Improved Optimization Algorithm for Constellation Mappings of Wavelet-Coded Communication Systems

E. A. Santos, E. C. Gurjão[1] and F. M. de Assis
Federal University of Campina Grande
Campina Grande - PB, Brazil
Email: eline.santos@ee.ufcg.edu.br, ecandeia@dee.ufcg.edu.br, fmarcos@dee.ufcg.edu.br
[1]Bolsista da CAPES - Proc. no 3826/11-2

***Abstract*— Wavelet channel coding (WCC) has been proposed by Tzannes at al. [1] as an approach to overcome fading effects in wireless communications channels. As can be seen in [2], WCC has good performance and simple decoding algorithm when compared to other wireless transmission schemes, surpassed in terms of bit error rate (BER) performance the space-time block coding (STBC) system proposed by Tarokh at al. [3] with similar complexity. Due to the non-uniform probability distribution of Wavelet symbols, special modulation schemes must be employed in WCC systems. In this work we improve an iterative methodology presented in [4] to obtain with small computational cost, zero mean constellations that outperforms the special PSK constellations designed in [2], and we extend our previous work [5] by adding a symmetry restriction to the constellation design algorithm and by using a BER metric instead of symbol error rate metric for performance evaluation.**

***Index Terms*— wavelet, channel, coding, constellation, fading, performance.**

I. Introduction

Wavelet channel coding has been proposed by Tzannes at al. [1] as an approach to overcome fading effects in wireless communications channels. This technique is based on the orthogonality between rows of a *wavelet coefficient matrix* (WCM). Each encoded symbol, called *wavelet symbol*, is produced by a weighted sum of several message bits, and the information contained in each message bit is spread out in several wavelet symbols. This feature gives an intrinsic time-diversity to the encoded sequence.

Good performance associated with simple decoding algorithm is a remarkable advantage of wavelet coding compared to other wireless transmission schemes. For example, the proposed wavelet encoding system in [2], surpassed in terms of bit error rate (BER) performance the space-time block coding (STBC) system proposed by Tarokh at al. [3] with similar complexity.

Due to the non-uniform probability distribution of Wavelet symbols, the adopted modulation scheme has a great impact on the overall performance of the communication system. In [2] and [6] was showed that special constellations must improve the system BER performance. In those works, the constellations were obtained by optimization via genetic algorithm (GA) to numerically minimize a semi-analytical formula for BER. The proposed optimization has a high computational cost, due to the "global" nature of the GA, and the obtained constellations in [2] have mean away from zero.

In this work we apply an iterative methodology presented in [4] to obtain with low computational cost, zero mean constellations that outperforms the special PSK constellations designed in [2] and [6]. We extend our previous work [5] by adding a symmetry restriction to the constellation design algorithm and by using a BER metric instead of symbol error rate metric for performance evaluation.

The remaining of this paper is organized as follows. In Section II we describe the wavelet coding and decoding processes. The design methodology to optimize the constellation maps for wavelet-coded systems is presented in Section III. In Section IV simulation results are presented and discussed. Finally, Section V contains some concluding remarks.

II. Wavelet Encoding

A *wavelet coefficient matrix* (WCM) of order m and genus g is defined as follows

$$\mathbf{A} = \begin{pmatrix} a_0^0, & \cdots, & a_{mg-1}^0 \\ a_0^1, & \cdots, & a_{mg-1}^1 \\ \vdots & \cdots & \vdots \\ a_0^{m-1} & \cdots, & a_{mg-1}^{m-1} \end{pmatrix} \tag{1}$$

In this work we use a WCM whose coefficients $(a_k^j) \in \{+1, -1\}$, and its rows satisfy the modified wavelet scaling conditions [1], [7]:

$$\sum_{k=0}^{mg-1} a_k^j = m\sqrt{g}\delta_{0,j}, \ 0 \leq j \leq m-1 \tag{2}$$

$$\sum_{k=0}^{mg-1} a_{[k+ml]}^j a_{[k+ml']}^{j'} = mg\delta_{j,j'}\delta_{l,l'}, \ 0 \leq j, j' \leq m-1$$
$$0 \leq l, l' \leq m-1 \tag{3}$$

where $\delta_{j,j'}$ is the Kronecker delta and the notation $[k+ml]$ stands for $k+ml \mod mg$

TABLE I

WAVELET CHANNEL CODING EXAMPLE

1	2	3	4	$\cdots$	8	9	10	$\cdots$
$x_1a_0^0$	$x_1a_1^0$	$x_1a_2^0$	$x_1a_3^0$	$\cdots$	$x_1a_7^0$			
$x_2a_0^1$	$x_2a_1^1$	$x_2a_2^1$	$x_2a_3^1$	$\cdots$	$x_2a_7^1$			
		$x_3a_0^0$	$x_3a_1^0$	$\cdots$	$x_3a_5^0$	$x_3a_6^0$	$x_3a_7^0$	
		$x_4a_0^1$	$x_4a_1^1$	$\cdots$	$x_4a_5^1$	$x_4a_6^1$	$x_4a_7^1$	
				$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$
y_1	y_2	y_3	y_4	$\cdots$	y_8	y_9	y_{10}	$\cdots$

Equation (3) states that the rows of a WCM with order m are mutually orthogonal at shifts of length lm. It also states that each row is orthogonal to itself shifted by lm for $0 < l \leq g-1$. These orthogonality conditions are the basis of wavelet channel coding.

Consider the information bits $\{x_n\}$, with $x_n \in \{+1, -1\}$, and an order m, genus g wavelet coefficient matrix $A = (a_k^j)$. In the wavelet coding process, bits are multiplied by distinct rows of a WCM, the results of these multiplications are shifted by m and added to generate the wavelet symbols[1]. This coding procedure is illustrated in Table I for a order 2, genus 4 WCM.

The wavelet symbol y_n produced at time n, it is equal to the sum of the *n*-th column of encoded message bit values in Table I and it is therefore not restricted to values ± 1. It can be expressed as [2]:

$$y_{pm+q} = \sum_{j=0}^{m-1} \sum_{l=0}^{g-1} a^j_{lm+q} x_{(p-l)m+j} \tag{4}$$

and it takes values in the set $\{-mg, -mg+2, \ldots, -mg+2k, \ldots, 0, \ldots, mg-2, mg\}$.

If the information bits are equiprobable, the wavelet symbols are distributed according to:

$$Pr\left(y_n = 2k - mg\right) = \binom{mg}{k} (0.5)^{mg},\ 0 \leq k \leq mg. \tag{5}$$

In the encoding process, m information bits are encoded in m wavelet symbols and m signaling intervals are used to send them, so this system has spectral efficiency of 1bit/s/Hz.

The information bits $\{x_n\}$ can be recovered from the received symbol sequence by using a bank of m correlators of length mg matched to the m rows of the WCM. In absence of errors, the output of the correlator matched to the row a^j at time $i = m(g+p) - 1$ can be expressed as:

$$\begin{aligned} z_i^j &= \sum_{k=0}^{mg-1} a^j_{(mg-1)-k} y_{i-k} \\ &= \sum_{k=0}^{mg-1} \sum_{j'=0}^{m-1} \sum_{l=0}^{g-1} a_k^j \left(a^{j'}_{k-lm} x_{j'+lm+i-(mg-1)} \right). \end{aligned} \tag{6}$$

[1]The wavelet encoder can be represented by filter banks as showed in [2]

Using the condition (3), it can be verified that except when $j = j'$ and $l = 0$ the terms of Equation (6) cancels, then

$$z_i^j = x_{j+i-(mg-1)} \sum_{k=0}^{mg-1} a_k^j a_k^j = mg x_{j+i-(mg-1)}. \tag{7}$$

Note that there is a delay of $mg - 1$ when decoding the first bit, thereafter, m symbols entering the decoder will produce m information bits.

The value of z_i^j is used to decide about the bit $x_{j+i-(mg-1)}$ by comparing this value with a threshold set to zero, in order to minimize the probability of error, since $x_n \in \{+1, -1\}$. The simplicity of correlative decoding is one of the main advantages of the wavelet coding technique [1].

III. OPTIMIZATION OF CONSTELLATIONS FOR WAVELET CODING

As expressed in Equation (5) encoded symbols are non-equiprobable, and the constellation mapping used to modulate the wavelet symbols to have a strong impact on the performance of this coding scheme.

In [6] and [2] genetic algorithm (GA) was used to search a PSK constellation that minimizes the BER of the wavelet encoding system. Compared to traditional PSK, the obtained constellation improves the system performance, but the used GA did not take into account the zero-mean constraint, a necessary property of any optimal (in terms of minimal SER) constellation with constrained average energy [4].

We can improve the performance of the constellations obtained in [6] and [2] simply translating them toward zero mean, and scaling them up to their original average energy.

Motivated by the good results obtained by the pairwise optimization for non-uniformly distributed sources in AWGN channels presented in [4], in this work, we apply this method, with some adjustments, to design constellations for wavelet encoded systems and to extend its application to flat fading channels.

A. Problem Formulation

The source generates equiprobable bits, and the transmitter uses a WCM to obtain wavelet symbols, distributed according to (5). The obtained symbols are mapped in a M-ary two-dimensional (2-D) modulation scheme. Modulated signals are transmitted over a flat Rayleigh fading channel. In the receiver the AWGN noise is added to the received signal and the demodulation is by *maximum a posteriori* (MAP) decoding.

To find the optimum constellation, the search space to be considered is continuous and it consists of all collections of points $\{\vec{s_1}, \vec{s_2}, \ldots, \vec{s_M}\}$ satisfying

- a zero mean constraint: $\sum_{i=1}^{M} p_i \vec{s_i} = 0$, and
- an average power constraint: $\sum_{i=1}^{M} p_i ||\vec{s_i}||^2 = E$,

where the average energy per symbol, E, is given. Note that $E = E_b$ to the wavelet coding present here.

Each wavelet symbol is mapped to a signal point, $\vec{s_i}$, in some initial M-ary constellation, where $\vec{s_i} = (s_{i,x}, s_{i,y})$. During the optimization, the arrangement of the points is changed in order to achieve the lowest symbol error rate (SER) at a given signal-to-noise ratio (SNR) E_b/N_0, where E_b is the average energy per bit.

For a given constellation that satisfies the constraints, it is not possible to adjust the position of a single point while adhering to the two constraints. However, taking any pair of points it is possible move it around while still adhering to the constraints. So, it was proposed in [4] a pairwise optimization, that it is explained below.

Select two points $\vec{s_1}$ and $\vec{s_2}$, the zero mean constraint implies that

$$p_1\vec{s_1} + p_2\vec{s_2} = -\sum_{i=3}^{M} p_i\vec{s_i} \tag{8}$$

And the average energy constraint implies that

$$p_1||\vec{s_1}||^2 + p_2||\vec{s_2}||^2 = E - \sum_{i=3}^{M} p_i||\vec{s_i}||^2 \tag{9}$$

Solving the equations (8) and (9) for $\vec{s_2}$ we have

$$\left(s_{2,x} - \frac{p_1 a_x}{(p_1+p_2)}\right)^2 + \left(s_{2,y} - \frac{p_1 a_y}{(p_1+p_2)}\right)^2 = r^2 \tag{10}$$

Where r^2:

$$r^2 = \frac{p_1(E-d)}{p_2(p_1+p_2)} - \frac{p_1^3}{p_2(p_1+p_2)^2}(a_x^2 + a_y^2). \tag{11}$$

whith $\vec{a} = -\frac{\sum_{i=3}^{M} p_i\vec{s_i}}{p_1}$ and $d = \sum_{i=3}^{M} p_i||\vec{s_i}||^2$

And $\vec{s_1}$ is given by $\vec{s_1} = \vec{a} - c\vec{s_2}$, thus

$$s_{1,x} = a_x - c \cdot s_{2,x} \tag{12}$$
$$s_{1,y} = a_y - c \cdot s_{2,y} \tag{13}$$

where $c = \frac{p_2}{p_1}$.

With equation (10), we have a circle centered at $\left(\frac{p_1 a_x}{p_1+p_2}, \frac{p_1 a_y}{p_1+p_2}\right)$ with radius r, on which $\vec{s_2}$ may travel while adhering to the constraints, and $\vec{s_1}$ is given by equation (12) also adhering to the constraints.

Then, the problem of searching over four variables $(s_{1,x}, s_{1,y}, s_{2,x}, s_{2,y})$ to each selected pair is reduced to searching over a single variable, θ, which is the angle parameterizing the circle for $\vec{s_2}$. For a given value of θ, the value of $\vec{s_2}$ is defined, and $\vec{s_1}$ is related by equation (12).

To evaluate the potential constellations, it is used the union upper bound on the SER P_s given by Equation (14), which is fairly tight for medium to high SNRs [4]. The union bound can be inaccurate for low SNRs, but it is simpler than other tighter bounds. To further improve system performance the tight upper and lower bounds of [8] and [9] can also be used.

$$\begin{aligned} P_s &= \sum_{u=1}^{M} P\left(\epsilon|\vec{s_u}\right) P\left(\vec{s_u}\right) \\ &= \sum_{u=1}^{M} P\left(\bigcup_{i\neq u} \epsilon_{iu}|\vec{s_u}\right) P\left(\vec{s_u}\right) \\ &\leq \sum_{u=1}^{M}\sum_{i\neq u} P\left(\epsilon_{iu}|\vec{s_u}\right) P\left(\vec{s_u}\right) \end{aligned} \tag{14}$$

where $P\left(\epsilon_{iu}|\vec{s_u}\right)$ is the probability that $\vec{s_i}$ has a larger MAP decoding metric than $\vec{s_u}$ given that $\vec{s_u}$ was sent, i.e., $P(\vec{s_i}|\vec{r}) \geq P(\vec{s_u}|\vec{r})$. Where $\vec{r} = \alpha\vec{s_u} + \vec{n}$ is the received vector signal, α is a Rayleigh distributed amplitude fading variable with second moment $2\sigma^2$, and $\vec{n}$ is a noise vector with zero-mean uncorrelated Gaussian distributed components, each with variance $N_0/2$.

We assume that α, $\vec{s_u}$, and $\vec{n}$ are independent from each other, and the fading variable α can be correctly estimated at the received signal. So, conditioned to a value of α, $P\left(\epsilon_{iu}|\alpha, \vec{s_u}\right)$ is given by:

$$P\left(\epsilon_{iu}|\alpha, \vec{s_u}\right) = Q\left(\frac{\alpha\|\vec{s}_i - \vec{s}_u\|}{\sqrt{2N_0}} + \frac{\sqrt{2N_0}\ln(P\left(\vec{s_u}\right)/P\left(\vec{s_i}\right))}{2\alpha\|\vec{s}_i - \vec{s}_u\|}\right) \tag{15}$$

And, $P\left(\epsilon_{iu}|\vec{s_u}\right)$ is given by [9]:

$$\begin{aligned} P\left(\epsilon_{iu}|\vec{s_u}\right) &= \mathbb{E}_\alpha\left[P\left(\epsilon_{iu}|\alpha, \vec{s_u}\right)\right] \\ &= \int_0^\infty P\left(\epsilon_{iu}|\alpha, \vec{s_u}\right) \frac{\alpha}{\sigma^2}\exp\left(-\frac{\alpha^2}{2\sigma^2}\right) d\alpha \end{aligned}$$

$$= \begin{cases} \frac{1}{2}\left(1 - \frac{1}{\tau_{iu}}\right)\exp\left[-\frac{\omega_{iu}}{2}\left(1-\tau_{iu}\right)\right], & \text{if } \omega_{iu} \geq 0 \\ 1 - \frac{1}{2}\left(1 + \frac{1}{\tau_{iu}}\right)\exp\left[-\frac{\omega_{iu}}{2}\left(1+\tau_{iu}\right)\right], & \text{if } \omega_{iu} < 0 \end{cases} \tag{16}$$

where $\omega_{iu} = \ln\left[P(\vec{s_u})/P(\vec{s_i})\right]$, $\tau_{iu} = \sqrt{\left(\sigma^2 d_{iu}^2 + 2N_0\right)/\left(\sigma^2 d_{iu}^2\right)}$, and $d_{iu} = ||\vec{s_i} - \vec{s_u}||$ (where $||\cdot||$ is the Euclidean norm). Note that, as shown by Equation 16, $P\left(\epsilon_{iu}|\vec{s_u}\right)$ admits a closed-form expression, unlike the case when the channel is AWGN (with no fading) [9], resulting in a more speedy optimization.

At each iteration only the pair of points $\vec{s_1}$ and $\vec{s_2}$ changes its position on the constellation map, so we can ignore the terms in equation (14) for $u \neq 1,2$ and $i \neq 1,2$ as they will remain constant. The objective function to be minimized for each pair is:

$$\begin{aligned} F_{12} &= \sum_{i\neq 1} P\left(\epsilon_{i1}\right) P\left(\vec{s_1}\right) + \sum_{i\neq 2} P\left(\epsilon_{i2}\right) P\left(\vec{s_2}\right) \\ &\quad + \sum_{u=3}^{M} P\left(\vec{s_u}\right)\left(P\left(\epsilon_{1u}\right) + P\left(\epsilon_{2u}\right)\right) \end{aligned} \tag{17}$$

Since the wavelet symbols of the same absolute value are equally likely, it is natural that there is a symmetry in the constellation map. This symmetry constraint was added to the optimization algorithm.

Thus, only pairs of positive symbols can be selected at each iteration, after determining the positions of the selected pair of points, the positions of negative symbols are obtained by symmetry with respect to the axis x. And the constellation is adjusted, before starting the new iteration, to satisfy the zero mean and average power constraints.

B. Algorithm

The optimization follows the steps presented in Algorithm 1. In this work, the initial constellation used were constellation with zero mean and average energy per symbol $E = 1$.

Input: objective-function, symbols probabilities
Output: optimum or sub-optimum constellation map
Initialization:

- Configure some initial constellation, ensuring it adheres to zero mean, average energy and symmetry constraints.

Iteration:
WHILE STOP = FALSE **DO**

1) Randomly (uniformly) select a pair of points $(\vec{s_1}, \vec{s_2})$ corresponding to positive wavelet symbols from the under optimization constellation .
2) Calculate the constrained circles from (10) and (12).
3) Find the new positions of $(\vec{s_1}, \vec{s_2})$ by minimizing (17), and to set this position in the constellation.
4) Obtain by symmetry with respect to the x-axis, the positions of negative wavelet symbols corresponding to the selected pair of points.
5) Adjust the constellation to satisfy the constraints.
6) If the constellation stabilizes, STOP = TRUE

END

Algorithm 1: Optimization Algorithm for constellation maps

The proposed algorithm is an exhaustive search, so in Step 4 the angle θ that parametrize the circle is set to be 0 relative to the x-axis, and take discrete steps counter-clockwise, each one of three degree. At each step of θ, F_{12} is calculated using the corresponding points $\vec{s_1}$ and $\vec{s_2}$ on their respective circles, and the design SNR (E_b/N_0), which is set as a constant.

The upper bound on SER given by Equation (14) is used to determine the stabilization in Step 5. In contrast with the algorithm used in [4] where the stabilization is determined by visual inspection. After each 100 iterations the upper bound value is verified, if it has a variation less than 10^{-10} and number of iterations performed is larger then minimum value established, the condition STOP becomes TRUE. We performed at least 3000 iterations for each optimization.

IV. Simulation Results

The results were obtained by Monte Carlo simulation of systems using the proposed pairwise optimized (PO) constellations (which are denoted by M-PO, where M is the number of constellation points). For performance comparison we also present numerical results for systems using special PSK constellations (which are denoted special M-PSK) obtained by using genetic algorithm (GA) [2] and [6].

We assume slowly-varying frequency-nonselective fading channel modeled by a Rayleigh distribution. We also assume perfect channel estimation at the receiver. The Maximum a Posteriori (MAP) criteria was used to detect the wavelet symbols at the receiver. The wavelet-coded systems were simulated with flat real WCM's with dimensions 2x8 and 2x128.

All pairwise optimized constellation presented here were obtained for an average energy constraint $E = 1$ and SNR = 4 dB.

The pairwise optimized constellation for wavelet coding with a 2x8 WCM is shown in Figure 1. The signals are labelled with the values of the corresponding wavelet symbol, in this case we have 9 wavelet symbols.

Figure 2 shows the BER curves *versus* E_b/N_0 for wavelet coding with 2x8 WCM. It includes the curve for the 9-PO constellation, the special 9-PSK [6] and the special 9-PSK translated and normalized to have zero mean and energy average 1.

At first, it can be verified that the translation and normalization applied to the special 9-PSK constellation improves its performance in about 2 dB. The 9-PO constellation attains a gain in the E_b/N_0 on the order of 1 dB over the special 9-PSK constellation for values of BER below 10^{-2} and 2 dB for values below 10^{-4}. These gains were obtained because imposing zero-mean it is possible enlarge the minimum Euclidean distance of the constellation as well as hold the original average energy.

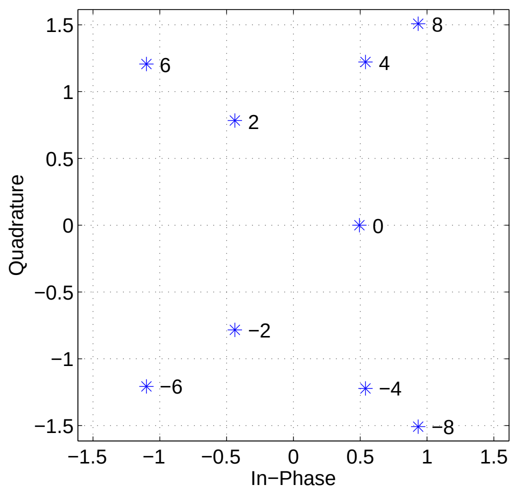

Fig. 1. Pairwise optimized constellation for a 2x8 WCM

We next consider a wavelet coding system with a 2x128 WCM. It should be noticed that if each wavelet symbol is mapped to a constellation point, as in the previous case for a 2x8 WCM, an increase in the dimension $m \times mg$ of the WCM can lead to performance degradation, due to the crowding of the constellation, since we have $mg + 1$ wavelet symbols for a $m \times mg$ WCM.

In order to overcome this problem, it was used a truncation scheme as in [2], [6], where more than one wavelet symbols

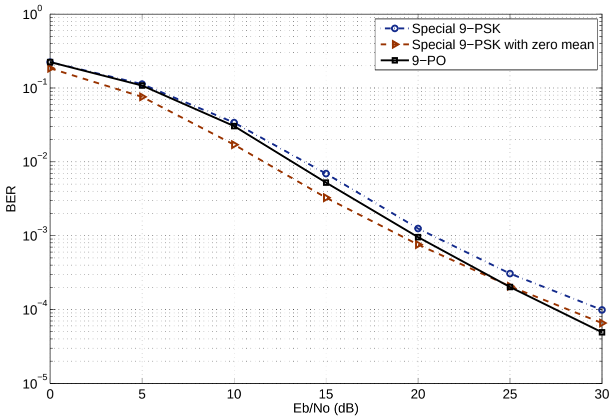

Fig. 2. BER performance of constellations for a 2x8 WCM

TABLE II

TRUNCATION SCHEME FOR A 2X128 WCM

Truncated Set	Key Symb.	Truncated Set	Key Symb.
		$\{14, 16, 18\}$	16
$\{0\}$	0	$\{-14, -16, -18\}$	-16
$\{2, 4, 6\}$	4	$\{20, 22, 24\}$	22
$\{-2, -4, -6\}$	-4	$\{-20, -22, -24\}$	-22
$\{8, 10, 12\}$	10	$\{26, 28, 30, ..., 128\}$	28
$\{-8, -10, -12\}$	-10	$\{-26, -28, -30, ..., -128\}$	28

are mapped to one constellation signal.

For the wavelet coding system with a 2x128 WCM we adopted the truncation scheme present in [2], it is shown in Table II. This truncation scheme reduces the number of constellations signals from 129 to 11. The pairwise optimized constellation is shown in Figure 3.

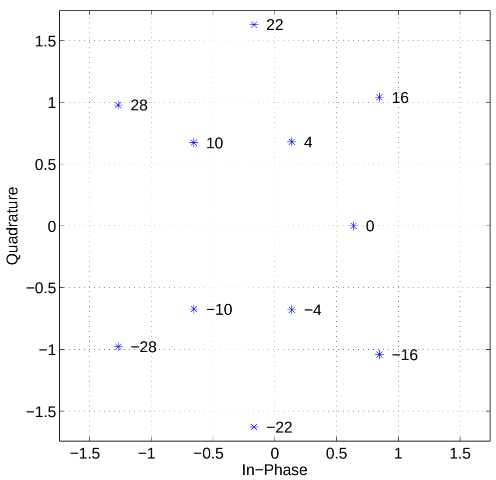

Fig. 3. Pairwise optimized constellation for wavelet coding with 2x128 WCM

The obtained BER curves for wavelet coding with 2x128 WCM are in Figure 4. It can be observed from Figure 4 that the 11-PO constellation presents performance close to the special 11-PSK and performer better for BER below 10^{-5}. Again, it can be verified that the special 11-PSK constellation with zero mean presents a gain over the special 11-PSK with mean different from zero.

The performance of this wavelet coded system with the 11-PO constellation are also compared with a space-time block coding (STBC) system. In this STBC system, at each time slot n, symbols c_n^i, $i = 1, 2$ are modulated by BPSK signals and transmitted simultaneously from two antennas [3]. This system also provide spectral efficiency of 1 bit/s/Hz. The 11-PO constellation presents a gain on the order of 2 dB over the STBC system for a BER of 10^{-5}.

So, the pairwise optimized constellations present similar or better performance than the special PSK constellation.

V. CONCLUSION

In this work, we propose a pairwise optimization to design constellations for a wavelet-coded system over a flat fading Rayleigh channel. The simulation results shown that the pairwise optimized constellations exhibit a good performance, in terms of BER, compared to the special PSK constellations. This methodology is more simple than the search by using a genetic algorithm (GA) to numerically minimize a semi-analytical formula for BER proposed in [2]. However, it requires additional demodulation complexity due to the asymmetry of the pairwise constellations.

The proposed algorithm is an adaptation of the iterative methodology presented in [4] to the problem of design constellation maps to wavelet-coded systems. The algorithm presented [4] was designed to numerically minimize a SER bound for an AWGN channel, here we used a SER bound for a flat fading Rayleigh channel, the reason is because the wavelet channel coding has no gain in AWGN channel [1] and in this case the SER bound admits a closed-form expression.

It should be pointed out that the constellations were optimized considering a SER bound. Note that, to minimize the SER do not necessarily implies in to minimize BER. In particular, to wavelet coding, a demodulation error between wavelet symbols with close values may result in no bit errors, while a demodulation error between wavelet symbols with distant values may cause a string of errors [1]. So, we extend our previous work [5] by adding a symmetry restriction to the constellation design algorithm.

With the symmetry restriction, the constellations obtained presented a good BER performance, even though be projected to optimize a SER bound.

REFERENCES

[1] M.A. Tzannes and M.C Tzannes, "Bit-by-bit channel coding using wavelets," in *Global Telecommunications Conference, 1992. Conference Record., GLOBECOM '92. Communication for Global Users., IEEE*, 6-9 1992, pp. 684 –688 vol.2.

[2] L. F. Q. Silveira, L. G. Q. Silveira, F. M. de Assis, and E.L. Pinto, "Analysis and optimization of wavelet-coded communication systems," *IEEE Transactions on Wireless Communications*, vol. 8, no. 2, pp. 563 –567, feb. 2009.

[3] V. Tarokh, H. Jafarkhani, and A. Calrdebank, "Space-time block coding for wireless communications: performance results," *IEEE J. Selected Areas on Communications*, vol. 17, no. 3, pp. 451–460, Mar. 1999.

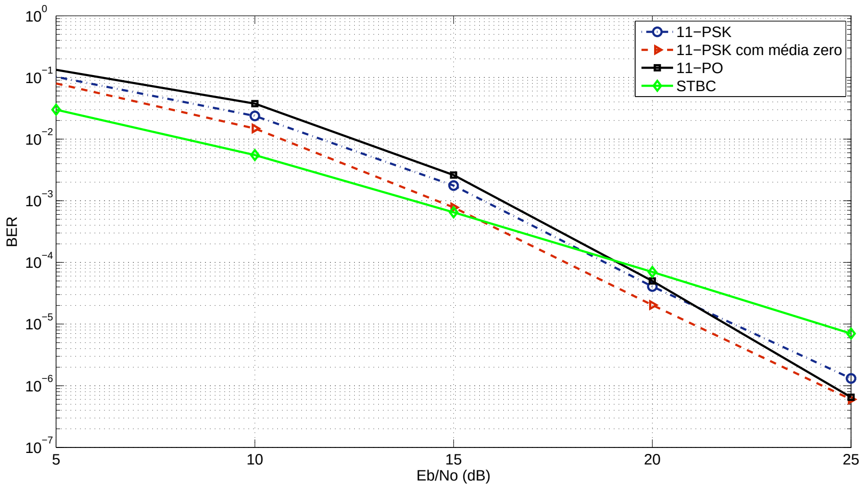

Fig. 4. BER performance of constellations for a 2x128 WCM

[4] Brendan Moore; Glen Takahara and Fady Alajaji, “Pairwise optimizastion of modulation constellations,” in *IEEE Pacific Rim Conference on Communications, Computers and Signal Processing*, August 2009.

[5] E. C. Gurjão E. A. Santos and F. M. de Assis, “Projeto de constelações de sinais para codificação wavelet,” in *XXIX Simpósio Brasileiro de Telecomunicações*, 2011.

[6] Luiz Felipe de Queiroz Silveira, *Wavelet-coded system analysis over time-varying flat fading channels (in Portuguese)*, Ph.D. thesis, DEE-UFCG, 2006.

[7] H. L.; WELLS RESNIKOFF, *Wavelet Analysis, The Scalable Structure of Information*, Springer, 1998.

[8] H. Kuai, F. Alajaji, and G. Takahara, “Tight error bounds for nonuniform signaling over awgn channels,” *IEEE Transactions on Information Theory*, vol. 46, no. 7, pp. 2712 –2718, nov 2000.

[9] Libo Zhong, F. Alajaji, and G. Takahara, “Error analysis for nonuniform signaling over rayleigh fading channels,” *IEEE Transactions on Communications*, vol. 53, no. 1, pp. 39 – 43, jan. 2005.