Coded Cooperation with Single Parity-Check Turbo-Product Codes over Fast Fading Channels

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***Abstract*—This paper describes a new coded cooperation scheme based on single parity-check turbo-product codes. The channel soft-information from the source and the relays are also combined in a novel way at the destination. The scheme has simple encoding and decoding, and unveils potential for large cooperation gains in fast fading channels.**

Keywords—coded cooperation; cooperation diversity; SPCTPC

I. INTRODUCTION

In cooperative communications with decode-and-forward (DF) relaying [1-4], a number of relay nodes collaborate by adopting the same channel code as the source, with or without incremental redundancy, which is named coded cooperation [4], or a DF relaying with repetition coding [1], respectively. If the relays use a different channel code, the scheme is called DF relaying with parallel coding. Parallel coding can also be accomplished if the relays interleave the recovered information bits before re-encoding with the same code, as is the case in distributed turbo coding (DTC) [2-3]. Important soft-information (SI) is lost in DTC, since the relays perform hard decisions. To remedy this problem, a soft-decode and forward (SDF) approach has been introduced [3]. In [1] the authors propose a DF relaying with partial repetition coding, where the relay repeats a fraction of the coded message. The destination then performs maximum-ratio combining (MRC) of the common part of the message transmitted by both the source and the relay.

Here we propose a novel coded cooperation scheme with a new DF relaying approach. We consider serial concatenated multidimensional single parity-check product codes (SPCPC) [5][6]. The relays decode and re-encode part of the received codeword where systematic bits are located. Forwarding takes place if no error is detected. Differently from any of the above-mentioned schemes, the destination forms soft channel information from the relayed transmissions and adds it to the common part of the soft channel information obtained from the direct transmission. It is shown that the enhanced SI yields attractive performance, while the encoding and decoding complexities are kept small due to short block lengths and low complexity of SPCPCs.

II. BACKGROUND

A. Single parity-check product code construction

A D-dimensional single parity-check product code (SPCPC) [5] is constructed as follows. For $d = 1, 2, \ldots, D$, the information bits are arranged in a D-dimensional hypercube whose d-th dimension has message length $k_d = n_d - 1$. All the blocks along the d-th dimension are encoded by a $(n_d, n_d - 1, 2)$ SPC code. After encoding in all dimensions, check-on-checks are also inserted. The resulting SPCPC has message length κ, codeword length ν, code rate r, and minimum distance $\delta_{\min}$ given by the product of the corresponding parameters in all dimensions. When identical single parity-check component codes $(n_d, k_d) = (n, n - 1)$ are used in all dimensions, the SPCPC has $\kappa = (n - 1)^D$, $\nu = n^D$, $r = (1 - 1/n)^D$ and $\delta_{\min} = 2^D$.

B. Serial concatenation of single parity-check product codes

We consider the serial concatenation of two systematic SPCPCs separated by an interleaver, as in [6]. However, differently from [6], we allow for the two SPCPCs to have distinct number of dimensions, as long as the outer SPCPC codeword has length equal to the message length of the inner SPCPC. As a consequence, the codeword lengths (n_o and n_i) and message lengths (k_o and k_i) of the component SPCs must comply with

$$
\begin{aligned}
&(n_o, k_o) = (n_o, n_o - 1), \text{ and} \\
&(n_i, k_i) = \left(\sqrt[D_i]{n_o^{D_o}} + 1, \sqrt[D_i]{n_o^{D_o}}\right),
\end{aligned}
\tag{1}
$$

where the subscripts "o" and "i" stand for "outer" and "inner", respectively, and D_o, D_i are the dimensions of the corresponding SPCPCs. The overall code rate is $r = k_o^{D_o} / n_i^{D_i}$. Also differently from [6], the turbo decoding of the concatenated code does not rely on iterated message passing between SPCPC decoders. Instead, a codeword can be turbo-decoded in two ways: i) as a codeword of the D_i-dimensional inner code C_i followed by a turbo decoding of the D_o-dimensional outer code, or ii) as a D_o-dimensional outer code C_o alone. In the former case, soft output values from the inner decoding is passed just once for the outer decoding. In the latter case, only part of the received codeword is processed by the outer turbo decoder alone.

C. Iterative decoding of the single parity-check product codes

When an SPCPC is iteratively decoded, it is referred to as a *single parity-check turbo-product code* (SPCTPC). In the soft-input soft-output (SISO) decoding process, the input log-likelihood ratio (LLR) $L'(\hat{b})$ is composed of the channel LLR, $L_c(x)$, and the *a priori* LLR, $L(b)$, for all received symbols. The a posteriori LLR $L(\hat{b})$ is composed of $L'(\hat{b})$ and the extrinsic information $L_e(\hat{b})$. In each iteration $L_e(\hat{b})$ is fed back to the decoder input as an updated $L(b)$. For binary antipodal signaling over a flat fading channel, the soft input is given by

$$L'(\hat{b}) = L_c(x) + L(b) = \frac{2\sqrt{E}}{\sigma^2} gx + L(b) \qquad (2)$$

where $E = rE_b$ is the average symbol energy, with E_b being the average message bit energy, $x = \pm g\sqrt{E} + \eta$ is the detector output affected by the channel gain g and corrupted by an additive white Gaussian noise (AWGN) η with variance $\sigma^2 = N_0/2$, being N_0 the unilateral noise power spectral density. The SISO decoding computes an LLR of a given bit according to [5]:

$$L\left(\sum_j \oplus b_j\right) = 2\operatorname{arctanh}\left[\prod_j \tanh\left(\frac{L(b_j)}{2}\right)\right] \qquad (3)$$

III. The Proposed Coded Cooperation Scheme

Consider the cooperation scenario illustrated in Fig. 1. The source transmits a codeword composed of two segments (using two time slots): the first contains "$C_o\pi$", which stands for an interleaved codeword of the outer code, C_o, and the second contains "PC_i", the parities of the inner codeword C_i. The relay receives this transmission and decodes the first segment as a codeword of C_o. If all check equations verify, the relay re-encodes the decoded codeword of C_o and transmits a fraction of it to the destination, in the second segment, denoted by "$C_{op}\pi$". This re-encoded part contains a fraction of the systematic information bits from the source node, including or not (depending on that fraction) some parity bits belonging to the first segment. The part marked with an "X" identifies the relay's first segment, as the relay may act as a source node having its own data.

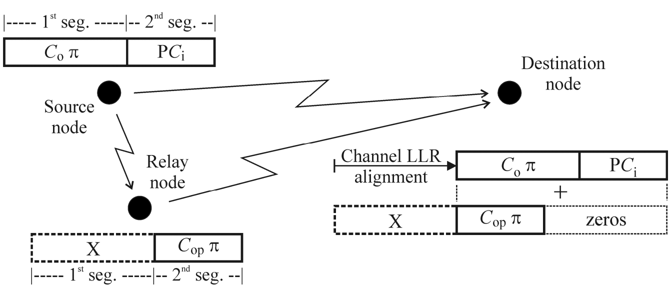

Fig. 1. Coded cooperation scenario with one cooperating relay.

The ratio between the lengths of the second segment and the sum of the lengths of the first and second ones is known as the cooperation ratio R_c. We define an effective cooperation ratio $R_{cE} = R_c$ if $R_c \leq 50\%$, $R_{cE} = 50\%$ otherwise (when the second segment is smaller than the first, i.e. $R_c < 50\%$, only a fraction of the re-encoded codeword of C_o fits in the second segment).

For any number of relays, the destination receives a corrupted codeword from the source and retains it until it is aligned with the second segment transmitted from the relays. This is necessary because the diversity bits are in the first segment.

The channel LLR at the destination is the sum of the LLRs from the source and relays, these last ones computed only for the fraction of the re-encoded C_o transmitted in the second segment. This new form of cooperation was inspired by the fact that the adopted symbol-by-symbol decoding algorithm can output words that are not codewords of the product code, since it works by exploring only the dependencies among coded bits of the component SPCs. This suggests that, if we are interested in increasing the reliability of message bits, it is intuitive to produce diversity where these bits are grouped, i.e., where the bits of C_o are. We have tested the conventional cooperation with diversity in the parity bits of C_i, i.e. in the second segment, but only marginal gains have been achieved.

Since a number of iterations close to the number of dimensions is enough for convergence of the decoding algorithm [5], after (D_i+1) iterations the inner decoder outputs its vector of LLRs with length equal to the length of the outer code codeword. From a de-interleaved version of these LLRs, the outer decoder performs (D_o+1) iterations and computes the soft output $L(\hat{b})$, from which the message bits are estimated. When only the first segment is available for decoding, only the outer decoder is used.

Notice that the above-described approach differs from conventional coded cooperation strategies, in which the second segment would contain the re-encoded parities of the inner codeword, i.e. diversity would be produced in these parities. To the best of the author's knowledge, the proposed form of channel encoding, diversity and soft combining, though very simple, have not been jointly considered in the literature so far.

IV. Numerical Results and Comments

We have considered three scenarios: 1) *no cooperation* – the destination has two segments from the source to perform decoding, i.e. it has the complete D_i-dimensional code to decode; 2) *source cooperating with a relay and the relay not cooperating* – the destination has only the first segment to recover the message bits from the source, i.e. it has the D_o-dimensional code to decode; 3) *one or two relays cooperating with the source* – here, improved cooperation diversity is achieved; the complete D_i-dimensional code is considered again. It is not of concern here the specifics of any coded cooperation algorithm. Consequently, no comparisons were made in this sense.

There are several codes complying with (1), giving some freedom in choosing the effective cooperation ratio R_{cE}, the code length ν, and the code rate r. In Fig. 2 we present bit error rate (BER) results for the code $(8,7)^2+(5,4)^3 = (125,49)$, which has R_{cE} = 48.8% and r = 0.392. The source-relay channels are AWGN channels with E_b/N_0 = 30 dB, simulating quasi-error-free transmissions. The source-destination and relay-destination channels are orthogonal flat Rayleigh fading channels, with independent and identically distributed gains in a symbol-by-symbol basis, simulating a fast fading, and $E\{g^2\}$ = 1. The average E_b/N_0 for the relay-destination links are fixed in 5.5 dB. Channel gains are known by the decoder when applicable; the modulation is BPSK with coherent detection.

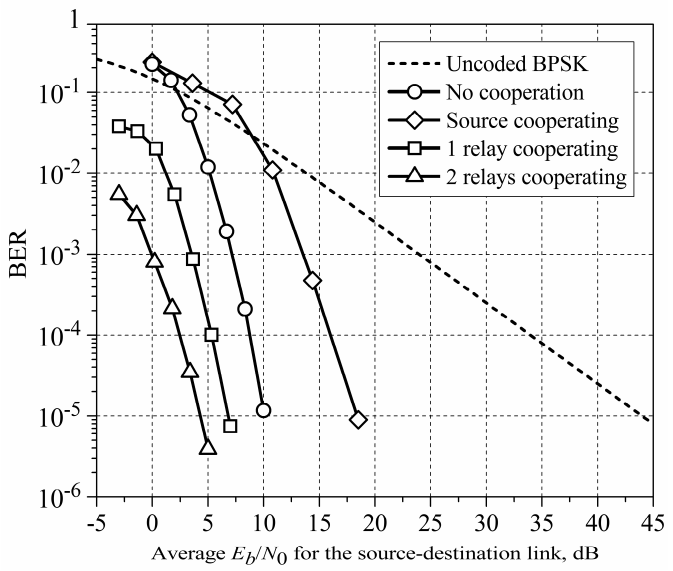

Fig. 2. Numerical Results.

It can be noticed in Fig. 2 that large coding gains can be obtained: 3.5 dB with one relay, and 6 dB with two relays, for a wide range of BER values. Considering that E_b/N_0 for the relay-destination links is half the value enough for a BER of 10^{-5} at the destination, with no cooperation, these are attractive results. Potentially, actual cooperation algorithms combined with different code parameters, different number of cooperating relays and dynamic signal-to-noise ratio scenarios can further increase the cooperation gains beyond those in Fig. 2. It is expected that relatively smaller gains will be produced for the most powerful codes complying with (1), a typical behavior of codes that tend to be capacity-achieving ones.

V. Concluding Remarks and Future Work

A new way of combining the channel soft-information, along with a new coded cooperation scheme based on SPCTPC unveiled potential for large cooperation gains in a fast fading channel environment. Moreover, the simple encoding and decoding of the SPCTPC is an attractive advantage from the implementation viewpoint. Nevertheless, some open questions still remains, mainly related to the actual cooperation gains that could be achieved under a real cooperation algorithm and a more realistic scenario in terms of possible erroneous decoding in the cooperation decision process. Some future work in this direction might be: i) the use of a cooperation decision method that allows for mutual cooperation between users. In this case it will be possible to analyze if the effect of worsening the performance due to the cooperation of the *source* node can be reduced by the cooperation of one or more *relays*; ii) making the encoders and decoders work with larger dimension codes. This can improve the capability of verifying the decoding correctness and, thereby, improve system performance.

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