Analysis of the Burst Gated Impulsive Noise Effect in Optimum Receivers

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***Abstract*— This article presents a new approach for analysing the effect of the gated impulsive noise in optimum receivers based on the maximum likelihood criteria. In this approach, the impulsive noise behavior is characterized as the modulation of a white Gaussian process by the product of two discrete random processes.**

***Index Terms*— Gaussian noise, impulsive noise, gated noise, optimum receivers**

I. Introduction

Impulsive noise is an interesting research subject for the telecommunications area and has been studied since 1950. The mathematical characterization of its harmful effects on several communication systems, such as ADSL (*Asymmetrical Digital Subscriber Line*) [1], PLC (*Power Line Communications*) [2], wireless communications [3], digital television [4] and radar systems [5] aims at developing techniques to mitigate its effects.

An impulsive noise model has been used in digital television, called Gated Additive Gaussian Noise (GAWGN), due to its facility of implementation and analysis, as well as its adherence to the measured data [6], [7]. Recently, some authors have applied the GAWGN impulsive noise to evaluate the digital television systems performance [8]. One of the main advantages of the GAWGN noise is its simulation facility compared to other models presented in the literature.

This article presents a model for impulsive noise as well as a stochastic analysis using auxiliary gating signals, which is new according to the authors knowledge. The influence of its effects on an optimum receiver, based on the maximum *a posteriori* probability (MAP), is presented and evaluated using the new expressions obtained for the Bit Error Probability (BEP) of M-QAM modulation schemes.

The total noise, represented by $\eta(t)$, is composed as a sum of two parts. The first one, denoted by $\eta_g(t)$, is characterized by a Gaussian process with null mean and variance σ_g^2. The second one, denoted by $\eta_i(t)$, is composed by a Gaussian process with null mean and variance σ_i^2 modulated by the product of two independent discrete random signals $c_1(t)$ and $c_2(t)$. The signal $c_1(t)$ takes the values 0 and 1, at random, and $c_2(t)$ can take, at random, different discrete values when $c_1(t)$ is 1. When $c_1(t)$ takes values different from 0, one can say that there is a noisy burst. The occurrence of $c_2(t)$ in the intervals in which $c_1(t)$ is different from 0 defines the random pulses. The occurrence of impulsive noise in the burst intervals defines the gated impulsive noise.

II. Problem Characterization

The case in which the random modulating signal $c(t)$ is a product of two discrete random signals, $c(t) = c_1(t)c_2(t)$, can be used to model a type of impulsive noise in which the noisy signals occur in bursts. It may occur, for example, the case in which $c_1(t)$ toggles its amplitude in a time interval T while the amplitude of $c_2(t)$ remains fixed. If both $c_1(t)$ and c_2 toggle their amplitude at random and take different values in a discrete set, then the probability density function of the random process $c(t)$ can be written as [9].

$$f_{c(t)}(c) = \sum_k \sum_l p_{c_1(t)}(c_{1k}) p_{c_2(t)}(c_{2l}) \delta(c - c_{2l}c_{1k}). \quad (1)$$

Therefore, if the total noise expression $\eta(t)$ is written as

$$\eta(t) = \eta_g(t) + c_1(t)c_2(t)\eta_i(t) \quad (2)$$

then the pdf of $\eta(t)$ can be written as

$$f_{\eta(t)}(z) = \sum_k \sum_l \frac{p_{c_1(t)}(c_{1k}) p_{c_2(t)}(c_{2l})}{|c_{1k}||c_{2l}|} \times \int_{-\infty}^{\infty} f_{\eta_i(t)}\left(\frac{\eta - \eta_g}{c_{1k}c_{2l}}\right) f_{\eta_g(t)}(\eta_g) d\eta_g. \quad (3)$$

Fig. 1 illustrates a sample function of $\eta(t)$ obtained with the Simulink® software, whose diagram is shown in Fig. 2. It is possible to note the random behavior of both the amplitude and duration of the noisy bursts.

If $\eta_g(t)$ and $\eta_i(t)$ represent Gaussian noisy processes with null mean and variances σ_g^2 and σ_i^2, so the pdf of $\eta(t)$ can be written as

$$f_{\eta(t)}(\eta) = \sum_k \sum_l \frac{p_{c_1(t)}(c_{1k}) p_{c_2(t)}(c_{2l})}{\sqrt{2\pi(\sigma_g^2 + \sigma_i^2 c_{1k}^2 c_{2l}^2)}} \times \exp\left(-\frac{\eta^2}{2(\sigma_g^2 + \sigma_i^2 c_{1k}^2 c_{2l}^2)}\right). \quad (4)$$

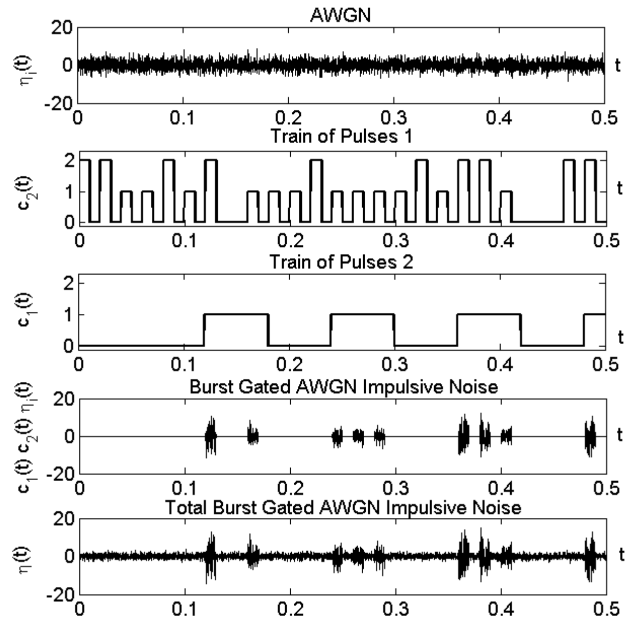

Fig. 1. Sample function of the composite additive noise $\eta(t)$ obtained using Simulink®.

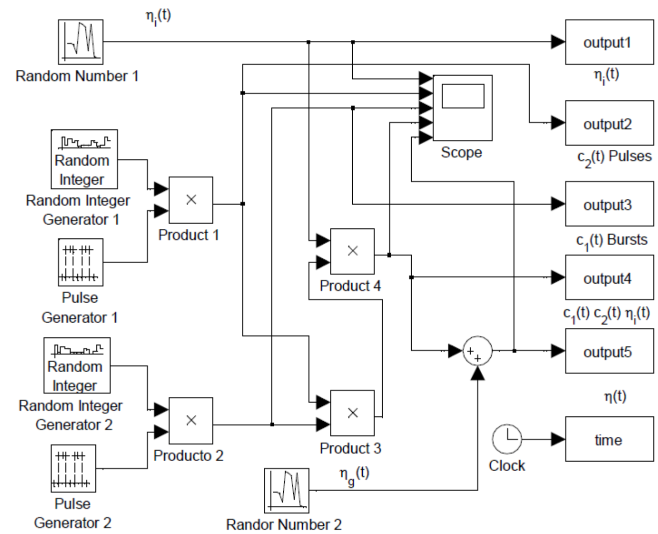

Fig. 2. Simulink® diagram used to simulate a sample of the composite noise $\eta(t)$

III. Expressions of the BEP for an M-QAM Scheme

The symbol error probability (SEP) calculation of an optimum MAP receiver, for an M-QAM (M-ary Quadrature Amplitude Modulation) modulation scheme, can be obtained using the M-PAM (M-ary Pulse Amplitude Modulation) modulation. The M-PAM constellation has M symbols equally spaced by a distance d along a straight line. In this scheme, a wrong decision is made by the optimum receiver when the Euclidean distance between the received signal r and one of the M symbols of the M-PAM constellation exceeds half the distance between two neighbor symbols. Therefore, the SEP of the M-PAM scheme can be written as

$$P_M = \frac{1}{M}(M-1)P\left[|r-s_m| > d\sqrt{\frac{1}{2}\mathcal{E}_g}\right], \tag{5}$$

in which $\mathcal{E}_g$ represents the energy of a pulse $g(t)$ in a signaling interval T. Regarding the composite noise $\eta(t)$, the SEP P_M can be evaluated by integrating the pdf $f_{\eta(t)}$ in the interval that corresponds to the restriction $|r-s_m| > d\sqrt{\frac{1}{2}\mathcal{E}_g}$.

As the expected value of $\eta(t)$ is null, its pdf is symmetric and therefore the SEP P_M can be written as

$$\begin{aligned} P_M = \sum_k \sum_l \frac{p_{(t)}(c_{1k})p_{c_2(t)}(c_{2l})}{\sqrt{2\pi(\sigma_g^2+\sigma_i^2 c_{1k}^2 c_{2l}^2)}} \\ \times \int_{d\sqrt{\mathcal{E}_g/2}}^{\infty} \exp\left(-\frac{\eta^2}{2(\sigma_g^2+\sigma_i^2 c_{1k}^2 c_{2l}^2)}\right). \end{aligned} \tag{6}$$

The bit error probability (BEP) of the M-PAM scheme can be written, after some algebraic manipulations, in terms of the bit energy to permanent noise energy ratio γ_g and the bit energy to impulsive noise energy ratio γ_i. Thus, the BEP P_b can be expressed as

$$\begin{aligned} P_b = \frac{2(M-1)}{M} \sum_k \sum_l p_{(t)}(c_{1k})p_{c_2(t)}(c_{2l}) \\ \times Q\left(\sqrt{\frac{6\log_2(M)\gamma_i\gamma_g}{(M^2-1)(\gamma_i+\gamma_g c_{1k}^2 c_{1l}^2)}}\right). \end{aligned} \tag{7}$$

Equation 7 provides the BEP of the optimum receiver for a noise category in which both $c_1(t)$ and $c_2(t)$ can assume different discrete amplitudes. To characterize the impulsive gated noise it is necessary to control the probability distribution of $c_1(t)$, so $P[c_1(t) = 1] >> P[c_1(t) = 0]$. This consideration ensures that $c_1(t)$ is 1 for enough time so that the pulses of $c_2(t)$ can occur. This method allows evaluating the random appearance and disappearance of the noise bursts. While the bursts controlled by $c_1(t)$ are present, the pulses controlled by $c_2(t)$ may either occur or not and take different discrete amplitudes with certain probability distribution. When the product $c_1(t)c_2(t)$ modulates the Gaussian noise $\eta_i(t)$, a noise with null mean and variance according to the amplitudes of $c_2(t)$ appears.

The SEP of the M-QAM scheme can be calculated by the formula

$$P_M = 1-(1-P_{\sqrt{M}})^2, \tag{8}$$

in which $P_{\sqrt{M}}$ can be calculated from Equation 6 substituting M by $\sqrt{M}$. The BEP can be calculated by the expression that results from this substitution process. However, this expression is not appropriate when one needs to evaluate the fading effect on the BEP. A more appropriate expression was developed using the BEP of the M-QAM scheme subject to Gaussian noise, $P_M(e|\gamma)$, proposed in [10].

$$P_M(e|\gamma) = \frac{1}{\log_2 \sqrt{M}} \sum_{k=1}^{\log_2 \sqrt{M}} P_b(k), \tag{9}$$

in which $P_b(k)$ can be written as

$$P_b(k) = \frac{1}{\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \Bigg\{ w(i,k,M) \times \operatorname{erfc}\left((2i+1)\sqrt{\frac{3\log_2 M\gamma}{2(M-1)}} \right) \Bigg\}, \tag{10}$$

and the coefficients $w(i,k,M)$ are given by

$$w(i,k,M) = (-1)^{\left\lfloor \frac{i2^{k-1}}{\sqrt{M}} \right\rfloor} \cdot \left(2^{k-1} - \left\lfloor \frac{i \cdot 2^{k-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor \right), \tag{11}$$

where $\gamma = E_b/N_0$ denotes the SNR per bit and $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

Regarding the M-QAM scheme, the BEP $P_b(k)$ can be written as

$$P_b(k) = \frac{2}{\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \Bigg\{ w(i,k,M) \times \sum_m \sum_l p_{c_1(t)}(c_{1m}) p_{c_2(t)}(c_{2l}) \times \mathrm{Q}\left(\sqrt{\frac{3(2i+1)^2 \log_2 M}{(M-1)} \frac{\gamma_g \gamma_i}{(\gamma_i + \gamma_g c_{1m}^2 c_{2l}^2)}} \right) \Bigg\}. \tag{12}$$

IV. Numerical Evaluation of the BEP

Fig. 3 shows the BEP of the 64-QAM scheme subject to the composite noise model $\eta(t)$ for different values of the signal to permanent noise ratio γ_g. In this first group of curves, the modulating signals $c_1(t)$ and $c_2(t)$ take, at random, five levels and the probability distributions are as follows

$$\begin{cases} p_{c_1(t)}(-2) = 0.25 \\ p_{c_1(t)}(-1) = 0.15 \\ p_{c_1(t)}(0) = 0.2 \\ p_{c_1(t)}(1) = 0.15 \\ p_{c_1(t)}(2) = 0.15 \end{cases} \qquad \begin{cases} p_{c_2(t)}(-3) = 0.2 \\ p_{c_2(t)}(-1) = 0.2 \\ p_{c_2(t)}(0) = 0.2 \\ p_{c_2(t)}(1) = 0.2 \\ p_{c_2(t)}(3) = 0.2 \end{cases} \tag{13}$$

Note, from the graphics, that when $\gamma_i = 10$dB and $\gamma_g = 5$dB the energy of the permanent noise η_g is larger than the energy of the impulsive noise η_i modulated by $c_1(t)c_2(t)$. When γ_i increases and becomes larger than 30dB, the energy of the impulsive noise η_i decreases relative to the energy of the permanent noise η_g and in this case the BEP decreases. This first amplitude distribution for $c_1(t)$ and $c_2(t)$ is mainly to show the versatility of the model, because both $c_1(t)$ as $c_2(t)$ are discrete processes that can take any set of values. The larger are the values taken by $c_1(t)$ and $c_2(t)$ the larger is noise variance of $c_1(t)c_2(t)\eta_i(t)$.

Fig. 4 shows curves of the BEP considering an equiprobable probability distribution for $c_1(t)$. Regarding $c_2(t)$, the probability of the null level is higher. These distributions are

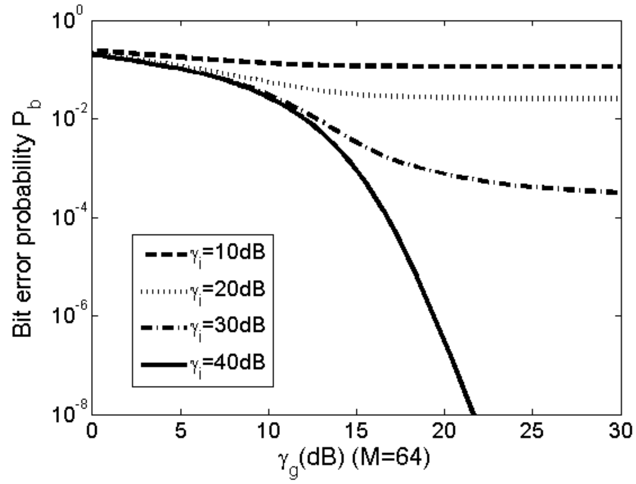

Fig. 3. BEP of the 64-QAM scheme subject to composite noise for different values of the signal to permanent noise ratio

$$\begin{cases} p_{c_1(t)}(-2) = 0.2 \\ p_{c_1(t)}(-1) = 0.2 \\ p_{c_1(t)}(0) = 0.2 \\ p_{c_1(t)}(1) = 0.2 \\ p_{c_1(t)}(2) = 0.2 \end{cases} \qquad \begin{cases} p_{c_2(t)}(-3) = 0.10 \\ p_{c_2(t)}(-1) = 0.15 \\ p_{c_2(t)}(0) = 0.50 \\ p_{c_2(t)}(1) = 0.15 \\ p_{c_2(t)}(3) = 0.10 \end{cases} \tag{14}$$

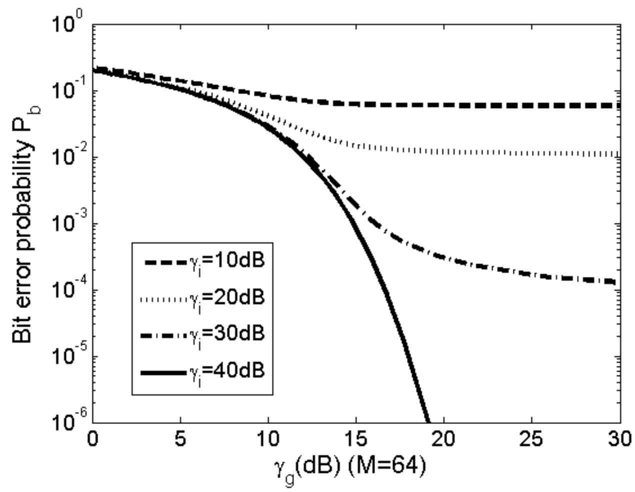

Fig. 4. BEP of the 64-QAM scheme subject to composite noise for different values of the signal to permanent noise ratio

In the second case, the probability distribution of $c_2(t)$ indicates that the zero level is more likely than the other values, which are equiprobable. This means that the probability of the product $c_1(t)c_2(t)$ take null value is larger than the situation shown in Fig. 4. Thus, the action of the impulsive noise $\eta_i(t)$ is less frequent and contributes to reduce the BEP. This is not a mechanism for noise control, but a way to model the emergence random variations in its amplitude.

Fig. 5 shows the curves of BEP of the M-QAM scheme subject to the composite noise $\eta(t)$ for different values of the order M. The signal to impulsive noise ratio, γ_i, is fixed at 30dB and the probability distribution levels of $c_1(t)$ and $c_2(t)$ is given by

$$\left\{\begin{array}{l} p_{c_1(t)}(-2)=0.2 \\ p_{c_1(t)}(-1)=0.2 \\ p_{c_1(t)}(0)=0.2 \\ p_{c_1(t)}(1)=0.2 \\ p_{c_1(t)}(2)=0.2 \end{array}\right. \quad \left\{\begin{array}{l} p_{c_2(t)}(-3)=0.10 \\ p_{c_2(t)}(-1)=0.15 \\ p_{c_2(t)}(0)=0.50 \\ p_{c_2(t)}(1)=0.15 \\ p_{c_2(t)}(3)=0.10 \end{array}\right. \tag{15}$$

One can see in Fig. 5 that the largest constellations provide

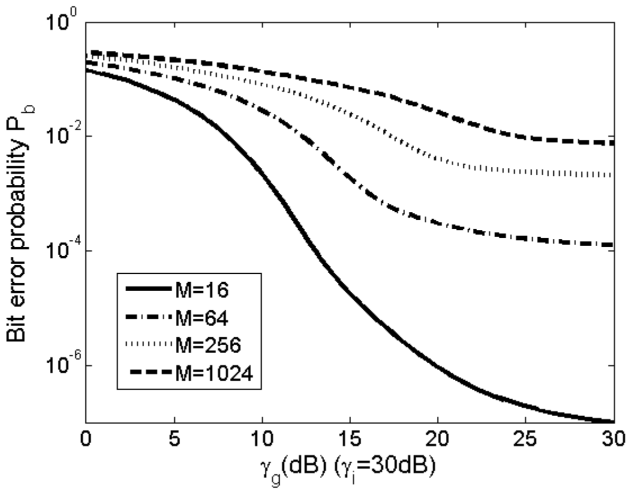

Fig. 5. BEP of the M-QAM scheme subject to composite noise for different values of the order M and for $\gamma_i = 30$dB

larger spectral efficiency but are more susceptible to the noise effects, especially because as the noise power increases, the decision regions of the receiver are less defined. This is what happens in this case, in which γ_g is less than γ_i and so the permanent noise power is larger than the power of impulsive noise modulated by $c_1(t)c_2(t)$.

Fig. 6 shows curves of BEP for the 64-QAM scheme subject to the composite noise for the case in which a modulating signal, $c_1(t)$, can take only two values. This case characterizes a situation that may occur more frequently, in practice, than the case in which the two signals take multiple values. When a modulating signal switches between zero and one with equal probability, the signal $c_2(t)$ can switch between multiple discrete values with a given distribution, such as

$$\left\{\begin{array}{l} p_{c_1(t)}(0)=0.5 \\ p_{c_1(t)}(1)=0.5 \end{array}\right. \quad \left\{\begin{array}{l} p_{c_2(t)}(0)=0.25 \\ p_{c_2(t)}(1)=0.25 \\ p_{c_2(t)}(2)=0.25 \\ p_{c_2(t)}(3)=0.25 \end{array}\right. \tag{16}$$

In Fig. 7 the BEP curves were plotted considering that the probability distribution of $c_1(t)$ can take the null value more frequently than the probability of taking the unit value. This case characterizes the situation in which the modulated impulsive noise with null amplitude is longer. The probability

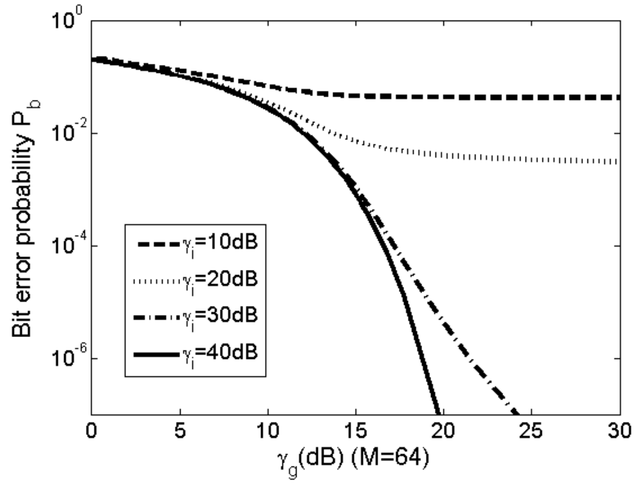

Fig. 6. BEP of the 64-QAM scheme subject to composite noise for different values of γ_i.

distribution of $c_1(t)$ and $c_2(t)$ for this case is given by

$$\left\{\begin{array}{l} p_{c_1(t)}(0)=0.7 \\ p_{c_1(t)}(1)=0.3 \end{array}\right. \quad \left\{\begin{array}{l} p_{c_2(t)}(0)=0.4 \\ p_{c_2(t)}(1)=0.2 \\ p_{c_2(t)}(2)=0.2 \\ p_{c_2(t)}(3)=0.2 \end{array}\right. \tag{17}$$

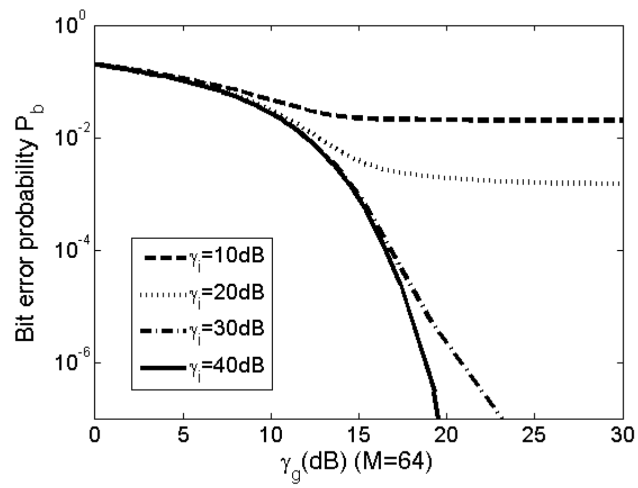

Fig. 7. BEP of the 64-QAM scheme subject to composite noise for different values of γ_i and different distributions for c_1 and $c_2(t)$.

One can notice in Fig. 7, regarding the curve corresponding to $\gamma_i = 10$dB, that there is a reduction in the BEP at $\gamma_g = 30$dB, relative to value of BEP in Fig. 6. As the amplitude levels of the signals in both cases are the same, one can say that the lower value of BEP at 30dB is due the the probability distribution of of $c_1(t)$ and $c_2(t)$, more specifically the fact that $c_1(t) = 0$ is more likely than $c_1(t) = 1$. Furthermore, within the range in which $c_1(t) = 1$, $c_2(t) = 0$ with probability 0.4, which characterizes a milder action of the modulated impulsive noise.

V. CONCLUSION

This article presents a new approach for mathematical analysis of a noise that consist of a zero mean Gaussian process with variance σ_g^2 and a component called gated impulsive noise, composed by a zero mean Gaussian process with variance σ_i^2 modulated by the product of two discrete processes $c_1(t)$ and $c_2(t)$.

The influence of the composite noise in the M-QAM modulation scheme was evaluated by means of the expressions of BEP developed considering a maximum likelihood optimum receiver. The BEP curves show that increasing the energy of the impulse noise deteriorates the performance of the receiver, but depends on the probability distribution of the discrete processes $c_1(t)$ and $c_2(t)$.

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