Design and Analysis of an Efficient Simulation Scheme for α-μ Fading Channels

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***Abstract*— Capitalizing on a simple relationship between the α-μ and Nakagami-m fading models, we design a new efficient simulation scheme for α-μ fading channels that (i) allows for arbitrary real values of the fading parameters α and μ, (ii) exactly matches the α-μ first-order statistics, (iii) and closely approximates the α-μ second-order statistics. In addition, we provide a detailed analysis of the first- and second-order statistics associated to our new simulator. More specifically, we derive closed-form expressions for important channel statistics, namely, level crossing rate, average fade duration, second-order probability density function of the channel envelope, and joint probability density function of the channel envelope and its time derivative. The envelope autocorrelation function is also obtained, but in an integral form. Numerical examples are given that attest the good performance of our design. To our best knowledge, no simulation scheme for α-μ fading channels has been proposed yet that allows for non-integer or non-half-integer values of the fading parameter μ.**

***Index Terms*— fading channels, α-μ fading, simulation.**

I. Introduction

The α-μ distribution [1] is a generalized statistical model that accounts for the joint effect of two fundamental radio-fading phenomena: the clustering of multipath waves and the nonlinearity of the propagation medium. In this distribution, the nonlinearity of the propagation medium is represented by a parameter $\alpha > 0$, in a way that the sum of squared multipath components equals not the square of the resulting signal amplitude, but this amplitude to the power of α. The clustering of waves, in its turn, is represented by the parameter $\mu > 0$, which is related to the number of multipath clusters that compose the resulting signal. Through these two shape parameters (α and μ), the α-μ distribution covers a vast range of propagation conditions, from light to moderate to severe fading, including as special cases one-sided Gaussian, Rayleigh, Nakagami, and Weibull.

The simulation of the α-μ fading channel may be challenging, depending on the value of the parameter μ. Originally, in the physical fading model behind the derivation of the α-μ distribution, 2μ equals the number of multipath Gaussian clusters that compose the resulting signal [1]. Therefore, in principle, it would only make sense to consider integer or half-integer values of μ, in which case the original α-μ fading model itself can be readily used as a simulation scheme. Here, we call this scheme the classical α-μ simulator. But there exist many reasons to consider non-integer and non-half-integer values of μ as well, including the following. First, the many statistical expressions derived from the original α-μ fading model have no mathematical constraints to be used for any real value of $\mu > 0$. Second, in practice, if the parameter μ is to be empirically estimated from field measurements, real values of μ will certainly occur. Third, the α-μ fading model—as any other fading model available—has inherent limitations for being indeed an approximate solution to the so-called random phasor problem, and these limitations can be made less stringent by allowing μ to be real-valued. Fourth, non-integer values of multipath clusters have been extensively reported in the literature (see, for instance, [2] and the references therein). However, as argued before, when μ is non-integer or non-half-integer, the original α-μ fading model has no meaning and thus cannot be used as a simulation scheme. In fact, to the best of our knowledge, no simulation scheme for α-μ fading channels has been reported yet that allows for any real values of μ. This is still an open problem.

In this work, we design a solution to the above problem by exploiting a simple relationship between the α-μ and Nakagami fading models. As mentioned before, in the α-μ model, the α-th power of the signal amplitude equals the sum of 2μ squared, zero-mean, independent identically distributed (i.i.d.) Gaussian multipath components [1]. Similarly, as well known, in the Nakagami model with fading parameter m, the squared signal amplitude equals the sum of $2m$ squared, zero-mean, i.i.d. Gaussian multipath components [3], [4]. Therefore, by raising a Nakagami process with $m = \mu$ to the power of $2/\alpha$, we can generate an α-μ process with parameters α and μ. As a result, using such a simple transformation, we can capitalize on an existing simulation scheme for Nakagami fading channels in order to readily build a corresponding simulation scheme for α-μ fading channels.

In principle, the above framework allows us to use any Nakagami simulator available as a basis to the α-μ simulator. Naturally, the better the Nakagami simulator used, the better the expected performance of the resulting α-μ simulator. We propose the use of a recent, highly-efficient Nakagami simulator that (i) allows for arbitrary real values of the fading parameter m, (ii) matches the exact Nakagami first-order statistics, (iii) and closely approximates the Nakagami second-order statistics. The scheme is based on a combination of two simulation techniques called random mixture [5] and rank matching [6]. Accordingly, it is called random-mixture-rank-matching (RM^2) Nakagami simulator [7]. As far as we know,

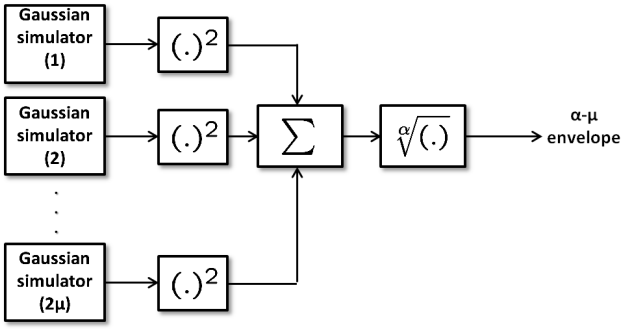

Fig. 1. The classical α-μ simulator.

the RM^2 method provides the best-match solution currently available for Nakagami fading channel simulation.

In the following, we capitalize on the RM^2 Nakagami simulator to design an efficient simulation scheme for α-μ fading channels that (i) allows for arbitrary real values of the fading parameters α and μ, (ii) exactly matches the α-μ first-order statistics, (iii) and closely approximates the α-μ second-order statistics. Each of these features is inherited from a corresponding feature of the underlying Nakagami simulator. We also provide a detailed analysis of the first- and second-order statistics associated to our new simulator. More specifically, we derive closed-form expressions for important channel statistics, namely, level crossing rate (LCR), average fade duration (AFD), second-order probability density function (PDF) of the channel envelope, and joint probability density function of the channel envelope and its time derivative. The envelope autocorrelation function (ACF) is also obtained, but in an integral form. Numerical examples are given that attest the good performance of our design.

The rest of the paper is organized as follows. In Section II, the classical α-μ simulator and its corresponding first- and second-order statistics are revisited. In Section III, a simple relationship between the α-μ and Nakagami fading models is provided as the basis to the design and analysis of our new α-μ simulator, which is then described in Section IV. Sample examples and performance comparisons between the classical and proposed simulators are presented in Section V. Finally, Section VI summarizes the paper.

II. The Classical α-μ Simulator Revisited

In the original physical fading model behind the derivation of the α-μ distribution [1], the channel envelope R is represented as the α-root of the sum of 2μ squared, zero-mean, i.i.d. Gaussian multipath components X_i ($i = 1, \ldots, 2\mu$), i.e.

$$R = \sqrt[\alpha]{\sum_{i=1}^{2\mu} X_i^2}. \tag{1}$$

Of course, this representation is valid only for integer and half-integer values of μ, and, in these cases, it can be readily used as a simulation scheme. We call this scheme the classical α-μ simulator. It is illustrated in Fig. 1.

Although the classical α-μ simulator is limited to integer and half-integer values of μ, it allows the derivation of analytical expressions for many important channel statistics with no constraints to be used for any real values of μ. Next, we reproduce some of these statistics. We shall use them as a benchmark to assess the performance of our new simulator.

Departing from (1), the PDF of R can be obtained as [1]

$$f_R(r; \alpha, \mu, \hat{r}) = \frac{\alpha \mu^{\mu} r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\Gamma(\mu)} \exp\left(-\mu \frac{r^{\alpha}}{\hat{r}^{\alpha}}\right), \tag{2}$$

where $\alpha > 0$ is a power parameter, $\mu = E^2[R^{\alpha}]/V[R^{\alpha}] > 0$ is the inverse of the normalized variance of R^{α}, $\hat{r} = \sqrt[\alpha]{E(R^{\alpha})}$ is the α-root mean value, $\Gamma(\cdot)$ is the gamma function, and the variance of each underlying Gaussian process X_i in (1) is $\hat{r}^{\alpha}/(2\mu)$. ($E[\cdot]$ denotes expectation, $V[\cdot]$ variance.) Note that the α-μ envelope PDF is specified by three parameters: α, μ, and $\hat{r}$. The corresponding cumulative distribution function (CDF) can be then obtained as [1]

$$F_R(r; \alpha, \mu, \hat{r}) = 1 - \frac{\Gamma(\mu, \mu r^{\alpha}/\hat{r}^{\alpha})}{\Gamma(\mu)}, \tag{3}$$

where $\Gamma(a, b) = \int_0^b t^{a-1} \exp(-t) dt$ is the incomplete gamma function.

In this work, we also address important second-order statistics of the communication channel, namely LCR, AFD, second-order PDF of R, joint PDF of R and its time derivative $\dot{R}$, and ACF of R. In particular, for simplicity, we focus on the scenario with isotropic scattering and omnidirectional reception. In such a case, the joint PDF of R and $\dot{R}$ is given by [1]

$$\begin{aligned} f_{R,\dot{R}}(r, \dot{r}; \alpha, \mu, \hat{r}) = {} & \frac{\alpha^2 \mu^{\mu+0.5} r^{\alpha(\mu+0.5)-2}}{\sqrt{2\pi}\omega \hat{r}^{\alpha\mu+0.5}\Gamma(\mu)} \\ & \times \exp\left(\frac{-\mu\alpha^2 r^{\alpha-2}\dot{r}^2}{2\omega^2 \hat{r}^{\alpha}} - \frac{\mu r^{\alpha}}{\hat{r}^{\alpha}}\right), \end{aligned} \tag{4}$$

where ω is the maximum Doppler shift in radians per second. Using (3) and (4), the LCR and AFD of the classical α-μ simulator can be derived respectively as [1]

$$N_R(r; \alpha, \mu, \hat{r}) = \frac{\omega \mu^{\mu-0.5} \rho^{\alpha(\mu-0.5)}}{\sqrt{2\pi}\Gamma(\mu) \exp(\mu\rho^{\alpha})} \tag{5}$$

$$T_R(r; \alpha, \mu, \hat{r}) = \frac{\sqrt{2\pi}\Gamma(\mu, \mu\rho^{\alpha}) \exp(\mu\rho^{\alpha})}{\omega \mu^{\mu-0.5} \rho^{\alpha(\mu-0.5)}}, \tag{6}$$

where $\rho \triangleq r/\hat{r}$ is the $\hat{r}$-normalized envelope.

The second-order envelope PDF of the classical α-μ simulator has been derived in [1] as

$$\begin{aligned} f_{R(t),R(t+\tau)}(r_1, r_2; \alpha, \mu, \hat{r}) = {} & \frac{\alpha^2 \mu^{\mu+1} \rho_1^{\frac{\alpha}{2}(\mu+1)-1} \rho_2^{\frac{\alpha}{2}(\mu+1)-1}}{(1-\delta(\tau))\delta^{\frac{\mu-1}{2}}\Gamma(\mu)} \\ & \times \exp\left(-\mu \frac{\rho_1^{\alpha} + \rho_2^{\alpha}}{1-\delta(\tau)}\right) I_{\mu-1}\left(\frac{2\mu\sqrt{\delta(\tau)\rho_1^{\alpha}\rho_2^{\alpha}}}{1-\delta(\tau)}\right), \end{aligned} \tag{7}$$

where t denotes time, τ is a time lag, $\rho_i \triangleq r_i/\hat{r}$, $i = 1, 2$, $I_v(.)$ is the modified Bessel function of the first kind and vth

order [8], and $\delta(\tau)$ is the power autocorrelation coefficient of each underlying Gaussian process G_i in (1). It can be shown that $\delta(\tau)$ is also the α-power autocorrelation coefficient of R, i.e. [1]

$$\delta(\tau) = \frac{C[R^\alpha(t), R^\alpha(t+\tau)]}{\sqrt{V[R^\alpha(t)]V[R^\alpha(t+\tau)]}}, \quad (8)$$

where $C[\cdot,\cdot]$ denotes covariance. The functional form of $\delta(\tau)$ depends on the scattering conditions [9]. In particular, for isotropic scattering and omnidirectional reception, $\delta(\tau) = J_0^2(\omega\tau)$, where $J_0(\cdot)$ is the Bessel function of first kind and zeroth order [8]. Finally, using (7), the ACF of R can be derived for the classical α-μ simulator as [1]

$$A_R(\tau;\alpha,\mu,\hat{r}) = \frac{\hat{r}^\alpha \Gamma^2\left(\mu+\frac{1}{\alpha}\right)}{\mu^{\frac{2}{\alpha}}\Gamma^2(\mu)} {}_2F_1\left(-\frac{1}{\alpha},\frac{1}{\alpha};\mu;\delta(\tau)\right), \quad (9)$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function [8].

III. A Simple Relationship between α-μ and Nakagami Fading

As highlighted in [1], there is a simple relationship connecting the α-μ envelope to the Nakagami envelope, as follows. In the Nakagami fading model, the channel envelope R_N with mean power $\Omega = E[R_N^2]$ and fading parameter $m = E^2[R_N^2]/V[R_N^2]$ is represented as the square root of the sum of $2m$ squared i.i.d. Gaussian multipath components X_i ($i = 1,\ldots,2m$) with zero mean and variance $\Omega/(2m)$, i.e. [3], [4]

$$R_N = \sqrt{\sum_{i=1}^{2m} X_i^2}. \quad (10)$$

Therefore, from (1) and (10), we note that a squared Nakagami envelope with parameters

$$m = \mu \text{ and } \Omega = \hat{r}^\alpha \quad (11)$$

equals the α-power of an α-μ envelope R with parameters α, μ, and $\hat{r}$, that is

$$R^\alpha = R_N^2. \quad (12)$$

Equivalently, Eq. (12) can be rewritten with R in terms of R_N,

$$R = R_N^{2/\alpha}, \quad (13)$$

or with R_N in terms of R,

$$R_N = R^{\alpha/2}. \quad (14)$$

In particular, inspired by Eq. (13), we can capitalize on any existing simulation scheme for Nakagami fading channels in order to readily build a corresponding simulation scheme for α-μ fading channels. This is attained simply by raising the output of the Nakagami simulator to the power of $2/\alpha$. In order to obtain a resulting α-μ envelope with desired parameters α, μ, and $\hat{r}$, we have to set the parameters of the Nakagami simulator according to (11). The general scheme is illustrated in Fig. 2.

In the next section, we shall use this scheme to design a new simulator for α-μ fading channels capitalizing on a recent, highly-efficient simulator for Nakagami fading channels, the

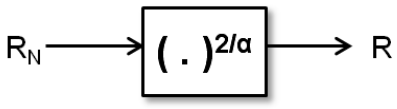

Fig. 2. Transformation of Nakagami envelope into α-μ envelope.

so-called RM^2 Nakagami simulator [7]. In order to allow the analysis of the second-order statistics of the new α-μ simulator to be performed directly in terms of the known second-order statistics of the RM^2 Nakagami simulator, it is crucial to derive general relationships between the second-order statistics of the Nakagami envelope R_N and those of the α-μ envelope generated from R_N via $R = R_N^{2/\alpha}$. In the following, these general relationships are presented for LCR, AFD, second-order envelope PDF, joint PDF of the envelope and its time derivative, and ACF.

From (14), we note that the α-μ envelope process crosses a given level r at the same rate at which the Nakagami envelope process crosses the level $r^{\alpha/2}$. In other words, the LCR $N_R(r;\alpha,\mu,\hat{r})$ of the α-μ envelope R can be written in terms of the LCR $N_{R_N}(r_N;m,\Omega)$ of the Nakagami envelope R_N as

$$N_R(r;\alpha,\mu,\hat{r}) = N_{R_N}(r^{\frac{\alpha}{2}};\mu,\hat{r}^\alpha). \quad (15)$$

Of course, the same relationship holds true for the AFD, i.e.

$$T_R(r;\alpha,\mu,\hat{r}) = T_{R_N}(r^{\frac{\alpha}{2}};\mu,\hat{r}^\alpha), \quad (16)$$

where $T_R(r;\alpha,\mu,\hat{r})$ and $T_{R_N}(r_N;m,\Omega)$ denote the AFD of the α-μ and Nakagami envelopes, respectively.

The joint PDF $f_{R,\dot{R}}(r,\dot{r};\alpha,\mu,\hat{r})$ of R and its time derivative $\dot{R}$ can be written in terms of the joint PDF $f_{R_N,\dot{R}_N}(r_N,\dot{r}_N;m,\Omega)$ of R_N and its time derivative $\dot{R}_N$ by using (14) and its time derivative

$$\dot{R}_N = \frac{\alpha}{2} R^{\frac{\alpha-2}{2}}\dot{R} \quad (17)$$

to perform a standard transformation of variables from $(R_N,\dot{R}_N)$ to $(R,\dot{R})$. After some algebraic manipulations, we obtain

$$f_{R,\dot{R}}(r,\dot{r};\alpha,\mu,\hat{r}) = \frac{\alpha^2}{4} r^{\alpha-2} f_{R_N,\dot{R}_N}\left(r^{\frac{\alpha}{2}}, \frac{\alpha}{2} r^{\frac{\alpha-2}{2}}\dot{r};\mu,\hat{r}^\alpha\right). \quad (18)$$

The second-order PDF $f_{R(t),R(t+\tau)}(r_1,r_2;\alpha,\mu,\hat{r})$ of R can be written in terms of the second-order PDF $f_{R_N(t),R_N(t+\tau)}(r_{N_1},r_{N_2};m,\Omega)$ of R_N by using (14) as $R_N(t) = R^{\alpha/2}(t)$ and $R_N(t+\tau) = R^{\alpha/2}(t+\tau)$ to perform a standard transformation of variables from $(R_N(t),R_N(t+\tau))$ to $(R(t),R(t+\tau))$. After some algebraic manipulations, we obtain

$$\begin{aligned} f_{R(t),R(t+\tau)}(r_1,r_2;\alpha,\mu,\hat{r}) &= \frac{\alpha^2}{4}(r_1 r_2)^{\frac{\alpha}{2}-1} \\ &\times f_{R_N(t),R_N(t+\tau)}(r_1^{\alpha/2}, r_2^{\alpha/2};\mu,\hat{r}^\alpha). \end{aligned} \quad (19)$$

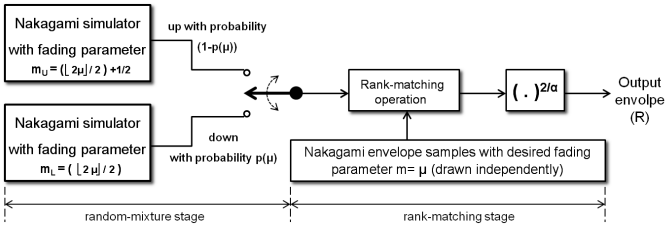

Fig. 3. RM^2 α-μ fading simulator.

Finally, using (19), the ACF of R can be generally written as

$$A_R(\tau;\alpha,\mu,\hat{r}) = \int_0^\infty \int_0^\infty r_1 r_2 f_{R(t),R(t+\tau)}(r_1,r_2;\alpha,\mu,\hat{r})dr_1 dr_2. \quad (20)$$

Depending on the second-order PDF of the input Nakagami simulator being used in the scheme described in Fig. 2, the resulting α-μ ACF in (20) may have a closed-form solution or not. In particular, if the classical Nakagami simulator is used, then the resulting α-μ ACF reduces to (9).

IV. The Proposed α-μ Simulator

In the context of the general framework presented in Fig. 2, we propose to use the recent highly-efficient RM^2 Nakagami fading simulator [7] in order to build a corresponding α-μ fading simulator. The proposed simulator is illustrated in Fig. 3. We call it RM^2 α-μ fading simulator. As shall be seen, the RM^2 α-μ simulator (i) allows for arbitrary real values of the fading parameters α and μ, (ii) exactly matches the α-μ first-order statistics, (iii) and closely approximates the α-μ second-order statistics. Each of these features is inherited from a corresponding feature of the underlying RM^2 Nakagami simulator. Next, we provide more detail on the operation of the proposed simulator. In addition, we derive many important second-order statistics associated to it, namely, LCR, AFD, second-order envelope PDF, joint PDF of the envelope and its time derivative, and ACF.

As mentioned before, the RM^2 scheme is based on a cascade combination of two simulation techniques called random mixture [5] and rank matching [6]. First, in the random-mixture stage, for a given desired fading parameter m, a process is obtained by drawing from a pair of different Nakagami processes with integer and half-integer fading parameters lower than or equal to m (fading parameter m_L) and greater than m (fading parameter m_U). In our case, from (11), $m = \mu$ and, correspondingly, $m_L = \mu_L$ and $m_U = \mu_U$, so that

$$m_L = \mu_L = \frac{\lfloor 2m \rfloor}{2} = \frac{\lfloor 2\mu \rfloor}{2} \quad (21)$$

$$m_U = \mu_U = \frac{\lfloor 2m \rfloor}{2} + \frac{1}{2} = \frac{\lfloor 2\mu \rfloor}{2} + \frac{1}{2}, \quad (22)$$

where $\lfloor \cdot \rfloor$ denotes floor. For instance, if the desired fading parameter is $m = \mu = 1.3$, then $m_L = \mu_L = 1$ and $m_U = \mu_U = 1.5$. Note that $\mu_L \leq \mu < \mu_U$. The Nakagami processes with fading parameters $m_L = \mu_L$ and $m_U = \mu_U$ can be in principle generated by any method available in the literature. Here, we assume they are generated via the classical method [3], [4], as in (10).

Note in Fig. 3 that the Nakagami process with lower fading parameter $m_L = \mu_L$ is drawn with probability $p(m) = p(\mu)$, and that with larger fading parameter $m_U = \mu_U$ is drawn with probability $[1-p(m)] = [1-p(\mu)]$. A central task is to design a suitable mixture probability $p(\mu)$ that renders the whole scheme a good approximation to the desired classical α-μ second-order statistics. This task has been performed in [7] but for the Nakagami case. Here, instead, just to gain insight on the potentials of the RM^2 scheme for α-μ fading channels, we pick the original design of $p(m)$ as performed in [5] by a moment-based approach, namely

$$p(\mu) = \frac{2\mu_L\,(\mu_U - \mu)}{\mu}. \quad (23)$$

The output of the random-mixture stage provides the input of the rank-matching stage [7]. In this second stage, a Nakagami output sequence is obtained from an input reference Nakagami sequence and a set of Nakagami samples, drawn independently. The output sequence is a mere rearrangement of these samples, in a way that the samples in the ouput sequence exactly match the rank of the samples in the input sequence, that is, their minima occur in the same position, their second minima occur in the same position, an so on. The operation is called rank matching, and the output sequence is said to be rank-matched to the input reference sequence. In the proposed scheme, because of the front random-mixture stage, the input reference sequence is either a Nakagami process with fading parameter $m_L = \mu_L$—with probability $p(\mu)$—or a Nakagami process with fading parameter $m_U = \mu_U$—with probability $[1-p(\mu)]$. In addition, following (11), for a desired set of α-μ parameters α, μ, and $\hat{r}$, the independent Nakagami samples have to be accordingly drawn with $m = \mu$ and $\Omega = \hat{r}^\alpha$. The rank-matching operation ensures that the Nakagami first-order statistics are attained in an exact manner [7]. Then, plugging (13)—or Fig. 2, equivalently—at the output of the rank-matching block, the α-μ sequence with desired parameters α, μ, and $\hat{r}$ is finally obtained with exact first-order statistics. The corresponding second-order statistics are derived next.

Let $N_{R,\mathrm{ref}}$ denote the reference Nakagami process with fading parameter $m_{\mathrm{ref}} = m_L$ or $m_{\mathrm{ref}} = m_U$, as required, generated by the random-mixture stage via the classical method. In addition, let N_R denote the Nakagami process with fading parameter m generated from $N_{R,\mathrm{ref}}$ via the rank-matching operation. In [10], it is proved that generating N_R from $N_{R,\mathrm{ref}}$ via the rank-matching operation is fully equivalent to generating N_R from $N_{R,\mathrm{ref}}$ via the well-known inversion transformation method [11, Eq. (7-157)]

$$R_N = F_{R_N}^{-1}(F_{R_N}(R_{N,\mathrm{ref}};m_{ref},\Omega);m,\Omega), \quad (24)$$

where $F_{R_N}^{-1}(u;m,\Omega)$ is the inverse CDF of the Nakagami envelope and $F_{R_N}(r;m,\Omega)$ is the CDF of the Nakagami

envelope. These are given by [3], [7]

$$F_{R_N}(r;m,\Omega) = 1 - \frac{\Gamma\left(m, \frac{mr^2}{\Omega}\right)}{\Gamma(m)} \tag{25}$$

$$F^{-1}_{R_N}(u;m,\Omega) = \sqrt{\frac{\Omega}{m} Q^{-1}(m, 1-u)}, \tag{26}$$

where $Q^{-1}(m,u)$ is the inverse of the regularized incomplete gamma function, i.e., it gives the solution for z in $u = \Gamma(m,z)/\Gamma(m)$. It can be computed in Mathematica by means of `InverseGammaRegularized[m,u]`.

In the analysis that follows, instead of (24), it is more convenient to use its inverse relationship, in which $R_{N,\text{ref}}$ is written in terms of R_N, i.e.

$$\begin{aligned} R_{N,\text{ref}} &= F^{-1}_{R_N}(F_{R_N}(R_N;m,\Omega); m_{\text{ref}}, \Omega) \\ &\triangleq h(R_N; m_{ref}, m, \Omega). \end{aligned} \tag{27}$$

In addition, it is also important to write the time derivative $\dot{R}_{N,\text{ref}}$ of $R_{N,\text{ref}}$ in terms of the time derivative $\dot{R}_N$ of R_N. This is attained by differentiating (27) with respect to time, which gives

$$\dot{R}_{N,\text{ref}} = h'(R_N; m_{ref}, m, \Omega)\dot{R}_N, \tag{28}$$

where $h'(\cdot;\cdot,\cdot,\cdot)$ denotes the first derivative of $h(\cdot;\cdot,\cdot,\cdot)$ as defined in (27). Using (25) and (26) into the definition of $h(\cdot;\cdot,\cdot,\cdot)$, this can be obtained, after some careful algebraic manipulations, as

$$h(r;m_{\text{ref}},m,\Omega) = \sqrt{\frac{\Omega}{m_{\text{ref}}} Q^{-1}\left(m_{\text{ref}}, \frac{\Gamma\left(m, \frac{mr^2}{\Omega}\right)}{\Gamma(m)}\right)}, \tag{29}$$

and its first derivative as

$$\begin{aligned} h'(r;m_{\text{ref}},m,\Omega) &= \frac{m^m r^{2m-1}\Gamma(m_{\text{ref}})}{m_{\text{ref}}^{1/2}\Omega^{m-1/2}\Gamma(m)} \\ &\times \exp\left[-\frac{mr^2}{\Omega} + Q^{-1}\left(m_{\text{ref}}, \frac{\Gamma\left(m, \frac{mr^2}{\Omega}\right)}{\Gamma(m)}\right)\right] \\ &\times Q^{-1}\left(m_{\text{ref}}, \frac{\Gamma\left(m, \frac{mr^2}{\Omega}\right)}{\Gamma(m)}\right)^{m_{\text{ref}}+1/2}. \end{aligned} \tag{30}$$

Now, based on (27)–(30), via standard transformation of variables, we are able to derive any statistics of R_N and $\dot{R}_N$ in terms of the corresponding statistics of $R_{N,\text{ref}}$ and $\dot{R}_{N,\text{ref}}$, which are those of the classical Nakagami simulator. Then, the derived statistics of R_N and $\dot{R}_N$ can be replaced into the general relationships between Nakagami and α-μ fading given in (15), (16), (18), (19), and (20), in order to finally obtain the required statistics of the proposed α-μ simulator. Such a procedure can be applied for each of the reference Nakagami processes with fading parameters $m_{\text{ref}} = m_L$ and $m_{\text{ref}} = m_U$. Then, because there is a random mixture of them in the proposed scheme, the overall statistics are given as a weighted sum of the individual statistics for $m_{\text{ref}} = m_L$ and $m_{\text{ref}} = m_U$, the weights being given by the mixture probabilities $p(\mu)$ and $[1-p(\mu)]$, respectively.

Take, for instance, the LCR and AFD. For a given $R_{N,\text{ref}}$ with fading parameter m_{ref}, the LCR and AFD of R_N are given by [7, Eqs. (27) and (28)]

$$N_{R_N}(r;m,\Omega) = N_{R_{N,\text{ref}}}\left(h\left(r;m_{\text{ref}},m,\Omega\right);m_{\text{ref}},\Omega\right) \tag{31}$$

$$T_{R_N}(r;m,\Omega) = T_{R_{N,\text{ref}}}\left(h\left(r;m_{\text{ref}},m,\Omega\right);m_{\text{ref}},\Omega\right). \tag{32}$$

Note that $N_{R_{N,\text{ref}}}(r;m,\Omega)$ and $T_{R_{N,\text{ref}}}(r;m,\Omega)$ are the LCR and AFD of the classical Nakagami simulator, given by (5) and (6), respectively, with $\alpha = 2$, $\mu = m$, and $\hat{r} = \sqrt[\alpha]{\Omega}$. Combining (31) and (32) with the corresponding mixture probabilities into (15) and (16), we then obtain the LCR and AFD of the proposed α-μ simulator as

$$\begin{aligned} N_R(r;\alpha,\mu,\hat{r}) &= p(\mu)N_{R_{N,\text{ref}}}(h(r^{\frac{\alpha}{2}};\mu_L,\mu,\hat{r}^\alpha);\mu_L,\hat{r}^\alpha) \\ &+ [1-p(\mu)]N_{R_{N,\text{ref}}}(h(r^{\frac{\alpha}{2}};\mu_U,\mu,\hat{r}^\alpha);\mu_U,\hat{r}^\alpha) \end{aligned} \tag{33}$$

$$\begin{aligned} T_R(r;\alpha,\mu,\hat{r}) &= p(\mu)T_{R_{N,\text{ref}}}(h(r^{\frac{\alpha}{2}};\mu_L,\mu,\hat{r}^\alpha);\mu_L,\hat{r}^\alpha) \\ &+ [1-p(\mu)]T_{R_{N,\text{ref}}}(h(r^{\frac{\alpha}{2}};\mu_U,\mu,\hat{r}^\alpha);\mu_U,\hat{r}^\alpha). \end{aligned} \tag{34}$$

For a given $R_{N,\text{ref}}$ with fading parameter m_{ref}, the joint PDF of R_N and $\dot{R}_N$ can be written in terms of the joint PDF of $R_{N,\text{ref}}$ and $\dot{R}_{N,\text{ref}}$ by using (27) and (28) to perform a standard transformation of variables from $(R_{N,\text{ref}}, \dot{R}_{N,\text{ref}})$ to $(R_N, \dot{R}_N)$. Performing this transformation and combining it with the corresponding mixture probabilities into (18), we then obtain, after some algebraic manipulations, the joint PDF of R and $\dot{R}$ for the proposed α-μ simulator as

$$f_{R,\dot{R}}(r,\dot{r};\alpha,\mu,\hat{r}) = \frac{\alpha^2}{4} r^{\alpha-2}\left\{p(\mu)f_1 + [1-p(\mu)]f_2\right\}, \tag{35}$$

$$\begin{aligned} f_1 &\triangleq h'^2(r^{\frac{\alpha}{2}};\mu_L,\mu,\hat{r}^\alpha) \\ &\times f_{R_{N,\text{ref}},\dot{R}_{N,\text{ref}}}\left(h(r^{\frac{\alpha}{2}};\mu_L,\mu,\hat{r}^\alpha), \right. \\ &\left. \frac{\alpha}{2} r^{\frac{\alpha-2}{2}}\dot{r}\; h'(r^{\frac{\alpha}{2}};\mu_L,\mu,\hat{r}^\alpha);\mu_L,\hat{r}^\alpha\right) \end{aligned} \tag{36}$$

$$\begin{aligned} f_2 &\triangleq h'^2(r^{\frac{\alpha}{2}};\mu_U,\mu,\hat{r}^\alpha) \\ &\times f_{R_{N,\text{ref}},\dot{R}_{N,\text{ref}}}\left(h(r^{\frac{\alpha}{2}};\mu_U,\mu,\hat{r}^\alpha), \right. \\ &\left. \frac{\alpha}{2} r^{\frac{\alpha-2}{2}}\dot{r}\; h'(r^{\frac{\alpha}{2}};\mu_U,\mu,\hat{r}^\alpha);\mu_U,\hat{r}^\alpha\right). \end{aligned} \tag{37}$$

In (36) and (37), $f_{R_{N,\text{ref}},\dot{R}_{N,\text{ref}}}(r,\dot{r};m,\Omega)$ is the joint PDF of the envelope and its time derivative for the classical Nakagami simulator, given by (4) with $\alpha = 2$, $\mu = m$, and $\hat{r} = \sqrt[\alpha]{\Omega}$.

Analogously, for a given $R_{N,\text{ref}}$ with fading parameter m_{ref}, the second-order PDF of R_N can be written in terms of the second-order PDF of $R_{N,\text{ref}}$ by using (27) to perform a standard transformation of variables from $(R_{N,\text{ref}}(t), R_{N,\text{ref}}(t+\tau))$ to $(R_N(t), R_N(t+\tau))$. Performing this transformation and combining it with the corresponding mixture probabilities

into (19), we then obtain, after some algebraic manipulations, the second-order PDF of R for the proposed α-μ simulator as

$$f_{R(t),R(t+\tau)}(r_1,r_2;\alpha,\mu,\hat{r}) = \frac{\alpha^2}{4}(r_1 r_2)^{\frac{\alpha}{2}-1} \times \{p(\mu)f_3 + [1-p(\mu)]f_4\} \quad (38)$$

$$f_3 \triangleq h'(r_1^{\frac{\alpha}{2}};\mu_L,\mu,\hat{r}^\alpha)h'(r_2^{\frac{\alpha}{2}};\mu_L,\mu,\hat{r}^\alpha) \times f_{R_{N,\mathrm{ref}}(t),R_{N,\mathrm{ref}}(t+\tau)}\Big(h(r_1^{\frac{\alpha}{2}};\mu_L,\mu,\hat{r}^\alpha), h(r_2^{\frac{\alpha}{2}};\mu_L,\mu,\hat{r}^\alpha);\mu_L,\hat{r}^\alpha\Big) \quad (39)$$

$$f_4 \triangleq h'(r_1^{\frac{\alpha}{2}};\mu_U,\mu,\hat{r}^\alpha)h'(r_2^{\frac{\alpha}{2}};\mu_U,\mu,\hat{r}^\alpha) \times f_{R_{N,\mathrm{ref}}(t),R_{N,\mathrm{ref}}(t+\tau)}\Big(h(r_1^{\frac{\alpha}{2}};\mu_U,\mu,\hat{r}^\alpha), h(r_2^{\frac{\alpha}{2}};\mu_U,\mu,\hat{r}^\alpha);\mu_U,\hat{r}^\alpha\Big). \quad (40)$$

In (39) and (40), $f_{R_{N,\mathrm{ref}}(t),R_{N,\mathrm{ref}}(t+\tau)}(r_1,r_2;m,\Omega)$ is the second-order PDF of the classical Nakagami simulator, given by (7) with $\alpha=2$, $\mu=m$, and $\hat{r}=\sqrt[\alpha]{\Omega}$.

Finally, using (38) into (20), the ACF of the proposed α-μ simulator can be expressed in an integral form. Unfortunately, it seems there is no closed-form solution to the ACF.

V. Numerical Examples and Comparisons

In this section, we give some sample plots in order to assess the performance of the proposed α-μ simulator when compared to the statistical expressions of the classical simulator. We focus on LCR and AFD, for they are most representative among the second-order statistics investigated. The LCR is presented in Fig. 4, the AFD in Fig. 5. Note that the statistics of the proposed simulator are shown in dashed lines, and those of the classical simulator are shown in solid lines. In addition, as a check, Monte Carlo simulation results for the proposed simulator are also shown in dotted lines. In the plots, we have used a fixed value of the parameter $m=1.75$ and varied the parameter μ as 0.75, 1.25, 1.75, 2.25 and 2.75. The corresponding values of the parameter α are 3.13532, 2.3743, 2, 1.76638, and 1.60229. (Please refer to [1] for details on the relationship between α and μ for a given value of m.) From Fig. 4, we see that, except for very low values of μ, the LCR of the proposed simulator is in very good agreement with the classical LCR, becoming practically indistinguishable from this as μ increases. From Fig. 5, a similar comment holds true for the AFD.

VI. Conclusions

We have designed and analyzed a new simulator for α-μ fading channels that (i) allows for arbitrary real values of the fading parameters α and μ, (ii) exactly matches the α-μ first-order statistics, (iii) and closely approximates the α-μ second-order statistics. To our best knowledge, no simulation scheme for α-μ fading channels has been proposed yet that allows for non-integer or non-half-integer values of the fading parameter μ. We are currently working on the redesign and optimization of the mixture probabilities at the random-mixture stage in order to further improve the performance of the proposed simulator.

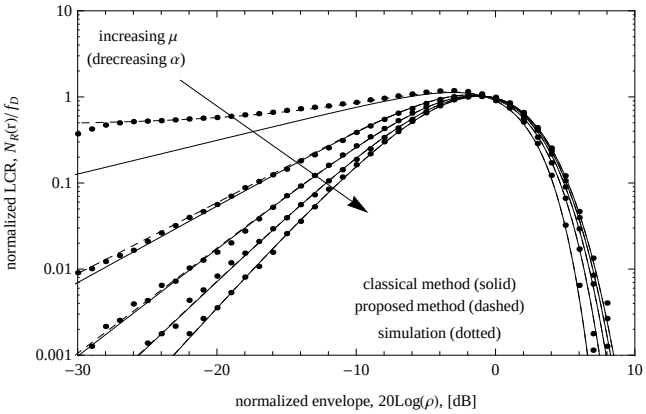

Fig. 4. Level crossing rate for the proposed α-μ simulator.

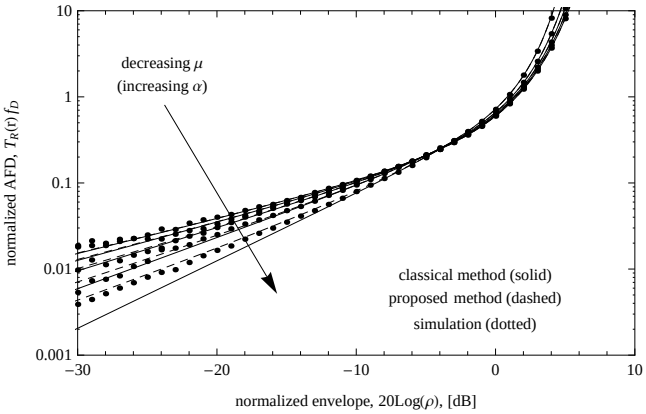

Fig. 5. Average fade duration for the proposed α-μ simulator.

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