & \text{, or} \\ \vdots & \text{, or} \\ \min\left(\frac{1+\gamma_{M-k+1}}{1+\rho_{2_{M-k+1}}}, \ldots, \frac{1+\gamma_M}{1+\rho_{2_M}}\right). \end{cases} \tag{53}$$

Assuming descending ordered NU parameters, it follows from (51) and (53) that

$$\frac{1}{1-\rho_{1_k}} < \begin{cases} \min\left(\frac{1+\gamma_1}{1+\rho_{2_1}}, \ldots, \frac{1+\gamma_k}{1+\rho_{2_k}}\right) & \text{, or} \\ \vdots & \text{, or} \\ \min\left(\frac{1+\gamma_{M-k+1}}{1+\rho_{2_{M-k+1}}}, \ldots, \frac{1+\gamma_M}{1+\rho_{2_M}}\right). \end{cases} \tag{54}$$

Determining the minimum in each line of (54) requires the knowledge of γ_i if $k \neq 1$. Thus, generally, one should consider

$$\begin{matrix} \gamma_1 > \frac{\rho_{1_k}+\rho_{2_1}}{1-\rho_{1_k}}, \ldots, \gamma_k > \frac{\rho_{1_k}+\rho_{2_k}}{1-\rho_{1_k}} & \text{, or} \\ \vdots & \text{, or} \\ \gamma_{M-k+1} > \frac{\rho_{1_k}+\rho_{2_{M-k+1}}}{1-\rho_{1_k}}, \ldots, \gamma_M > \frac{\rho_{1_k}+\rho_{2_M}}{1-\rho_{1_k}}. \end{matrix} \tag{55}$$

The conclusions about (34), (36), and (39) also apply here once the adherence with (50) and (52) to achieve the ideal performance metrics, considering (17b) and (33) for $\bar{Q}_{\mathrm{fa}} = 0$ and $\bar{Q}_{\mathrm{d}} = 1$, is equivalent to conforming with at least one group having k conditions in at least one of the lines of (55). Thus, the equality in each condition of (55) gives the generalized SNRw of the ith SU under the novel NU source as

$$\gamma_{\mathrm{w}_i} = (\rho_{1_k} + \rho_{2_i})(1-\rho_{1_k})^{-1}. \tag{56}$$

Applying $\gamma_i = \gamma$ for all i and $k = 1$, (54) becomes

$$\begin{matrix} \gamma > \frac{\rho_{1_1}+\rho_{2_1}}{1-\rho_{1_1}}, \text{or } \gamma > \frac{\rho_{1_1}+\rho_{2_2}}{1-\rho_{1_1}}, \text{or}, \ldots, \text{or} \\ \gamma > \frac{\rho_{1_1}+\rho_{2_{M-1}}}{1-\rho_{1_1}}, \text{or } \gamma > \frac{\rho_{1_1}+\rho_{2_M}}{1-\rho_{1_1}}. \end{matrix} \tag{57}$$

Since satisfying any condition in (57) meets the constraint on the SNR, one may simplify (57) to $\gamma > (\rho_{1_1} + \rho_{2_M})/(1-\rho_{1_1})$ to obtain the smallest generalized SNRw. Thus, for $k = 1$,

$$\gamma_{\mathrm{w}} = (\rho_{1_1} + \rho_{2_M})(1-\rho_{1_1})^{-1}. \tag{58}$$

Similarly, for $1 < k < M$, observe that (54) becomes

$$\gamma > \frac{\rho_{1_k}+\rho_{2_1}}{1-\rho_{1_k}}, \text{or } \gamma > \frac{\rho_{1_k}+\rho_{2_2}}{1-\rho_{1_k}}, \text{or}, \ldots, \text{or } \gamma > \frac{\rho_{1_k}+\rho_{2_{M-k+1}}}{1-\rho_{1_k}}, \tag{59}$$

being simplified to $\gamma > (\rho_{1_k} + \rho_{2_{M-k+1}})/(1-\rho_{1_k})$ for $k < M-k+1$, or to $\gamma > (\rho_{1_k} + \rho_{2_k})/(1-\rho_{1_k})$ for $k \geq M-k+1$, to obtain the smallest generalized SNRw. Thus, for $1 < k < M$,

$$\gamma_{\mathrm{w}} = \begin{cases} (\rho_{1_k} + \rho_{2_{M-k+1}})(1-\rho_{1_k})^{-1}, & \text{if } k < M-k+1 \quad (60a) \\ (\rho_{1_k} + \rho_{2_k})(1-\rho_{1_k})^{-1}, & \text{if } k \geq M-k+1. \quad (60b) \end{cases}$$

Finally, for $k = M$, (54) becomes $\gamma > (\rho_{1_M} + \rho_{2_1})(1-\rho_{1_M})^{-1}$, which leads to

$$\gamma_{\mathrm{w}} = (\rho_{1_M} + \rho_{2_1})(1-\rho_{1_M})^{-1}. \tag{61}$$

Similarly to what has been made to simplify (43) in the MAJ rule, (59) can be written as $\gamma > (\rho_{1_{M/2+1}} + \rho_{2_{M/2}})/(1-\rho_{1_{M/2+1}})$

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when M is even, or as $\gamma > (\rho_{1_{\lfloor M/2+1\rfloor}} + \rho_{2_{\lfloor M/2+1\rfloor}})/(1-\rho_{1_{\lfloor M/2+1\rfloor}})$ when M is odd. Thus,

$$\gamma_{\text{w}}=\begin{cases}(\rho_{1_{M/2+1}}+\rho_{2_{M/2}})(1-\rho_{1_{M/2+1}})^{-1}, \text{ if } M \text{ is even} & \text{(62a)}\\ (\rho_{1_{\lfloor M/2+1\rfloor}}+\rho_{2_{\lfloor M/2+1\rfloor}})(1-\rho_{1_{\lfloor M/2+1\rfloor}})^{-1}, \text{ if } M \text{ is odd.} & \text{(62b)}\end{cases}$$

If $\rho_i = \rho$ for all i, the above generalized SNRw expressions convert into

$$\gamma_{\text{w}} = (\rho_1 + \rho_2)(1 - \rho_1)^{-1}. \quad (63)$$

Applying $\rho_{2_i} = \rho_i - 1$ and $\rho_{1_k} = (\rho_k - 1)/\rho_k$ in (56), for $\rho_k, \rho_i \geq 1$, (56) becomes

$$\gamma_{\text{w}_i} = \rho_k \rho_i - 1, \quad (64)$$

which is the SNRw of the ith SU when combining novel NU source with traditional NU model, and turns to $\gamma_{\text{w}_i} = \rho_i^2 - 1$ with $\rho_i = \rho$ for all i, consistently with (30). On the other hand, applying $\rho_{2_k} = \rho_k$ and $\rho_{1_i} = \rho_i$ in (56), for $0 \leq \rho_k, \rho_i < 1$, it follows that

$$\gamma_{\text{w}_i} = (\rho_k + \rho_i)(1 - \rho_i)^{-1}, \quad (65)$$

which is the SNRw of the ith SU under the novel NU source and model, and becomes $\gamma_{\text{w}_i} = 2\rho_i/(1 - \rho_i)$ with $\rho_i = \rho$ for all i, consistently with (31).

The conclusion from nCSS regarding the more conservative SNRw under the novel NU source is also valid here: Observe that "Eq. (65) $\geq$ Eq. (48)" for $0 \leq \{\rho_i, \rho_k\} < 1$, and that "Eq. (64) $\geq$ Eq. (49)" for $\{\rho_i, \rho_k\} \geq 1$. Moreover, "Eq. (48) $\geq$ Eq. (49)" for $0 \leq \{\rho_i, \rho_k\} < 1$ and $\rho_i, \rho_k \geq 1$, respectively, and "Eq. (64) $\geq$ Eq. (48)" for $\rho_i, \rho_k \geq 1$ and $0 \leq \{\rho_i, \rho_k\} < 1$, respectively.

C. Performance metrics and SNRw in CSS with SD fusion and NU

1) Traditional NU source combined with traditional or novel NU model: Under NU, the mean and variance of T in (12) are $\hat{\mu}_0 = \frac{1}{M}\sum_{i=1}^{M}\frac{\hat{\sigma}^2_{\text{v}_i}}{\sigma^2_{\text{v}_i}}$ and $\hat{\sigma}^2_0 = (M^2N)^{-1}\sum_{i=1}^{M}\frac{\hat{\sigma}^4_{\text{v}_i}}{\sigma^4_{\text{v}_i}}$ under $\mathcal{H}_0$, and $\hat{\mu}_1 = \frac{1}{M}\sum_{i=1}^{M}\left(\hat{\sigma}^2_{\text{v}_i}/\sigma^2_{\text{v}_i} + \gamma_i\right)$ and $\hat{\sigma}^2_1 = (M^2N)^{-1}\sum_{i=1}^{M}\left(\hat{\sigma}^2_{\text{v}_i}/\sigma^2_{\text{v}_i} + \gamma_i\right)^2$ under $\mathcal{H}_1$. Therefore, the global performance metrics for a given value of $\hat{\sigma}^2_{\text{v}_i}$ are (with $\tau = \lambda$)

$$\begin{cases} Q_{\text{fa}} = Q\left(\dfrac{\tau - \hat{\mu}_0}{\hat{\sigma}_0}\right) = Q\left(\dfrac{M\tau - \sum_{i=1}^{M}\hat{\sigma}^2_{\text{v}_i}/\sigma^2_{\text{v}_i}}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M}\hat{\sigma}^4_{\text{v}_i}/\sigma^4_{\text{v}_i}}}\right) \text{ and} & \text{(66a)}\\ Q_{\text{d}} = Q\left(\dfrac{\tau - \hat{\mu}_1}{\hat{\sigma}_1}\right) = Q\left(\dfrac{M\tau - \sum_{i=1}^{M}\left(\hat{\sigma}^2_{\text{v}_i}/\sigma^2_{\text{v}_i} + \gamma_i\right)}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M}\left(\hat{\sigma}^2_{\text{v}_i}/\sigma^2_{\text{v}_i} + \gamma_i\right)^2}}\right). & \text{(66b)}\end{cases}$$

Averaging these metrics over all possible values of $\hat{\sigma}^2_{\text{v}_i}$ according to the PDF given in (16) yields[1]

$$\begin{cases} \bar{Q}_{\text{fa}} = \int_{\mathbf{a}}^{\mathbf{b}} Q\left(\dfrac{M\lambda - \sum_{i=1}^{M} x_i\sigma^{-2}_{\text{v}_i}}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M} x_i^2\sigma^{-4}_{\text{v}_i}}}\right) f_{\hat{\sigma}^2_{\text{v}_M},\ldots,\hat{\sigma}^2_{\text{v}_1}}(\mathbf{x})d^M\mathbf{x} \text{ and} & \text{(67a)}\\ \bar{Q}_{\text{d}} = \int_{\mathbf{a}}^{\mathbf{b}} Q\left(\dfrac{M\lambda - \sum_{i=1}^{M}(x_i\sigma^{-2}_{\text{v}_i}+\gamma_i)}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M}(x_i\sigma^{-2}_{\text{v}_i}+\gamma_i)^2}}\right) f_{\hat{\sigma}^2_{\text{v}_M},\ldots,\hat{\sigma}^2_{\text{v}_1}}(\mathbf{x})d^M\mathbf{x}. & \text{(67b)}\end{cases}$$

Applying (18) in (67a), it follows that the condition $M\lambda - \sum_{i=1}^{M} x_i/\sigma^2_{\text{v}_i} > 0$ must be met to allow $\bar{Q}_{\text{fa}_i} = 0$. Hence, $\lambda > \frac{1}{M}\sum_{i=1}^{M} x_i/\sigma^2_{\text{v}_i}$. Applying (15), then $x_i = b_i = (1 + \rho_{2_i})\sigma^2_{\text{v}_i}$ to guarantee $\bar{Q}_{\text{fa}} = 0$ in the whole NU range. Likewise, applying (18) in (67b), the condition $M\lambda - \sum_{i=1}^{M}(x_i/\sigma^2_{\text{v}_i} + \gamma_i) < 0$ must be met to achieve $\bar{Q}_{\text{d}} = 1$. This leads to $\lambda < \frac{1}{M}\sum_{i=1}^{M}(x_i/\sigma^2_{\text{v}_i} + \gamma_i)$, where $x_i = a_i = (1 - \rho_{1_i})\sigma^2_{\text{v}_i}$ to guarantee $\bar{Q}_{\text{d}} = 1$ in the whole NU range. Hence, to guarantee $\bar{Q}_{\text{fa}} = 0$ and $\bar{Q}_{\text{d}} = 1$, the conditions $\lambda > (1/M)\sum_{i=1}^{M}(1 + \rho_{2_i})$ and $\lambda < (1/M)\sum_{i=1}^{M}(1 - \rho_{1_i} + \gamma_i)$ must be respectively met, meaning that

$$\sum_{i=1}^{M}(1 + \rho_{2_i}) < \lambda < \sum_{i=1}^{M}(1 - \rho_{1_i} + \gamma_i), \quad (68)$$

which leads to

$$\sum_{i=1}^{M}\gamma_i > \sum_{i=1}^{M}(\rho_{2_i} + \rho_{1_i}). \quad (69)$$

An equality in (69) gives the generalized SNRw $\gamma_{\text{w}} = \sum_{i=1}^{M}(\rho_{2_i} + \rho_{1_i})$.

The inequality in (69) reveals an important advantage of CSS with SD fusion, namely, some SUs may even operate at an SNR below their individual SNRw's, as long the sum of all individual SNRs exceeds γ_{w}.

If $\gamma_i = \gamma$ for all i, (69) can be simplified to $\gamma > \frac{1}{M}\sum_{i=1}^{M}(\rho_{2_i} + \rho_{1_i})$ and, therefore,

$$\gamma_{\text{w}} = \frac{1}{M}\sum_{i=1}^{M}(\rho_{2_i} + \rho_{1_i}), \quad (70)$$

which becomes $\gamma_{\text{w}} = \rho_2 + \rho_1$ if $\rho_i = \rho$ for all i, consistently with (47).

If $\rho_{1_i} = \rho_{2_i} = \rho_i$ in (70), for $0 \leq \rho_i < 1$, this generalized SNRw is particularized to the case of combining the traditional NU source with the novel NU model. That is,

$$\gamma_{\text{w}} = \frac{2}{M}\sum_{i=1}^{M}\rho_i, \quad (71)$$

which becomes (23) when $M = 1$ or $\rho_i = \rho$ for all i. Likewise, using $\rho_{1_i} = (\rho_i - 1)/\rho_i$ and $\rho_{2_i} = \rho_i - 1$ in (70), $\rho_i \geq 1$, the SNRw under the traditional NU source and model is given by

$$\gamma_{\text{w}} = \frac{1}{M}\sum_{i=1}^{M}(\rho_i^2 - 1)/\rho_i, \quad (72)$$

which converts to (24) when $M = 1$ or $\rho_i = \rho$ for all i.

The required N is derived by eliminating τ from (66a) and (66b) and using $\hat{\sigma}^2_{\text{v}_i} = (1 + \rho_{2_i})\sigma^2_{\text{v}_i}$ in (66a) and

[1] In our derivations, given a vector function $f(\cdot)$ and constant vectors $\mathbf{a} = (a_1, \ldots, a_M)$ and $\mathbf{b} = (b_1, \ldots, b_M)$, we use the shorthand notation $\int_{\mathbf{a}}^{\mathbf{b}} f(\mathbf{x})d^M\mathbf{x}$ for the multiple integral $\int_{a_1}^{b_1}\cdots\int_{a_M}^{b_M} f(x_1, \ldots, x_M)dx_M \cdots dx_1$.

$\hat{\sigma}^2_{\mathrm{v}_i} = (1-\rho_{1_i})\sigma^2_{\mathrm{v}_i}$ in (66b). Thus, for $\gamma_i = \gamma$ for all i, one can obtain

$$N = \frac{\left(Q^{-1}(Q_{\mathrm{fa}})\sqrt{\sum_{i=1}^{M}(1+\rho_{2_i})^2} - Q^{-1}(Q_{\mathrm{d}})\sqrt{\sum_{i=1}^{M}(1-\rho_{1_i}+\gamma)^2}\right)^2}{M^2\left[\gamma - \frac{1}{M}\sum_{i=1}^{M}(\rho_{2_i}+\rho_{1_i})\right]^2}, \tag{73}$$

where $N\to\infty$ as $\gamma\to\frac{1}{M}\sum_{i=1}^{M}(\rho_{2_i}+\rho_{1_i})$, which is consistent with (70). Hence, $\gamma>\frac{1}{M}\sum_{i=1}^{M}(\rho_{2_i}+\rho_{1_i})$. Adopting $\rho_i = \rho$ for all i in (73), notice that it becomes

$$N = \frac{\left[Q^{-1}(P_{\mathrm{fa}})(1+\rho_2) - Q^{-1}(P_{\mathrm{d}})\,(1-\rho_1+\gamma)\right]^2}{M\left[\gamma-(\rho_1+\rho_2)\right]^2}. \tag{74}$$

Comparing (74) with (25), it can be concluded that CSS with SD fusion does not lower the SNRw when $\rho_i = \rho$ and $\gamma_i = \gamma$ for all i, but produces an M-fold reduction in N to yield a given target performance [10]. Observe that (74) converts to (25) if $M = 1$ as expected.

2) Novel NU source combined with traditional or novel NU model: Under the novel NU source, $\hat{\sigma}^2_{\mathrm{v}_i}$ is at the decision threshold. Thus, T in (12) has mean $\hat{\mu}_0 = (1/M)\sum_{i=1}^{M}\sigma^2_{\mathrm{v}_i}/\hat{\sigma}^2_{\mathrm{v}_i}$ and variance $\hat{\sigma}^2_0 = (M^2N)^{-1}\sum_{i=1}^{M}\sigma^4_{\mathrm{v}_i}/\hat{\sigma}^4_{\mathrm{v}_i}$ under $\mathcal{H}_0$, and mean $\hat{\mu}_1 = (1/M)\sum_{i=1}^{M}(\sigma^2_{\mathrm{v}_i}/\hat{\sigma}^2_{\mathrm{v}_i})\,(1+\gamma_i)$ and variance $\hat{\sigma}^2_1 = (M^2N)^{-1}\sum_{i=1}^{M}(\sigma^4_{\mathrm{v}_i}/\hat{\sigma}^4_{\mathrm{v}_i})\,(1+\gamma_i)^2$ under $\mathcal{H}_1$. Thus, for a given $\hat{\sigma}^2_{\mathrm{v}_i}$ value (with $\tau = \lambda$)

$$\begin{cases} Q_{\mathrm{fa}} = Q\left(\frac{\tau-\hat{\mu}_0}{\hat{\sigma}_0}\right) = Q\left(\frac{M\tau-\sum_{i=1}^{M}\sigma^2_{\mathrm{v}_i}/\hat{\sigma}^2_{\mathrm{v}_i}}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M}\sigma^4_{\mathrm{v}_i}/\hat{\sigma}^4_{\mathrm{v}_i}}}\right) \text{ and} & (75a)\\ Q_{\mathrm{d}} = Q\left(\frac{\tau-\hat{\mu}_1}{\hat{\sigma}_1}\right) = Q\left(\frac{M\tau-\sum_{i=1}^{M}\sigma^2_{\mathrm{v}_i}/\hat{\sigma}^2_{\mathrm{v}_i}\,(1+\gamma_i)}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M}\sigma^4_{\mathrm{v}_i}/\hat{\sigma}^4_{\mathrm{v}_i}\,(1+\gamma_i)^2}}\right). & (75b)\end{cases}$$

Averaging (75a) and (75b) over all possible values of $\hat{\sigma}^2_{\mathrm{v}_i}$ via the truncated PDF in (16), yields

$$\begin{cases} \bar{Q}_{\mathrm{fa}} = \int_{\mathbf{a}}^{\mathbf{b}} Q\left(\frac{M\lambda-\sum_{i=1}^{M}\sigma^2_{\mathrm{v}_i}/x_i}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M}\sigma^4_{\mathrm{v}_i}x_i^{-2}}}\right) f_{\hat{\sigma}^2_{\mathrm{v}_M},\ldots,\hat{\sigma}^2_{\mathrm{v}_1}}(\mathbf{x})d^M\mathbf{x} \text{ and} & (76a)\\ \bar{Q}_{\mathrm{d}} = \int_{\mathbf{a}}^{\mathbf{b}} Q\left(\frac{M\lambda-\sum_{i=1}^{M}\sigma^2_{\mathrm{v}_i}(1+\gamma_i)/x_i}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M}\sigma^4_{\mathrm{v}_i}(1+\gamma_i)^2x_i^{-2}}}\right) f_{\hat{\sigma}^2_{\mathrm{v}_M},\ldots,\hat{\sigma}^2_{\mathrm{v}_1}}(\mathbf{x})d^M\mathbf{x}. & (76b)\end{cases}$$

Applying (18) in (76a), the condition $M\lambda-\sum_{i=1}^{M}\sigma^2_{\mathrm{v}_i}/x_i > 0$ must be met to achieve $\bar{Q}_{\mathrm{fa}_i} = 0$. Thus, $\lambda > \frac{1}{M}\sum_{i=1}^{M}\sigma^2_{\mathrm{v}_i}/x_i$, where $x_i = a_i = (1-\rho_{1_i})\sigma^2_{\mathrm{v}_i}$ to guarantee $\bar{Q}_{\mathrm{fa}} = 0$ in the whole NU range. Likewise, applying (18) in (76b), it can be seen that $M\lambda-\sum_{i=1}^{M}\sigma^2_{\mathrm{v}_i}\,(1+\gamma_i)\,/x_i < 0$ must be met to yield $\bar{Q}_{\mathrm{d}} = 1$, that is, $\lambda < \frac{1}{M}\sum_{i=1}^{M}\sigma^2_{\mathrm{v}_i}(1+\gamma_i)\,/x_i$, where $x_i = b_i = (1+\rho_{2_i})\sigma^2_{\mathrm{v}_i}$ to guarantee $\bar{Q}_{\mathrm{d}} = 1$ in the whole NU range. Hence, to have $\bar{Q}_{\mathrm{fa}} = 0$ and $\bar{Q}_{\mathrm{d}} = 1$, notice that $\lambda > (1/M)\sum_{i=1}^{M}(1-\rho_{1_i})^{-1}$ and $\lambda < (1/M)\sum_{i=1}^{M}(1+\gamma_i)(1+\rho_{2_i})^{-1}$ must be met, respectively, meaning that $\sum_{i=1}^{M}(1-\rho_{1_i})^{-1} < \lambda < \sum_{i=1}^{M}(1+\gamma_i)(1+\rho_{2_i})^{-1}$. Then, it follows that

$$\sum_{i=1}^{M}\gamma_i(1+\rho_{2_i})^{-1} > \sum_{i=1}^{M}(1-\rho_{1_i})^{-1} - \sum_{i=1}^{M}(1+\rho_{2_i})^{-1}. \tag{77}$$

If one consider $\gamma_i = \gamma$ for all i in (77), then $\gamma > \left(\sum_{i=1}^{M}(1-\rho_{1_i})^{-1} - \sum_{i=1}^{M}(1+\rho_{2_i})^{-1}\right)/\sum_{i=1}^{M}(1+\rho_{2_i})^{-1}$. Therefore, the generalized SNRw in each SU is

$$\gamma_{\mathrm{w}} = \frac{\sum_{i=1}^{M}(1-\rho_{1_i})^{-1} - \sum_{i=1}^{M}(1+\rho_{2_i})^{-1}}{\sum_{i=1}^{M}(1+\rho_{2_i})^{-1}}. \tag{78}$$

When $\rho_i = \rho$ for all i is applied in (78), the generalized SNRw coincides with that in (63), *i.e.*,

$$\gamma_{\mathrm{w}} = (\rho_2+\rho_1)(1-\rho_1)^{-1}. \tag{79}$$

Using $\rho_{1_i} = (\rho_i-1)/\rho_i$ and $\rho_{2_i} = \rho_i - 1$ in (78) for $\rho_i \geq 1$, this generalized SNRw specializes to the case in which the novel NU source is combined with the traditional NU model, yielding

$$\gamma_{\mathrm{w}} = \left(\sum_{i=1}^{M} 1/\rho_i\right)^{-1}\sum_{i=1}^{M}(\rho_i^2-1)/\rho_i, \tag{80}$$

which agrees with (30) when $M = 1$ or $\rho_i = \rho$ for all i, that is, $\gamma_{\mathrm{w}} = \rho^2 - 1$. On the other hand, applying $\rho_{1_i} = \rho_{2_i} = \rho_i$ in (78), for $0 \leq \rho_i < 1$, yields

$$\gamma_{\mathrm{w}} = \left(\sum_{i=1}^{M}(1+\rho_i)^{-1}\right)^{-1} 2\sum_{i=1}^{M}\rho_i(1-\rho_i^2)^{-1}, \tag{81}$$

such that (81) is the SNRw under the novel NU source and model. Notice that for $M = 1$ or $\rho_i = \rho$ for all i, (81) converts to (31), which is consistent with [13, Eq. (28)].

Finally, the required N for given Q_{fa} and Q_{d} is derived by eliminating τ from (75a) and (75b), using $\hat{\sigma}^2_{\mathrm{v}_i} = (1-\rho_{1_i})\sigma^2_{\mathrm{v}_i}$ in (75a) and $\hat{\sigma}^2_{\mathrm{v}_i} = (1+\rho_{2_i})\sigma^2_{\mathrm{v}_i}$ in (75b). With this and $\gamma_i = \gamma$ for all i,

$$N = \frac{\left(Q^{-1}(Q_{\mathrm{fa}})\sqrt{\frac{\sum_{i=1}^{M}(1-\rho_{1_i})^{-2}}{\left[\sum_{i=1}^{M}(1+\rho_{2_i})^{-1}\right]^2}} - Q^{-1}(Q_{\mathrm{d}})(1+\gamma)\sqrt{\frac{\sum_{i=1}^{M}(1+\rho_{2_i})^{-2}}{\left[\sum_{i=1}^{M}(1+\rho_{2_i})^{-1}\right]^2}}\right)^2}{\left(\gamma - \frac{\sum_{i=1}^{M}(1-\rho_{1_i})^{-1}-\sum_{i=1}^{M}(1+\rho_{2_i})^{-1}}{\sum_{i=1}^{M}(1+\rho_{2_i})^{-1}}\right)^2}, \tag{82}$$

where $N\to\infty$ as $\gamma\to\left(\sum_{i=1}^{M}(1-\rho_{1_i})^{-1}/\sum_{i=1}^{M}(1+\rho_{2_i})^{-1}\right) - 1$, consistently with (78). Thus, $\gamma > \sum_{i=1}^{M}(1-\rho_{1_i})^{-1}/\sum_{i=1}^{M}(1+\rho_{2_i})^{-1} - 1$. Adopting $\rho_i = \rho$ for all i in (82), it becomes

$$N = \frac{\left[Q^{-1}(Q_{\mathrm{fa}})\frac{1+\rho_2}{1-\rho_1} - Q^{-1}(Q_{\mathrm{d}})\,(1+\gamma)\right]^2}{M\left[\gamma-(\rho_1+\rho_2)(1-\rho_1)^{-1}\right]^2}, \tag{83}$$

such that when compared with (32) leads to the same conclusions regarding (74).

The conclusions from nCSS and CSS with HD fusion once again apply here regarding the most conservative SNRw: See that "Eq. (81) ≥ Eq. (71)", "Eq. (80) ≥ Eq. (72)" and "Eq. (71) ≥ Eq. (72)", for $0 \leq \rho_i < 1$ and $\rho_i \geq 1$, respectively, and "Eq. (80) ≥ Eq. (71)" for $\rho_i \geq 1$ and $0 \leq \rho_i < 1$, respectively. Comparing these equations with those derived in Appendix A unveils that (7) leads to the same SNRw expressions obtained with (12), except under the novel NU source and model. In this case, "Eq. (81) ≥ Eq. (94)", with the equality holding if $\rho_i = \rho$ for all i. That is why we state that (12) can lead to better detection performances than (7) when

SUs have unequal noise powers and to a more conservative SNRw under different NU parameters when (12) is subjected to the novel NU source and model. For the other NU combinations, see that "Eq. (72) = Eq. (88)" under the traditional NU source and model, "Eq. (71) = Eq. (87)" under the traditional NU source and novel NU model, and "Eq. (80) = Eq. (93)" under the novel NU source and traditional NU model. Notice that the SNRw expressions derived with (12) hold for $\gamma_i = \gamma$ for all i, while those derived with (7) for $\sigma^2_{\mathrm{v}_i} = \sigma^2_{\mathrm{v}}$ and $\gamma_i = \gamma$ for all i (see Appendix A). Thus, (12) also leads to more general SNRw expressions in CSS with SD fusion.

IV. Numerical Results

This section presents theoretical, empirical, and Monte Carlo simulation results, aiming at validating the theoretical derivations and gaining insight into the performance and the SNRw of the ED in nCSS, in CSS with HD fusion under the k-out-of-M rule, and in CSS with SD fusion with the novel and traditional test statistic computation.

We assume a primary network with a single PU transmitter and a secondary network with $M = 5$ SUs in CSS, arbitrarily setting the PUs received signal powers at the SUs as $\sigma^2_{\mathrm{s}_1}, \sigma^2_{\mathrm{s}_2}, \ldots, \sigma^2_{\mathrm{s}_5} = 0.9153, 1.0310, 0.9552, 1.0819, 1.0166$, with $\sigma^2_{\mathrm{v}_i} = \sigma^2_{\mathrm{s}_i}/\gamma_i$ and $\gamma_i \neq \gamma_j$ for $i, j = 1, \ldots, M$, $i \neq j$, to guarantee heterogeneous SUs in terms of PU signal and noise powers, where γ_i is the average SNR of the ith SU. We apply the algorithms proposed in [13] to achieve our empirical SNRw results. Monte Carlo simulations counted with a PU activity of 50% in the on state and 50% in the off state, from a total of 50,000 simulation runs, for estimating the probabilities of false alarm and detection. The number of samples collected by each SU in the Monte Carlo simulations was $N = 2000$ per sensing run. Performance analyses follow using receiver operating characteristic (ROC) curves and measures of the area below them, which we refer to as the area under the curve (AUC). Wolfram Mathematica software, version 13, carried out all numerical computations.

Fig. 1 shows theoretical and simulated ROCs from nCSS ($M = 1$ SU) for an average SNR $\gamma = -12$ dB in the absence and presence of NU for some values of ρ. Simulation results are those with asterisk marks, and theoretical results are those with dotted, dashed, and solid lines. The dotted line identifies the NU-free ROC, whereas the solid and dashed lines identify the ROCs under NU for the combinations among traditional and novel NU sources and models. NU parameters are $\rho = 1.06, 1.09, 1.12, 1.15$ for the traditional NU model and $\rho = 0.06, 0.09, 0.12, 0.15$ for the novel NU model. We chose the first set of ρ as the second set plus one, *i.e.*, $\rho = \{0.06, 0.09, 0.12, 0.15\} + 1$. This intentional setting ensures the same deviation from the NU-free cases for each element of both sets, *i.e.*, $\rho = 1$ and $\rho = 0$, respectively, thus allowing fair comparisons of the results of Fig. 1 or Table I. Yet, see that these values guarantee heterogeneous SUs also in terms of NU levels. Fig. 1 shows the high sensitivity of the ED to NU, even for values of ρ close to the NU-free case for both NU models. It also shows perfect agreement between simulated and theoretical results and unveils that the traditional NU source and model lead to better performance metrics than the novel NU source and model, especially for larger values of ρ.

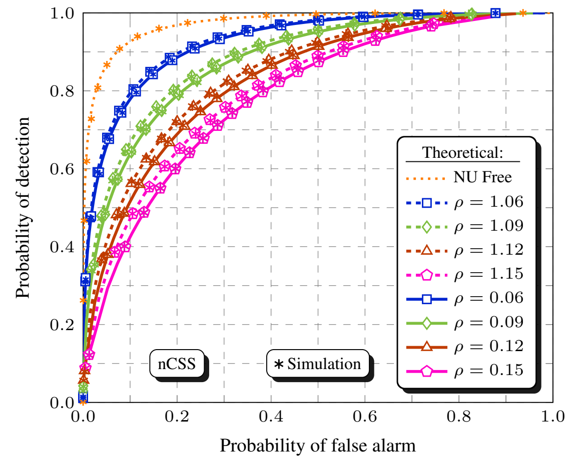

Fig. 1. ROCs from nCSS ($M = 1$ SU) with $\gamma = -12$ dB in the absence of NU (dotted line) and under NU for the traditional (dashed lines), with $\rho = 1.06, 1.09, 1.12, 1.15$, and novel (solid lines), with $\rho = 0.06, 0.09, 0.12, 0.15$, NU source and model.

Complementing Fig. 1, Table I shows the SNRw in dB and AUCs (from theoretical ROCs) for $\rho = 1.06, 1.09, 1.12, 1.15$ and $\rho = 0.06, 0.09, 0.12, 0.15$, for all combinations of NU sources and models. The AUCs give a more precise view than the ROCs on performances, besides showing the combinations not shown in Fig. 1. The shaded cells highlight the AUCs from the ROCs shown in Fig. 1 under NU and the SNRw. See from these cells that using the novel NU source and model is the most conservative approach since it gives the largest SNRw values.

Fig. 2 shows ED ROC curves from nCSS, CSS with HD fusion under the k-out-of-M rules OR ($k = 1$), MAJ ($k = \lfloor M/2 + 1 \rfloor = 3$), and AND ($k = M = 5$), and CSS with SD fusion under the traditional (7) and novel (12) T. The novel NU source and model have been applied, with NU parameters and average SNRs at the SUs respectively given by $\rho_1, \rho_2, \ldots, \rho_5 = 0.02, 0.015, 0.012, 0.01, 0.0075$ and $\gamma_1, \gamma_2, \ldots, \gamma_5 = -13, -14, -15, -16, -17$ dB. Theoretical and Monte Carlo simulation results show complete agreement again, with SD outperforming HD as expected. The MAJ rule outperforms the OR and AND in this scenario, and the superiority of the novel T in (12) over the traditional one in (7) is easily noticeable when SUs have unequal nominal noise powers. Based on the nCSS results, notice that the adopted SNRs prevailed over the NU levels in this scenario despite the well-known sensitivity of ED to NU. For instance, these results show that the SU_1 outperformed the remaining ones despite its highest NU level.

Fig. 3 shows the results of SNR versus the number N of samples required to yield probabilities of false alarm and detection of 0.1 and 0.9, respectively, at a given SU or the FC. It considers the absence and presence of NU and delivers the empirical SNRw in nCSS and CSS (marks on the SNR axis). Under NU, the setup is identical to that of Fig. 2 regarding NU source, model, and ρ. In the absence or presence of NU

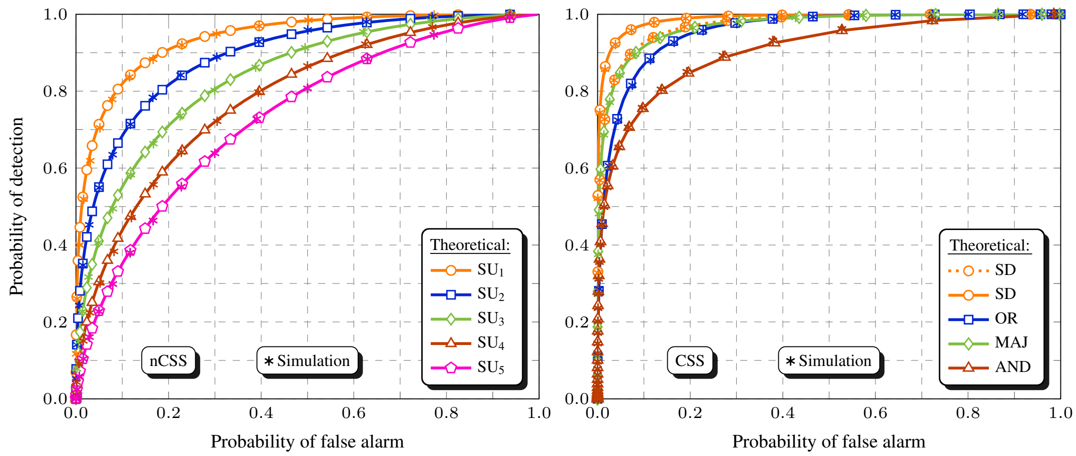

Fig. 2. ROCs from nCSS and CSS ($M = 5$ SUs) with SD under the traditional (7) (dotted line) and novel (12) (solide line) T, and HD fusion under the rules OR ($k = 1$), MAJ ($k = 3$), and AND ($k = 5$), for the novel NU source and model, NU parameters $\rho_1, \rho_2, \ldots, \rho_5 = 0.02, 0.015, 0.012, 0.01, 0.0075$, and average SNRs $\gamma_1, \gamma_2, \ldots, \gamma_5 = -13, -14, -15, -16, -17$ dB.

TABLE I
SNRw, IN DECIBELS, AND AUCs, IN nCSS FOR THE TRADITIONAL AND NOVEL NU SOURCE AND MODEL COMBINATIONS (WITH $\rho = 1.06, 1.09, 1.12, 1.15$ OR $\rho = 0.06, 0.09, 0.12, 0.15$ FOR THE TRADITIONAL OR NOVEL NU MODEL).

	Traditional Source	Novel Source
Traditional Model	$\gamma_w = -5.5217$; AUC = 0.9143	$\gamma_w = -4.9147$; AUC = 0.9126
	$\gamma_w = -6.4370$; AUC = 0.8456	$\gamma_w = -5.9448$; AUC = 0.8446
	$\gamma_w = -7.6304$; AUC = 0.7663	$\gamma_w = -7.2561$; AUC = 0.7674
	$\gamma_w = -9.3329$; AUC = 0.6902	$\gamma_w = -9.0798$; AUC = 0.6931
Novel Model	$\gamma_w = -5.2288$; AUC = 0.9105	$\gamma_w = -4.5230$; AUC = 0.9092
	$\gamma_w = -6.1979$; AUC = 0.8344	$\gamma_w = -5.6427$; AUC = 0.8346
	$\gamma_w = -7.4473$; AUC = 0.7463	$\gamma_w = -7.0377$; AUC = 0.7494
	$\gamma_w = -9.2082$; AUC = 0.6624	$\gamma_w = -8.9395$; AUC = 0.6676

($\rho_i = 0$ or $\rho_i > 0$ for all i, respectively), the nCSS curves are from (32), and the CSS ones are from (82) via (12) and from (95) via (7), with $\rho_{1_i} = \rho_{2_i} = \rho_i$ in all expressions. In CSS, the curve marked with circles and a dotted line refers to (7), whereas the one with asterisks and a solid line refers to (12). See in Fig. 3 that each empirical SNRw coincides with the theoretical one, which is the SNR value at the point in which $\{N, N_i\} \to \infty$. In the case of CSS, theoretical curves and empirical SNRw values are identical for (7) and (12), which is consistent with the fact that "Eq. (81) ≥ Eq. (94)". Notice that CSS reduces the required N to achieve given performance targets compared to nCSS as expected. A comparison between (83) and (32) or (74) and (25) unveils that CSS reduces N_i in M times.

Complementing Figs. 2 and 3 in a condensed way, Table II shows theoretical SNRw, γ_w, empirical SNRw, γ_w^e, and AUCs from nCSS and CSS with SD under the traditional (7) and novel (12) T. The table also gives theoretical SNRw and AUCs in CSS with HD under the OR, MAJ, and AND rules for all NU source and model combinations. The values of ρ and average SNRs are those used to plot Fig. 2. Results associated with Figs. 2 and 3 are those in the shaded cells. They confirm that the novel NU source and model combination is more conservative than the traditional NU source and model combination. The most and least conservative SNRw in HD

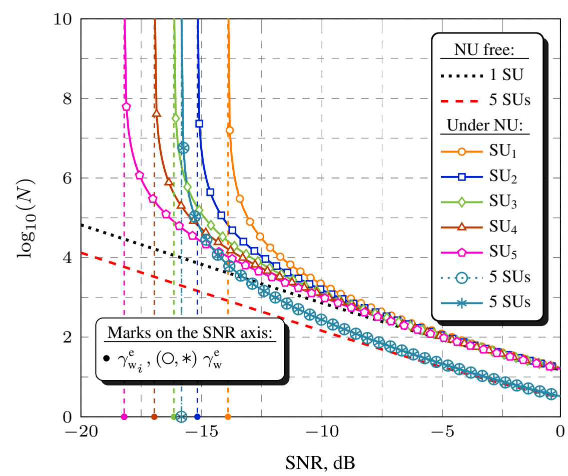

Fig. 3. Theoretical N versus SNR, and empirical SNRw (marks on the SNR axis) of the ED with $M = 5$ SUs, in nCSS ($\gamma_{w_i}^e$, $i = 1, \ldots, 5$) and CSS (γ_w^e) with SD under the traditional T in (7) (dotted line with circular marks) and the novel T in (12) (solid line with asterisk marks) for the novel NU source and model and NU parameters $\rho_1, \rho_2, \ldots, \rho_5 = 0.02, 0.015, 0.012, 0.01, 0.0075$.

correspond to the OR rule ($k = 1$) and the AND rule ($k = M$) since their expressions consider the maximum and minimum NU parameters, *i.e.*, ρ_1 and ρ_M, respectively. Table II also reinforces the overlap of the curves associated with CSS with SD in Fig. 3 once each value obtained with (7) in the shaded cell at the bottom of the table is practically equal to the corresponding one obtained with (12), meaning that "Eq. (81) ≈ Eq. (94)" for $\rho_1, \rho_2, \ldots, \rho_5 = 0.02, 0.015, 0.012, 0.01, 0.0075$. Finally, see that smaller SNRw values lead to larger AUCs (better detection performances) in CSS with SD. However, this does not hold in HD since the larger or smaller SNRw is proportional to ρ_1 and ρ_M, respectively, and performances follow the influence of the decision rule determined by k in the k-out-of-M rule. For instance, the MAJ rule attains superior detection performances than the OR and AND in this article, but the OR rule leads to the largest and the AND to the smallest SNRw. Comparing

γ_{w} with $\gamma_{\mathrm{w}}^{\mathrm{e}}$, note that theoretical and empirical results agree, which validates the theoretical SNRw expressions derived.

We highlight that we arbitrarily chose the settings of this section mainly to jointly ensure heterogeneous SUs and ease the graphical analyses, telling that one could freely try other setups.

V. Conclusions

This paper presented a comprehensive study of energy detection (ED) signal-to-noise ratio wall (SNRw) due to noise uncertainty (NU) in non-cooperative and cooperative spectrum sensing (CSS) with soft-decision (SD) and hard-decision fusion under the k-out-of-M rule for combinations of traditional and novel NU sources and models. It can be considered a compendium of conventional and novel possibilities regarding ED SNRw analyses, which studied performance metrics and derived the SNRw under these NU combinations. It also established a rule to map the novel into the traditional NU limits, allowing for comparisons involving the associated derivations. Analyses in CSS with SD considered the conventional and a novel test statistic computation that offers better performances under unequal noise powers and leads to a more conservative SNRw when considering different NU parameters and the novel NU source and model. Results showed that the most or the least conservative approach refers to the novel or the traditional NU source and model. The former approach also leads to more conservative performances and practical significance since it considers estimation errors of estimators in practice, thus revealing that the SNR limit imposed by NU can be tighter than reported in the literature. All empirical, simulation, and theoretical results are in accordance, which validates the derivations and the computer-aided calculations. Finally, notice that although this study had given more practical appeal in terms of NU modeling to the analyses, a step forward to bring an even more practical viewpoint can be revisiting it under fading, which is a work in progress.

Appendix A
SNR Walls of the ED with the Traditional Test Statistic Computation in Cooperative Spectrum Sensing with Soft Decision Fusion

This appendix presents the ED SNRw derivations with the conventional T in (7) under the traditional and novel NU source and model combinations. While serving for comparisons with the results presented throughout the text, it also confers a self-contained character to the article.

A. Traditional NU source combined with conventional or novel NU model

The means, variances, and global metrics of the traditional T in (7) are those in (8a), (8b), (9a), and (9b) in the absence of NU. Under NU, one must replace $\sigma_{\mathrm{v}_i}^2$ with $\hat{\sigma}_{\mathrm{v}_i}^2$ in these expressions, and use $\tau = \frac{\lambda}{M}\sum_{i=1}^{M}\sigma_{\mathrm{v}_i}^2$. After that, averaging Q_{fa} and Q_{d} over the PDF in (16) respectively yields

$$\begin{cases} \bar{Q}_{\mathrm{fa}} = \displaystyle\int_{\mathbf{a}}^{\mathbf{b}} Q\left(\frac{M\tau - \sum_{i=1}^{M} x_i}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M} x_i^2}}\right) f_{\hat{\sigma}_{\mathrm{v}_M}^2,\ldots,\hat{\sigma}_{\mathrm{v}_1}^2}(\mathbf{x})\, d^M\mathbf{x} \text{ and} & (84a)\\ \bar{Q}_{\mathrm{d}} = \displaystyle\int_{\mathbf{a}}^{\mathbf{b}} Q\left(\frac{M\tau - \sum_{i=1}^{M}(x_i+\sigma_{\mathrm{s}_i}^2)}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M}(x_i+\sigma_{\mathrm{s}_i}^2)^2}}\right) f_{\hat{\sigma}_{\mathrm{v}_M}^2,\ldots,\hat{\sigma}_{\mathrm{v}_1}^2}(\mathbf{x})\, d^M\mathbf{x}. & (84b)\end{cases}$$

Using the same procedure adopted in the corresponding main text, after applying (18) in (84a) and (84b) to guarantee $\bar{Q}_{\mathrm{fa}} = 0$ and $\bar{Q}_{\mathrm{d}} = 1$ in the whole NU range, the inequality

$$\sum_{i=1}^{M}(1+\rho_{2_i})\sigma_{\mathrm{v}_i}^2 < \lambda < \sum_{i=1}^{M}(1-\rho_{1_i}+\gamma_i)\sigma_{\mathrm{v}_i}^2 \quad (85)$$

must hold. This inequality is analogous to (68).

Assuming $\sigma_{\mathrm{v}_i}^2 = \sigma_{\mathrm{v}}^2$ for all i, one can write $\sum_{i=1}^{M}\gamma_i > \sum_{i=1}^{M}(\rho_{2_i}+\rho_{1_i})$ from (85), which is identical to (69). Moreover, with $\sigma_{\mathrm{v}_i}^2 = \sigma_{\mathrm{v}}^2$ and $\gamma_i = \gamma$ for all i in (85), (85) turns into (70), *i.e.*,

$$\gamma_{\mathrm{w}} = \frac{1}{M}\sum_{i=1}^{M}(\rho_{2_i}+\rho_{1_i}). \quad (86)$$

Thus, the remaining derivations are also equal to those of the corresponding main text. For example, using $\rho_{1_i} = \rho_{2_i} = \rho_i$, with $0 \le \rho_i < 1$, in (86), (86) becomes (72). That is,

$$\gamma_{\mathrm{w}} = \frac{2}{M}\sum_{i=1}^{M}\rho_i. \quad (87)$$

Similarly, using $\rho_{1_i} = (\rho_i - 1)/\rho_i$ and $\rho_{2_i} = \rho_i - 1$, with $\rho_i \ge 1$ in (86), (86) becomes (72), *i.e.*,

$$\gamma_{\mathrm{w}} = \frac{1}{M}\sum_{i=1}^{M}(\rho_i^2 - 1)/\rho_i. \quad (88)$$

B. Novel NU source combined with conventional or novel NU model

For the novel NU source, the means and variances of the test statistic in (7) follow (8a) and (8b), and $\hat{\tau} = (\lambda/M)\sum_{i=1}^{M}\hat{\sigma}_{\mathrm{v}_i}^2$. Thus, for a given fixed $\hat{\sigma}_{\mathrm{v}_i}^2$ value, Q_{fa} and Q_{d} can be written as

$$\begin{cases} Q_{\mathrm{fa}} = Q\left(\dfrac{\hat{\tau}-\mu_0}{\sigma_0}\right) = Q\left(\dfrac{M\hat{\tau} - \sum_{i=1}^{M}\sigma_{\mathrm{v}_i}^2}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M}\sigma_{\mathrm{v}_i}^4}}\right) \text{ and} & (89a)\\ Q_{\mathrm{d}} = Q\left(\dfrac{\hat{\tau}-\mu_1}{\sigma_1}\right) = Q\left(\dfrac{M\hat{\tau} - \sum_{i=1}^{M}(\sigma_{\mathrm{v}_i}^2+\sigma_{\mathrm{s}_i}^2)}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M}(\sigma_{\mathrm{v}_i}^2+\sigma_{\mathrm{s}_i}^2)^2}}\right). & (89b)\end{cases}$$

Averaging Q_{fa} and Q_{d} over all possible values of $\hat{\sigma}_{\mathrm{v}_i}^2$ via the PDF given in (16), it follows that

$$\begin{cases} \bar{Q}_{\mathrm{fa}} = \displaystyle\int_{\mathbf{a}}^{\mathbf{b}} Q\left(\frac{\lambda\sum_{i=1}^{M} x_i - \sum_{i=1}^{M}\sigma_{\mathrm{v}_i}^2}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M}\sigma_{\mathrm{v}_i}^4}}\right) f_{\hat{\sigma}_{\mathrm{v}_M}^2,\ldots,\hat{\sigma}_{\mathrm{v}_1}^2}(\mathbf{x})\, d^M\mathbf{x} \text{ and} & (90a)\\ \bar{Q}_{\mathrm{d}} = \displaystyle\int_{\mathbf{a}}^{\mathbf{b}} Q\left(\frac{\lambda\sum_{i=1}^{M} x_i - \sum_{i=1}^{M}(\sigma_{\mathrm{v}_i}^2+\sigma_{\mathrm{s}_i}^2)}{\frac{1}{\sqrt{N}}\sqrt{\sum_{i=1}^{M}(\sigma_{\mathrm{v}_i}^2+\sigma_{\mathrm{s}_i}^2)^2}}\right) f_{\hat{\sigma}_{\mathrm{v}_M}^2,\ldots,\hat{\sigma}_{\mathrm{v}_1}^2}(\mathbf{x})\, d^M\mathbf{x}. & (90b)\end{cases}$$

Applying (18) in (90a) and (90b), to guarantee $\bar{Q}_{\mathrm{fa}} = 0$ and $\bar{Q}_{\mathrm{d}} = 1$ in the whole NU range, see that $\sum_{i=1}^{M}\sigma_{\mathrm{v}_i}^2 / \sum_{i=1}^{M}(1 -$

TABLE II

THEORETICAL, γ_{W}, AND EMPIRICAL, $\gamma_{\mathrm{W}}^{\mathrm{E}}$, SNRW, AND AUCS (FROM THEORETICAL ROCS) FROM NCSS, CSS ($M = 5$ SUS) WITH SD FUSION UNDER THE TRADITIONAL (7) AND NOVEL (12) T, AND THEORETICAL SNRW AND AUCS IN CSS WITH HD FUSION UNDER THE OR, MAJ AND AND RULES, WITH $\rho_1, \rho_2, \ldots, \rho_5 = 0.02, 0.015, 0.012, 0.01, 0.0075$ AND $\gamma_1, \gamma_2, \ldots, \gamma_5 = -13, -14, -15, -16, -17$ FOR THE TRADITIONAL AND NOVEL NU SOURCE AND MODEL COMBINATIONS.

	nCSS					
	Traditional Source			Novel Source		
Traditional Model	$\gamma_{\mathrm{w}_1} = -14.0222$	$\gamma_{\mathrm{w}_1}^{\mathrm{e}} = -14.0234$	$\mathrm{AUC}_1 = 0.927322$	$\gamma_{\mathrm{w}_1} = -13.9362$	$\gamma_{\mathrm{w}_1}^{\mathrm{e}} = -13.9355$	$\mathrm{AUC}_1 = 0.926977$
	$\gamma_{\mathrm{w}_2} = -15.2610$	$\gamma_{\mathrm{w}_2}^{\mathrm{e}} = -15.2344$	$\mathrm{AUC}_2 = 0.882498$	$\gamma_{\mathrm{w}_2} = -15.1963$	$\gamma_{\mathrm{w}_2}^{\mathrm{e}} = -15.1953$	$\mathrm{AUC}_2 = 0.882303$
	$\gamma_{\mathrm{w}_3} = -16.2237$	$\gamma_{\mathrm{w}_3}^{\mathrm{e}} = -16.2500$	$\mathrm{AUC}_3 = 0.830432$	$\gamma_{\mathrm{w}_3} = -16.1719$	$\gamma_{\mathrm{w}_3}^{\mathrm{e}} = -16.1719$	$\mathrm{AUC}_3 = 0.830325$
	$\gamma_{\mathrm{w}_4} = -17.0113$	$\gamma_{\mathrm{w}_4}^{\mathrm{e}} = -17.0312$	$\mathrm{AUC}_4 = 0.777786$	$\gamma_{\mathrm{w}_4} = -16.9680$	$\gamma_{\mathrm{w}_4}^{\mathrm{e}} = -16.9727$	$\mathrm{AUC}_4 = 0.777730$
	$\gamma_{\mathrm{w}_5} = -18.2553$	$\gamma_{\mathrm{w}_5}^{\mathrm{e}} = -18.2812$	$\mathrm{AUC}_5 = 0.729369$	$\gamma_{\mathrm{w}_5} = -18.2228$	$\gamma_{\mathrm{w}_5}^{\mathrm{e}} = -18.2227$	$\mathrm{AUC}_5 = 0.729346$
Novel Model	$\gamma_{\mathrm{w}_1} = -13.9794$	$\gamma_{\mathrm{w}_1}^{\mathrm{e}} = -13.9844$	$\mathrm{AUC}_1 = 0.927150$	$\gamma_{\mathrm{w}_1} = -13.8917$	$\gamma_{\mathrm{w}_1}^{\mathrm{e}} = -13.8965$	$\mathrm{AUC}_1 = 0.926805$
	$\gamma_{\mathrm{w}_2} = -15.2288$	$\gamma_{\mathrm{w}_2}^{\mathrm{e}} = -15.2344$	$\mathrm{AUC}_2 = 0.882411$	$\gamma_{\mathrm{w}_2} = -15.1631$	$\gamma_{\mathrm{w}_2}^{\mathrm{e}} = -15.1660$	$\mathrm{AUC}_2 = 0.882216$
	$\gamma_{\mathrm{w}_3} = -16.1979$	$\gamma_{\mathrm{w}_3}^{\mathrm{e}} = -16.2109$	$\mathrm{AUC}_3 = 0.830385$	$\gamma_{\mathrm{w}_3} = -16.1455$	$\gamma_{\mathrm{w}_3}^{\mathrm{e}} = -16.1426$	$\mathrm{AUC}_3 = 0.830278$
	$\gamma_{\mathrm{w}_4} = -16.9897$	$\gamma_{\mathrm{w}_4}^{\mathrm{e}} = -16.9531$	$\mathrm{AUC}_4 = 0.777760$	$\gamma_{\mathrm{w}_4} = -16.9461$	$\gamma_{\mathrm{w}_4}^{\mathrm{e}} = -16.9531$	$\mathrm{AUC}_4 = 0.777704$
	$\gamma_{\mathrm{w}_5} = -18.2391$	$\gamma_{\mathrm{w}_5}^{\mathrm{e}} = -18.2812$	$\mathrm{AUC}_5 = 0.729358$	$\gamma_{\mathrm{w}_5} = -18.2064$	$\gamma_{\mathrm{w}_5}^{\mathrm{e}} = -18.2031$	$\mathrm{AUC}_5 = 0.729336$

	CSS in HD with the fusion rule OR ($k = 1$), MAJ ($k = 3$), and AND ($k = 5$)					
	Traditional Source			Novel Source		
	$k = 1$	$k = \lfloor M/2 + 1 \rfloor$	$k = M$	$k = 1$	$k = \lfloor M/2 + 1 \rfloor$	$k = M$
Traditional Model	$\gamma_{\mathrm{w}_1} = -14.0222$	$\gamma_{\mathrm{w}_1} = -15.0021$	$\gamma_{\mathrm{w}_1} = -15.6691$	$\gamma_{\mathrm{w}_1} = -13.9362$	$\gamma_{\mathrm{w}_1} = -14.9502$	$\gamma_{\mathrm{w}_1} = -15.6366$
	$\gamma_{\mathrm{w}_2} = -14.5869$	$\gamma_{\mathrm{w}_2} = -15.7222$	$\gamma_{\mathrm{w}_2} = -16.5212$	$\gamma_{\mathrm{w}_2} = -14.5009$	$\gamma_{\mathrm{w}_2} = -15.6704$	$\gamma_{\mathrm{w}_2} = -16.4887$
	$\gamma_{\mathrm{w}_3} = -14.9679$	$\gamma_{\mathrm{w}_3} = -16.2237$	$\gamma_{\mathrm{w}_3} = -17.1315$	$\gamma_{\mathrm{w}_3} = -14.8819$	$\gamma_{\mathrm{w}_3} = -16.1719$	$\gamma_{\mathrm{w}_3} = -17.0990$
	$\gamma_{\mathrm{w}_4} = -15.2431$	$\gamma_{\mathrm{w}_4} = -16.5954$	$\gamma_{\mathrm{w}_4} = -17.5943$	$\gamma_{\mathrm{w}_4} = -15.1571$	$\gamma_{\mathrm{w}_4} = -16.5436$	$\gamma_{\mathrm{w}_4} = -17.5618$
	$\gamma_{\mathrm{w}_5} = -15.6155$	$\gamma_{\mathrm{w}_5} = -17.1121$	$\gamma_{\mathrm{w}_5} = -18.2553$	$\gamma_{\mathrm{w}_5} = -15.5295$	$\gamma_{\mathrm{w}_5} = -17.0603$	$\gamma_{\mathrm{w}_5} = -18.2228$
	$\mathrm{AUC}_1 = 0.952032$	$\mathrm{AUC}_{\lfloor M/2+1 \rfloor} = 0.967339$	$\mathrm{AUC}_M = 0.888796$	$\mathrm{AUC}_1 = 0.952042$	$\mathrm{AUC}_{\lfloor M/2+1 \rfloor} = 0.967221$	$\mathrm{AUC}_M = 0.888589$
Novel Model	$\gamma_{\mathrm{w}_1} = -13.9794$	$\gamma_{\mathrm{w}_1} = -14.9485$	$\gamma_{\mathrm{w}_1} = -15.6067$	$\gamma_{\mathrm{w}_1} = -13.8917$	$\gamma_{\mathrm{w}_1} = -14.8961$	$\gamma_{\mathrm{w}_1} = -15.5740$
	$\gamma_{\mathrm{w}_2} = -14.5593$	$\gamma_{\mathrm{w}_2} = -15.6864$	$\gamma_{\mathrm{w}_2} = -16.4782$	$\gamma_{\mathrm{w}_2} = -14.4716$	$\gamma_{\mathrm{w}_2} = -15.6339$	$\gamma_{\mathrm{w}_2} = -16.4455$
	$\gamma_{\mathrm{w}_3} = -14.9485$	$\gamma_{\mathrm{w}_3} = -16.1979$	$\gamma_{\mathrm{w}_3} = -17.0997$	$\gamma_{\mathrm{w}_3} = -14.8608$	$\gamma_{\mathrm{w}_3} = -16.1455$	$\gamma_{\mathrm{w}_3} = -17.0670$
	$\gamma_{\mathrm{w}_4} = -15.2288$	$\gamma_{\mathrm{w}_4} = -16.5758$	$\gamma_{\mathrm{w}_4} = -17.5696$	$\gamma_{\mathrm{w}_4} = -15.1410$	$\gamma_{\mathrm{w}_4} = -16.5233$	$\gamma_{\mathrm{w}_4} = -17.5369$
	$\gamma_{\mathrm{w}_5} = -15.6067$	$\gamma_{\mathrm{w}_5} = -17.0997$	$\gamma_{\mathrm{w}_5} = -18.2391$	$\gamma_{\mathrm{w}_5} = -15.5189$	$\gamma_{\mathrm{w}_5} = -17.0472$	$\gamma_{\mathrm{w}_5} = -18.2064$
	$\mathrm{AUC}_1 = 0.951969$	$\mathrm{AUC}_{\lfloor M/2+1 \rfloor} = 0.967300$	$\mathrm{AUC}_M = 0.888744$	$\mathrm{AUC}_1 = 0.951979$	$\mathrm{AUC}_{\lfloor M/2+1 \rfloor} = 0.967179$	$\mathrm{AUC}_M = 0.888538$

	CSS with SD for the traditional T in (7)					
	Traditional Source			Novel Source		
Traditional Model	$\gamma_{\mathrm{w}} = -15.9146$	$\gamma_{\mathrm{w}}^{\mathrm{e}} = -15.9180$	AUC = 0.969912	$\gamma_{\mathrm{w}} = -15.8590$	$\gamma_{\mathrm{w}}^{\mathrm{e}} = -15.8594$	AUC = 0.969872
Novel Model	$\gamma_{\mathrm{w}} = -15.8838$	$\gamma_{\mathrm{w}}^{\mathrm{e}} = -15.8789$	AUC = 0.969889	$\gamma_{\mathrm{w}} = -15.8272$	$\gamma_{\mathrm{w}}^{\mathrm{e}} = -15.8301$	AUC = 0.969851

	CSS with SD for the novel T in (12)					
	Traditional Source			Novel Source		
Traditional Model	$\gamma_{\mathrm{w}} = -15.9180$	$\gamma_{\mathrm{w}}^{\mathrm{e}} = -15.9146$	AUC = 0.984992	$\gamma_{\mathrm{w}} = -15.8594$	$\gamma_{\mathrm{w}}^{\mathrm{e}} = -15.8590$	AUC = 0.984957
Novel Model	$\gamma_{\mathrm{w}} = -15.8789$	$\gamma_{\mathrm{w}}^{\mathrm{e}} = -15.8838$	AUC = 0.984961	$\gamma_{\mathrm{w}} = -15.8252$	$\gamma_{\mathrm{w}}^{\mathrm{e}} = -15.8272$	AUC = 0.984932

$\rho_{1_i})\sigma_{\mathrm{v}_i}^2 < \lambda < \sum_{i=1}^{M}(1+\gamma_i)\sigma_{\mathrm{v}_i}^2 / \sum_{i=1}^{M}(1+\rho_{2_i})\sigma_{\mathrm{v}_i}^2$, which leads to

$$\sum_{i=1}^{M}(1+\gamma_i)\sigma_{\mathrm{v}_i}^2 > \frac{\sum_{i=1}^{M}\sigma_{\mathrm{v}_i}^2 \sum_{i=1}^{M}(1+\rho_{2_i})\sigma_{\mathrm{v}_i}^2}{\sum_{i=1}^{M}(1-\rho_{1_i})\sigma_{\mathrm{v}_i}^2}. \quad (91)$$

If $\sigma_{\mathrm{v}_i}^2 = \sigma_{\mathrm{v}}^2$ for all i, one can write (91) as $\sum_{i=1}^{M}\gamma_i = M\sum_{i=1}^{M}(\rho_{2_i}+\rho_{1_i}) / \sum_{i=1}^{M}(1-\rho_{1_i})$. Besides, if $\sigma_{\mathrm{v}_i}^2 = \sigma_{\mathrm{v}}^2$ and $\gamma_i = \gamma$ for all i in (91), then $\gamma > \sum_{i=1}^{M}(\rho_{2_i}+\rho_{1_i}) / \sum_{i=1}^{M}(1-\rho_{1_i})$. Thus, the generalized SNRw in each SU is

$$\gamma_{\mathrm{w}} = \left(\sum_{i=1}^{M}(1-\rho_{1_i})\right)^{-1}\sum_{i=1}^{M}(\rho_{2_i}+\rho_{1_i}), \quad (92)$$

in which (92) specializes to (79) with $\rho_i = \rho$ for all i.

Using $\rho_{1_i} = (\rho_i - 1)/\rho_i$ and $\rho_{2_i} = \rho_i - 1$ in (92), for $\rho_i \geq 1$, the SNRw in the case of combining the novel NU source with the traditional NU model is identical to (80). That is,

$$\gamma_{\mathrm{w}} = \left(\sum_{i=1}^{M} 1/\rho_i\right)^{-1}\sum_{i=1}^{M}(\rho_i^2 - 1)/\rho_i. \quad (93)$$

On the other hand, applying $\rho_{1_i} = \rho_{2_i} = \rho_i$ in (92), for $0 \leq \rho_i < 1$, the SNRw in each SU is

$$\gamma_{\mathrm{w}} = \left(\sum_{i=1}^{M}(1-\rho_i)\right)^{-1} 2\sum_{i=1}^{M}\rho_i, \quad (94)$$

which is the SNRw under the novel NU source and model, as demonstrated in [13, Eq. (57)].

The required N is derived here by eliminating λ from (89a) and (89b), using $\hat{\sigma}_{\mathrm{v}_i}^2 = (1-\rho_{1_i})\sigma_{\mathrm{v}_i}^2$ in (89a) and $\hat{\sigma}_{\mathrm{v}_i}^2 = (1+\rho_{2_i})\sigma_{\mathrm{v}_i}^2$ in (89b). Thus, for $\sigma_{\mathrm{v}_i}^2 = \sigma_{\mathrm{v}}^2$ and $\gamma_i = \gamma$ for all i,

$$N = \frac{\left[Q^{-1}(Q_{\mathrm{fa}})\frac{\sum_{i=1}^{M}(1+\rho_{2_i})}{\sum_{i=1}^{M}(1-\rho_{1_i})} - Q^{-1}(Q_{\mathrm{d}})(1+\gamma)\right]^2}{M\left[\gamma - \frac{\sum_{i=1}^{M}(\rho_{2_i}+\rho_{1_i})}{\sum_{i=1}^{M}(1-\rho_{1_i})}\right]^2}, \quad (95)$$

where $N \to \infty$ as $\gamma \to \sum_{i=1}^{M}(\rho_{2_i}+\rho_{1_i}) / \sum_{i=1}^{M}(1-\rho_{1_i})$, which is consistent with (92). So, $\gamma > \sum_{i=1}^{M}(\rho_{2_i}+\rho_{1_i}) / \sum_{i=1}^{M}(1-\rho_{1_i})$. Notice that (95) converts into (83) if $\rho_i = \rho$ for all i.

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