Analysis of the Gated Impulsive Noise in Optimum Receivers

Évio A. Rocha
Universidade de Pernambuco – UPE
Instituto Federal de Educação, Ciência e Tecnologia de Pernambuco - IFPE
Recife – PE - Brazil
evio.araujo@ee.ufcg.edu.br

Wamberto J. L. de Queiroz
Universidade Federal de Campina Grande – UFCG
Intituto de Estudos Avançados em Comunicações - Iecom
Campina Grande – PB – Brazil
wamberto@dee.ufcg.edu.br

Marcelo S. Alencar
Universidade Federal de Campina Grande – UFCG
Intituto de Estudos Avançados em Comunicações - Iecom
Campina Grande – PB – Brazil
malencar@dee.ufcg.edu.br

***Abstract*—This paper presents a model for the modulated gated additive white Gaussian noise, and evaluates its effect on the performance of the optimal receiver designed subject to the maximum *a posteriori* probability criterion (MAP).**

***Index Terms*—Gated noise, Impulsive noise, Optimum receiver.**

I. INTRODUCTION

The Gated Additive White Gaussian Noise (GAWGN) model is often used to simulate the impulsive noise in digital television channels because it adheres to the experimental results, and it is easy to implement and analyze [1-3].

This model is used by the British Broadcasting Corporation (BBC) digital television research group to simulate the impulsive noise in plant facilities and telecommunication companies, to facilitate the the performance evaluation of the equipments that are used in digital television systems, and to foster the development of measures to combat the noise effects [2-7].

II. COMPOSED NOISE MODEL

This paper presents a model for the modulated gated additive white Gaussian noise [2], [6-8] and evaluates its effect on the performance of the optimal receiver designed under the maximum *a posteriori* probability criterion (MAP). The additive noise is composed of two parts. The first one is characterized as a Gaussian process with zero mean and variance σ_g^2, represented by $\eta_g(t)$. The second part is characterized as a Gaussian process with zero mean and variance σ_i^2, $\eta_i(t)$, modulated by a signal $c(t)$.

The composite noise can be written as

$$\eta(t) = \eta_g(t) + c(t)\eta_i(t). \qquad (1)$$

Figure 1 shows a sample function of the process $\eta(t)$ versus time. The modulating signal $c(t)$ is used in this analysis to characterize the addition of noise permanent $\eta_g(t)$ to component $\eta_i(t)$. Random variations in the amplitude of $c(t)$ directly alter the variance of the Gaussian noise modulated $\eta_i(t)$ whereas random variations in the duration of the pulses of $c(t)$ affect the instants in which $\eta_i(t)$ is added to $\eta_g(t)$. This behavior can simulate, for example, situations in which a signal s (t), constantly affected by a noisy component $\eta_g(t)$, is under attack by additional noisy components that might occur with random variations in amplitude. It can be observed in Fig.1 that for the instants in which $c(t) = 0$ the permanent noise $\eta_g(t)$ continues to attack the signal $s(t)$. When $c(t)$ takes nonzero values and modulates the noise $\eta_i(t)$, there is a noise represented by the term $c(t)\eta_i(t)$. The amplitude-modulated signal $c(t)\eta_i(t)$ can attack the signal $s(t)$ during the time of one or several symbols.

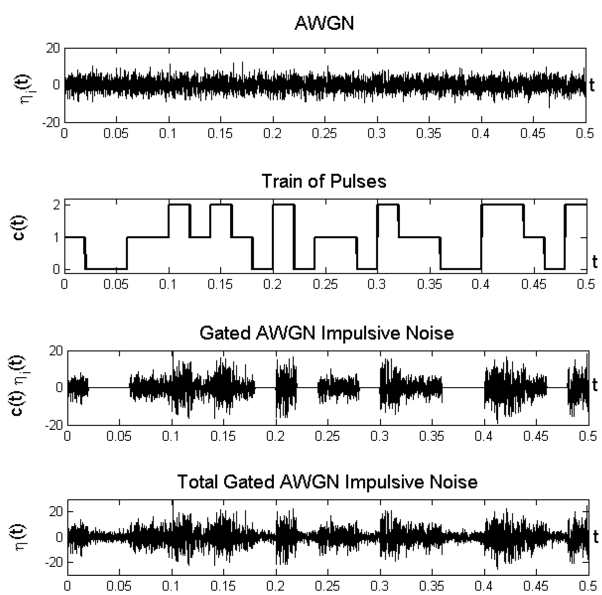


Fig. 1. Waveforms of gated AWGN noise.

The signal $c(t)$ can be used to characterize both the amplitude variation of the process $\eta_i(t)$ and the time intervals in which $\eta_i(t)$ is added to the noise $\eta_g(t)$. The signal $c(t)$ is a discrete random process characterized by a probability distribution and can be defined both in discrete time and continuous time. The noisy component $c(t)\eta_i(t)$ represents usual situations in communication systems in which switching mechanisms give rise to temporary noisy signals that add to the permanent noise.

Fig.2 shows the block diagram of the system used to simulate the noisy components of Fig.1.

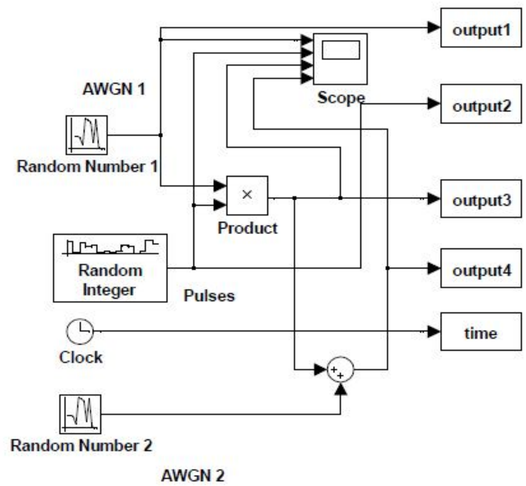


Fig.2. Simulink® diagram of the noise simulator GAWGN.

Therefore, if $s(t)$ represents a transmitted signal in a signaling interval T, then

$$r(t) = s(t) + \eta_g(t) + c(t)\eta_i(t) \quad (2)$$

represents the received signal affected by this random noise.

To evaluate the MAP receiver performance under the proposed switched noise, using the M-QAM modulation scheme, it is necessary to calculate the probability density function (pdf) of the noisy process $\eta(t)$. If $c(t)$ is a discrete process, then its pdf can be written as

$$f_{c(t)}(c) = \sum_k p_{c(t)}(c_k)\delta(c - c_k). \quad (3)$$

After some algebraic manipulations, it is possible to write the pdf of $\eta(t)$ as

$$f_\eta(\eta) = \sum_k \frac{p_{c(t)}(c_k)}{\sqrt{2\pi(\sigma_g^2 + \sigma_i^2 c_k^2)}} \exp\left(-\frac{\eta^2}{2(\sigma_g^2 + \sigma_i^2 c_k^2)}\right). \quad (4)$$

One can note that $f_\eta(\eta)$ can be written as a sum of different Gaussian pdfs, therefore

$$f_\eta(\eta) = f_{c_0}(\eta) + f_{c_1}(\eta) + f_{c_2}(\eta) + \cdots + f_{c_n}(\eta) + \cdots, \quad (5)$$

in which

$$f_{c_k}(\eta) = \frac{p_{c(t)}(c_k)}{\sqrt{2\pi(\sigma_g^2+\sigma_i^2 c_k^2)}} \exp\left(-\frac{\eta^2}{2(\sigma_g^2+\sigma_i^2 c_k^2)}\right). \quad (6)$$

Equation (5) is a weighted sum of Gaussian probability density functions with zero mean and variance $\sigma_g^2 + \sigma_i^2 c_k^2$. The constants c_k correspond to the amplitude levels that the signal $c(t)$ can take. In Fig. 1, for example, $c(t)$ takes three equiprobable levels with c_0=0, c_1 =1 and c_2 =2.

Fig. 2 presents the plots of three components of the pdf $f_\eta(\eta)$ composed by a sum of Gaussian pdfs, as shown in Eq. (). In this figure, $p_{c(t)}(0) = p_{c(t)}(1) = p_{c(t)}(2) = \frac{1}{3}$, $\sigma_g = 2$ and $\sigma_i = 3$.

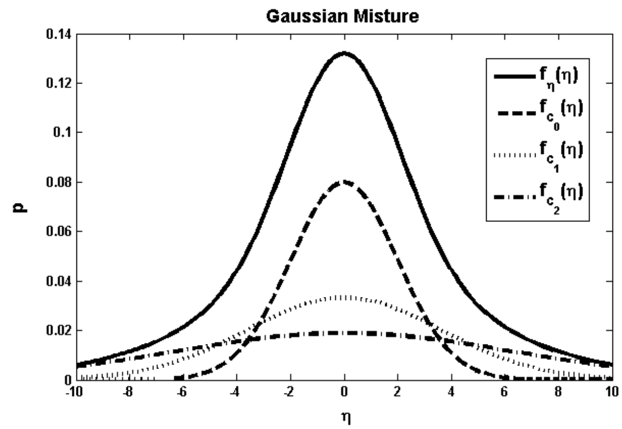


Fig. 2. Plots of different components of the mixed pdf $\boldsymbol{f_\eta(\eta)}$.

III. Evaluation of the Bit Error Probability

To calculate the symbol error probability (SEP) at the MAP receiver output, one should consider that the optimum receiver makes its decisions based on the minimization of the Euclidian distance between the vectors of transmitted and received signals [9].

If s_m represents, for example, the *m*-th signal of the signal space of on M-PAM constellation, then the received signal, affected by the impulsive noise is given by

$$r = s_m + \eta. \quad (7)$$

The signals s_m can be represented in the signal space by vectors whose elements are $s_m = \sqrt{\frac{1}{2}E_g}A_m$, $m = 1,2,\cdots,M$, in which E_g represents the energy of the basic pulse $g(t)$ associated to each signal s_m. The amplitudes A_m can be written as $A_m = (2m - 1 - M)d$, in which $m = 1,2,\cdots,M$ and d represents the Euclidian distance between two neighbor symbols in the M-PAM constellation. In such a modulation scheme, a detection error is caused when the optimum receiver calculates the distance from the received signal r to one of the M signals s_m and this distance exceeds half the distance between two symbols, or $|r - s_m| > d\sqrt{\frac{1}{2}E_g}$. The M-PAM constellation is one-dimensional, if its M symbols are equally spaced in a straight line, there will be *M*-1 decision intervals among the symbols. The average decision error probability is then calculated adding the errors in each *M*-1 intervals among the symbols s_m and then weighting by the number of symbols *M*. The SEP can be written as

$$P_M = \frac{1}{M}(M-1)P\left[|r - s_m| > d\sqrt{\frac{1}{2}E_g}\right]. \quad (8)$$

As the total noise η is characterized by a symmetric pdf, the SEP under this noise can be written as

$$P_M = \frac{2}{M}(M-1)\sum_k \frac{p_{c(t)}(c_k)}{\sqrt{2\pi(\sigma_g^2 + \sigma_i^2 c_k^2)}} \times \int_{d\sqrt{E_g/2}}^{\infty} \exp\left(-\frac{\eta^2}{2(\sigma_g^2 + \sigma_i^2 c_k^2)}\right) d\eta. \quad (9)$$

Applying the Q(x) function definition, one can write the M-PAM SEP expression as

$$P_M = \frac{2}{M}(M-1)\sum_k p_{c(t)}(c_k)\ \mathrm{Q}\left(\sqrt{\frac{d^2 E_g}{2(\sigma_g^2 + \sigma_i^2 c_k^2)}}\right). \quad (10)$$

Using the relationship between the energy of the pulse g(t), E_g, and the mean energy of the signal s_m, E_{av},

$$d^2 E_g = \frac{6}{M^2 - 1} E_{\mathrm{av}},$$

the bit error probability – BEP P_b is written as

$$P_b = \frac{2}{M}(M-1)\sum_k p_{c(t)}(c_k)\ Q\left(\sqrt{\frac{6\log_2(M)\gamma_g\gamma_i}{(M^2-1)(\gamma_i + \gamma_g c_k^2)}}\right), \quad (8)$$

in which $\gamma_g = \frac{E_b}{N_g}$ and $\gamma_i = \frac{E_b}{N_i}$ are the bit energy to noise spectrum density ratio and the bit energy to impulsive noise ratio. The SEP of the M-QAM scheme can be calculated using the formula

$$P_M = 1 - \left(1 - P_{\sqrt{M}}\right)^2, \quad (9)$$

in which $P_{\sqrt{M}}$ can be calculated using Equation (6). The main disadvantage of Equation (9) is the square term that complicates the BEP evaluation under fading. This difficulty can be overcome expressing P_b in Equation (8) in terms of the expressions obtained in [10], for the BEP evaluation under Gaussian noise. Using those expressions, one can write P_b as

$$P_b = \frac{1}{\log_2(\sqrt{M})}\sum_{k=1}^{\log_2(\sqrt{M})} P_b(k), \quad (10)$$

in which

$$P_b(k) = \frac{2}{\sqrt{M}}\sum_{i=1}^{(1-2^{-k})\sqrt{M}-1}\left\{w(i,k,M)\sum_l p_{c(t)}(c_l) \times Q\left(\sqrt{\frac{3(2i+1)^2\log_2(M)\gamma_g\gamma_i}{(M-1)(\gamma_i + \gamma_g c_l^2)}}\right)\right\}, \quad (11)$$

and

$$w(i,k,M) = (-1)^{\left\lfloor \frac{i2^{k-1}}{\sqrt{M}} \right\rfloor}\left(2^{k-1} - \left\lfloor \frac{i2^k - 1}{\sqrt{M}} + \frac{1}{2} \right\rfloor\right) \quad (12)$$

and the term $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. A similar procedure could be used to obtain BEP expressions for rectangular QAM evaluation under the proposed noise. An advantage of this expression is the absence of a quadratic power of the Q(x) function. As one can see in Equation (10), the probability $P_b(k)$ is written in terms of the probabilities $p_{c(t)}(c_l)$ of the switching signal $c(t)$. Therefore, as the absolute value of $c(t)$ increases so does the impulsive noise influence.

Figure 4 shows the behavior of the BEP of the M-QAM scheme under compound noise. In this figure the Signal to Impulsive Noise Ratio $\gamma_i = \frac{E_b}{N_i}$ assumes four distinct values and the order of M-QAM constellation is 64. For this figure a signal $c(t)$ with five random levels is considered, with the following probability distribution

$$\begin{cases} P\{c(t) = 0\} = p_{c(t)}(0) = 0.25 \\ P\{c(t) = 1\} = p_{c(t)}(1) = 0.15 \\ P\{c(t) = 2\} = p_{c(t)}(2) = 0.20 \\ P\{c(t) = 3\} = p_{c(t)}(3) = 0.15 \\ P\{c(t) = 4\} = p_{c(t)}(4) = 0.25 \end{cases}. \quad (13)$$

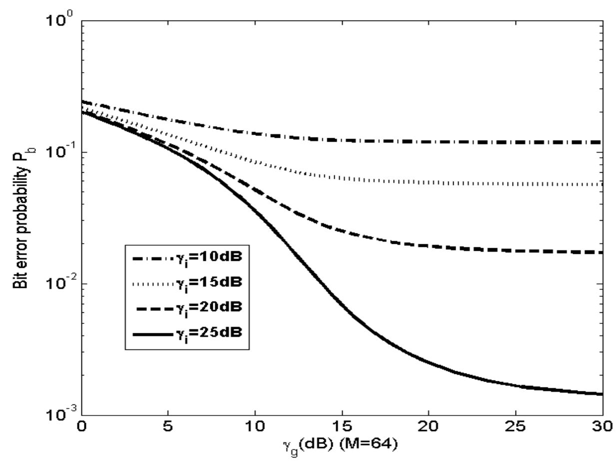


Fig. 4. Bit error probability of a 64-QAM modulation scheme under composed noise.

Note in Fig.4 that the probability of bit error decreases with the increasing of the signal to permanent noise ratio - $\gamma_g = \frac{E_b}{N_g}$, for fixed values of the signal impulse noise ration - $\gamma_g = \frac{E_b}{N_g}$. Since the signal energy is E_b when $\gamma_i > \gamma_g$ this means that $N_i < N_g$, that is, the energy of the impulse noise is smaller than the power of the permanent noise. When $\gamma_i > \gamma_g$ this means that $N_i > N_g$ and then the BEP decreases because the action of the impulse noise is not constant in the system, such as the permanent noise $\eta_g(t)$. Considering also the energy of bit fixed at E_b note that when $\gamma_{i1} = \frac{E_b}{N_{i1}} = 10$ dB < $\gamma_{i2} = \frac{E_b}{N_{i2}} = 25$ dB, the energy of impulsive noise $N_{i1} > N_{i2}$, and in this case as it was expected, the BEP increases. In the case in which γ_i=10 dB there is little reduction of the BEP when γ_g increases up to 30 dB.

Fig. 5 shows the BEP of the 64-QAM scheme under the compound noise considering a signal $c(t)$ with a different probability distribution. The Signal to Impulsive Noise Ratio $\gamma_i = \frac{E_b}{N_i}$ assumes the same values used in Fig.4. The probability distribution of $c(t)$ is given by

$$\begin{cases} P\{c(t) = 0\} = p_{c(t)}(0) = 0.25 \\ P\{c(t) = 1\} = p_{c(t)}(1) = 0.25 \\ P\{c(t) = 2\} = p_{c(t)}(2) = 0.20 \\ P\{c(t) = 3\} = p_{c(t)}(3) = 0.15 \\ P\{c(t) = 4\} = p_{c(t)}(4) = 0.15 \end{cases} . \quad (14)$$

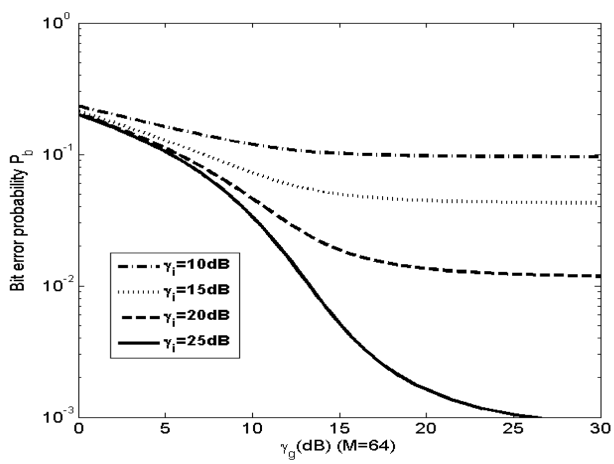
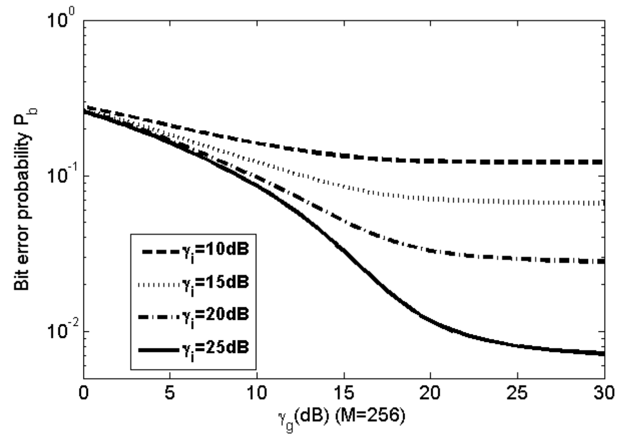


Fig. 5. Bit error probability of a 64-QAM modulation scheme under composed noise.

In this second case, the amplitude levels 0 and 1 of $c(t)$ are more probable than the amplitudes 3 and 4, which have probability 0.15. This behavior of $c(t)$, causes the modulated signal) $c(t)\eta_i(t)$ (which represents the impulsive noise component) to have a larger amplitude variation, with lower probability, and a smaller variation in amplitude, with higher probability. Note, in Fig. 5, that the curve decreases more than BEP in Fig. 4 wherein the amplitudes 4 and 0 of $c(t)$ are equiprobable. It is noticed that with a SNR of 25 dB (γ_g) it is possible to achieve a BEP of 10^{-3} with γ_i equal to 25 dB. In this case, $\gamma_g = \gamma_i$, $N_i = N_g$ and as $\eta_i(t)$ affects $s(t)$ with a smaller probability, the BEP can reach lower values.

Fig. 6 shows the behavior of the BEP 256-QAM scheme subject to the proposed noise. In this figure the Signal to Impulsive Noise Ratio $\gamma_i = \frac{E_b}{N_i}$ assumes four distinct values. The signal $c(t)$ has five random levels and the following probability distribution

$$\begin{cases} P\{c(t) = 0\} = p_{c(t)}(0) = 0.30 \\ P\{c(t) = 1\} = p_{c(t)}(1) = 0.25 \\ P\{c(t) = 2\} = p_{c(t)}(2) = 0.20 \\ P\{c(t) = 3\} = p_{c(t)}(3) = 0.15 \\ P\{c(t) = 4\} = p_{c(t)}(4) = 0.10 \end{cases} . \quad (15)$$



For the probability distribution shown in (15), the highest probability of occurrence was assigned to $c(t) = 0$, meaning that $c(t)$ spends more time at the zero amplitude level and consequently the likelihood of an attack by impulse noise on the signal $s(t)$ is higher. Because of the order of the constellation used, M = 256, the symbols damaged by noise are closer, and therefore the reception is more susceptible to errors. This is the reason why the BEP does not decrease, for a value $\gamma_g \leq 30$ dB, to less than 10^{-3}.

The evaluation of the influence of the order M in BEP is shown in Figure Fig. 7 for four values of M, considering $\gamma_i = 25$ dB.

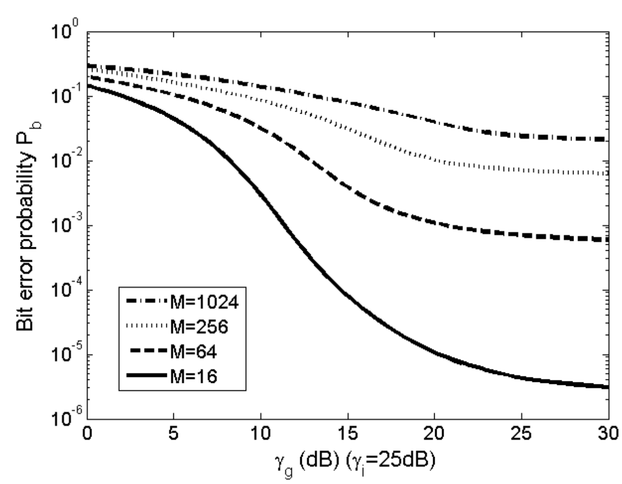


Fig. 7 Bit error probability of M-QAM schemes of different orders under composed noise.

The probability distribution of the possible amplitude levels of $c(t)$ é given by

$$\begin{cases} P\{c(t) = 0\} = p_{c(t)}(0) = 0.45 \\ P\{c(t) = 1\} = p_{c(t)}(1) = 0.15 \\ P\{c(t) = 2\} = p_{c(t)}(2) = 0.20 \\ P\{c(t) = 3\} = p_{c(t)}(3) = 0.10 \\ P\{c(t) = 4\} = p_{c(t)}(4) = 0.10 \end{cases} \quad (16)$$

Even when $c(t)$ spends more time with zero amplitude, one can see in Fig. 7 that for γ_i equals to 25 dB the BEP does not decrease below 10^{-3} for γ_g below 30 dB, for constellations with M above 64.

CONCLUSIONS

This article has presented a new analytical approach to evaluate the effect of additive noise modeled as a composition of white Gaussian process, $\eta_g(t)$, and another component, $\eta_i(t)$, called impulsive noise.

The term $c(t)\eta_i(t)$ can be seen as an amplitude modulation of $\eta_i(t)$ by $c(t)$, and characterizes the emergence of the noise $\eta_i(t)$ at random instants. The reception of the modulated signal M-QAM, corrupted by the compound noise, was evaluated by means of new expressions for the BEP, obtained using the optimum maximum likelihood receiver. The curves obtained have shown that the system performance depends on a function of the signal to permanent noise and signal to impulsive noise ratio.

REFERENCES

[1] C. Yih, Iterative Interference Cancellation for OFDM Signals With Blanking Nonlinearity in Impulsive Noise Channels, IEEE Signal Processing Letters, V.19, No. 3, March, 2012.

[2] J. Lago-Fernández and J. Salter. Modeling Impulsive Interference in DVB-T: Statistical Analysis, Test Waveform & Receiver Performance. BBC R & D White Paper WHP 080, 2004.

[3] J. Lago-Fernández and J. Salter. "Modeling Impulsive Interference in DVB-T". EBU Technical Review, 2004.

[4] D.A. Guimarães and G.G.R. Gomes. "Caracterização do Canal para o MI-SBTVD". Revista Telecomunicações, Dezembro, 2006.

[5] H.A. Suraweera and J. Armstrong. "Noise Bucket Effect for Impulse Noise in OFDM". Electronics Letters, 40(18):1156–1157, 2004.

[6] C. Akamine. Improving DVB-T for Brazilian Environments. IEEE, 2006.

[7] S. Matjka and H. Pavel "Initial Tests of DVB-T Receivers on Tolerance to Impulsive Interference" IEEE, 2011.

[8] E. R. Araújo, W. J. L. Queiroz and M. S. Alencar, Probabilidade de Erro de Bit em Sistemas de Comunicação Digital na Presença de Ruído Impulsivo Gaussiano Gatilhado, XXX Simpósio Brasileiro de Telecomunicações, Brasília, DF, Outubro 2012.

[9] A. Goldsmith. *Wireless Communications*. McGraw-Hill, 2000.

[10] K. Cho and D. Yoon, On the General BER Expressions of One and Two Dimensional Amplitude Modulations, IEEE Transactions on Communications, vol. 50, no. 7, pp. 1074-1080, July 2002.