Performance Modeling and Analysis of OFDM-TDMA Wireless Systems Based on Queueing Theory

Victor Hugo Teles Costa Escola de Engenharia Elétrica, Mecânica e de Computação - EMC Universidade Federal de Goiás CEP: 74605-010 Goiânia - GO - Brazil victor@vcosta.com.br Flávio Henrique Teles Vieira Escola de Engenharia Elétrica, Mecânica e de Computação - EMC Universidade Federal de Goiás CEP: 74605-010 Goiânia - GO - Brazil flavio@eee.ufg.br Gustavo Batista de Castro Souza Escola de Engenharia Elétrica, Mecânica e de Computação - EMC Universidade Federal de Goiás CEP: 74605-010 Goiânia - GO - Brazil gustavo2x4@gmail.com

Abstract— This paper presents a model based on queueing theory where the Kernel Density Estimation Method is used in order to evaluate the performance of the transmission link in OFDM-TDMA systems. From the implemented model, it is derived multiple QoS parameter estimators. The obtained results confirm that the proposed model is efficient in describing the link performance indicators. The use of Kernel Density Estimation to model the arrival process improves the QoS parameter estimates of the queueing model making their values very close to those obtained with simulations.

Index Terms—Kernel Density Estimation, OFDM, QoS, Queueing Analysis, TDMA, Wireless Systems.

I. INTRODUCTION

Nowadays, the OFDM (*Orthogonal Frequency Division Multiplexing* [1]) is used in various type of high-speed wireless networks such as Wi-Fi (IEEE 802.11 [2]), WiMAX (IEEE 802.16 [3]) and LTE [4]. Investments recently made in the development and deployment of these technologies in several countries led them to be very promising in the modern world of telecommunications.

This paper presents a model based on queueing theory using Kernel Density Estimation Method in order to evaluate the performance of the transmission link in OFDM-TDMA systems. While the transmission through an OFDM channel increases the transmission rate for wireless communications, the access by multiple users is ensured by TDMA (*Time Division Multiple Access* [5]). A queueing model can be established to the OFDM-TDMA mechanism [6]. To ensure greater fidelity in the representation, in this paper, we propose the use of a model based on Queueing Theory where the incoming traffic is modeled by the nonparametric Kernel Density Estimation Method [7]. From the implemented model, it is derived multiple QoS parameters, such as the average number of packets in the queue, the average delay for packet transmission and the buffer overflow probability.

The obtained model can be used not only for designing communication systems, but also as the basis of an user

admission control system in the sense that a new call can be admitted only if the quality of service for the other calls is maintained at a certain level.

A. Performance Analysis Based on Queue Theory

The queueing theory is important in performance analysis of transmission systems for capturing the dynamics of the communication channel.

Different studies on performance analysis address wireless communication systems based on OFDM in various aspects through the Theory of Queues. Zhang and Letaief [8] propose an algorithm for adaptive resource allocation in OFDM systems that considers the channel's physical conditions, the stochastic packet arrival process, various QoS requirements and user fairness at the link layer. Liu et al. [9] suggest an algorithm for resource allocation in multiuser OFDM systems which separates the subcarriers into several resource blocks and, then, allocates non-conflicted blocks to different users. Das, Carvalho and Prasad [10] compares the performance of OFDM systems with dynamically adaptable subcarrier bandwidth against standard OFDM systems with fixed subcarrier bandwidth. Niyato and Hossain [11] present a model based on Queueing Theory for different admission control strategies in OFDM wireless networks. Chen [12] suggests a model to evaluate the performance of the subcarrier-allocation system and derive expressions for calculating two important system performance measurements, the call blocking probability and the bandwidth utilization. Wunder and Zhou [13] propose bounds for delay and queue backlog for a large class of scheduling policies in the context of OFDM. The developed model is applied in the design of LTE systems. Bouchti, Kafhali and Haqiq [14] present a performance analysis model for OFDMA systems applied to the WiMAX environment.

These studies consider that the user traffic arrival can be described by the Poisson model due to its simplicity. However, it is a fact that real network traffic does not follow, in most cases, the behavior defined by this model, restricting the application of these works to real world situations [15]. In this paper, the nonparametric Kernel method was used to estimate the probability distribution of the packet arrival process, thus obtaining a more accurate model, applicable to real systems.

II. OFDM TRANSMISSION

In this work, we considered that the transmitter has information about the channel quality (signal-to-noise ratio, SNR) available at the time of transmission of a frame, which allows the use of adaptive modulation and coding. This information can be obtained through the pilot signal [16].

Through the use of adaptive modulation and coding, the maximum number of information bits per sampled symbol (Hz) which a subcarrier m can transmit to an user n during a OFDM symbol at time t can be written as a function of the SNR and the Bit Error Rate (BER). Although there are several approximations to this function, all of them are upper bounded by the following modulation level expression [17]:

$$c_{m,n}(t) = \left\lfloor \log_2 \left(1 + \frac{-1,5}{\ln(5P_{ber})} \gamma_{m,n}(t) \right) \right\rfloor$$
(1)

where $\gamma_{m,n}(t)$ is the instantaneous SNR on a OFDM symbol transmitted to an user n during t by a subcarrier m, and P_{ber} is the bit error rate.

Let C be the maximum modulation level that can be used. The SNR levels can be devided into C + 1 consecutive and disjoint intervals with boundaries defined as Γ_c , where c = 0, 1, ..., C. Each defined region is equivalent to a modulation level. Thus, the higher the SNR read, the higher will be the modulation level used. The boundaries that define these intervals do not depend on the users or on the subcarriers in which the transmission will occur, but only on the modulation level and the BER. Therefore, from the equation 1, these boundaries can be obtained as:

$$\Gamma_c = \frac{(2^c - 1)\ln(5P_{ber})}{-1,5}$$
(2)

The OFDM system transmits at the maximum modulation level as long as the bit error rate remains below the defined threshold. Thus, the modulation level c is used when $\Gamma_c \leq \gamma_{m,n} \leq \Gamma_{c+1}$. When the modulation level is zero (c = 0), no packet is transmitted.

We also considered that the transmission power is fixed and the channel undergoes fast Rayleigh fading. The time-invariant SNR average for a subcarrier m and user n is called $\bar{\gamma}_{m,n}$. The received SNR, $\gamma_{m,n}$, is then a random variable with the following probability density function:

$$p_{\gamma}\left(\gamma_{m,n}\right) = \frac{1}{\bar{\gamma}_{m,n}} \exp\left(-\frac{\gamma_{m,n}}{\bar{\gamma}_{m,n}}\right) \tag{3}$$

Thus, the probability of a modulation level c to be selected for a subcarrier m and user n is given as:

$$Pc_{m,n}(c) = \int_{\Gamma_c}^{\Gamma_{c+1}} p_{\gamma}(\gamma_{m,n}) \, \mathrm{d}\gamma_{m,n} = \exp\left(-\frac{\Gamma_c}{\bar{\gamma}_{m,n}}\right) - \exp\left(-\frac{\Gamma_{c+1}}{\bar{\gamma}_{m,n}}\right) \quad (4)$$

For each transmitted frame (with a duration T), the number of packets that can be transmitted (r) is defined as a function of the modulation level:

$$r\left(c\right) = \left\lfloor \frac{T \times c \times \Delta f}{L} \right\rfloor \tag{5}$$

where $\Delta f = \frac{B}{M}$, B is the transmission channel bandwidth, M is the number of subcarriers that compose the OFDM channel and L is the size (in bits) of the transmitted packet.

For an user n, the probability of transmitting in a particular packet rate r_m on a subcarrier m can be calculated as follows:

$$Pr_{m,n}(r_m) = \sum_{r_m = r(c)} Pc_{m,n}(c)$$
(6)

where c = 0, 1, 2, ..., C, $r_m = 0, 1, 2, ..., R$ and $R = \left\lfloor \frac{T \times C \times \Delta f}{L} \right\rfloor$. In addition, the probability mass function of the packet

In addition, the probability mass function of the packet transmission rate during a frame on a subcarrier m is obtained by the equation 7:

$$r_{m,n} = \begin{bmatrix} Pr_{m,n}(0) & Pr_{m,n}(1) & \cdots & Pr_{m,n}(R) \end{bmatrix}$$
(7)

Thus, the probability mass function of the total packet transmission rate (\mathbf{R}_n) , involving all M channel subcarriers dedicated to the user n, is given by the convolution of each subcarrier's probability mass function of the packet transmission rate:

$$\mathbf{R}_n = r_{1,n} * r_{2,n} * \dots * r_{M,n} \tag{8}$$

III. OFDM-TDMA TRANSMISSION MODEL

The system under consideration involves a base station connected to multiple subscriber stations, as shown in figure 1).

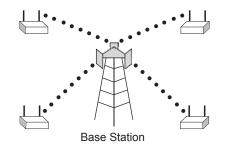


Fig. 1. Considered scenario: a base station connected to multiple subscriber stations

The scenario involves the downlink transmission on a channel accessed by multiple subscriber stations and controlled

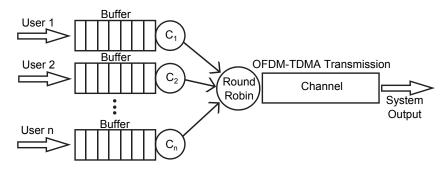


Fig. 2. OFDM-TDMA transmission model

by OFDM-TDMA. The time division based multiple access is implemented through the round-robin scheduling algorithm and it is assumed fairness condition among the users. There are N users sharing a channel composed by M subcarriers. Each user has its own buffer, the size of which is predetermined, where incoming traffic is stored. The model of this system is illustrated in figure 2.

It is worth noting that the OFDM transmission is performed through frames, which are considered the units of data transmission and comprises several OFDM symbols. A frame contains, besides the data, the metadata that maps the modulation techniques used against the transmitted symbols and the downlink and uplink mapping over the channel, in case of duplex communication.

Back to the scenario considerations, in a transmission cycle, the channel is sequentially allocated for each user during a time slot to send a frame, totalizing, thus, N transmitted frames at the end of the cycle. The length of the time interval for which the channel is dedicated to the considered user is Tseconds. The number of transmitted packets in a frame may be different depending on the modulation level used on each subcarrier. Short intervals are inserted between the transition from one user to another in order to avoid interference.

IV. FORMULATION OF THE QUEUEING MODEL

We propose an analytical model based on Discrete Time Markov Chain in order to analyze the performance of a particular user queue system. Thus, once defined the probability mass functions of the total packet transmission rate and of the incoming traffic (which is directly estimated from the Kernel method), the obtained model can be applied to the user queue.

The model assumes that the queue states are observed at the end of every frame transmission. The time interval inserted between the transmission of each frame is ignored in order to simplify the model, since it is considerably shorter than the time dedicated to a frame transmission. However, this interval may be considered in the model by dedicating a time slot in which the transmission rate is null and no user is served.

Each user incoming traffic is modeled from series of samples of real TCP/IP network traffic. The traffic trace must have its samples aggregated for each T seconds (where T is the length of the time slot in which a frame is transmitted). Thereafter, the Kernel method is applied and it is obtained

a probability density function f(v) that describes the user's packet arrival process. The cumulative distribution function of f(v) is referred to by F(v).

It is assumed a number V that corresponds to the maximum number of packets that can arrive during the time slot T. Thus, the probability of arriving V packets is defined as f(V) = 1 - F(V - 1).

The behavior of the user queue can be modeled as a quasibirth and death process [18] and, due to the discrete time domain, a transition probability matrix \mathbf{P} of a Markov Chain can be defined. This matrix is given by the equation 9, where R is the highest total transmission rate.

The matrix **P** is square and its rows represent the number of packets (x) in the queue, while the values $\mathbf{p}_{x,x'}$ represent the probability of the queue transitioning from the state x to x', in other words, it represent the probability that the queue contains x packets now and x' packets in the next time slot.

Once the transition matrix \mathbf{P} is defined, one can estimate the steady-state probabilities of the system to be found in each possible state and, thereafter, determine multiple QoS parameters.

For the OFDM-TDMA system, the state space of an user queue can be defined as:

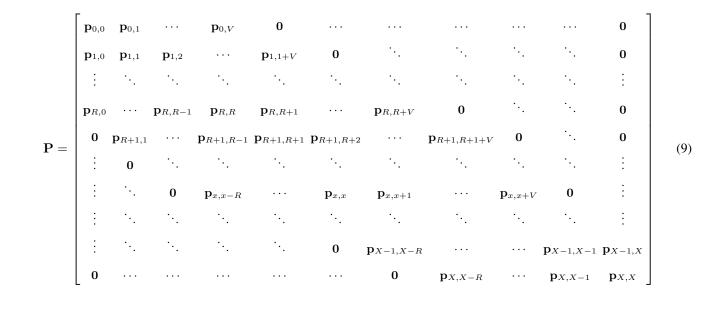
$$\Delta = \{ (\mathcal{X}_i, \mathcal{N}_i), 0 \le \mathcal{X}_i \le X, 1 \le \mathcal{N}_i \le N \}$$
(10)

where \mathcal{X}_i is the number of packets in the queue, X is the buffer size, \mathcal{N}_i corresponds to the user being served during the time slot and N refers to the number of users accessing the transmission channel.

Once the users are served sequentially and cyclically, the following transition matrix can represent the round-robin scheduling:

$$\mathbf{U} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$
(11)

The matrix U has $N \times N$ dimensions and each row of it represents the user who is being served at the moment. Thus, the row *n* represents the user *n* being served by the system in the current time slot. The probability of the user queue



transiting from state x to x' depends on which user is currently being served and it is determined by the diagonal matrix $\mathbf{V}_{x,x'}$ of size $N \times N$.

$$\left[\mathbf{V}_{x,x'}\right]_{i,i} = \begin{cases} \sum_{v-r=x'-x} f(v) [\mathbf{R}_n]_{r+1}, & i=n\\ f(x'-x), & i \neq n, x' \ge x \end{cases}$$
(12)

where $x-D \le x' \le x+A$, r = 0, 1, ..., R and v = 0, 1, ..., V. \mathbf{R}_n is the vector that contais the probability mass function of the total packet transmission rate for the user *n* (equation 8). $[\mathbf{V}_{x,x'}]_{i,i}$ indicates the element in row *i* and column *i* of the diagonal matrix $\mathbf{V}_{x,x'}$.

The $\mathbf{p}_{x,x'}$ values, which compose the system's transition matrix, are calculated as shown in equation 13. It is observed that each $\mathbf{p}_{x,x'}$, in this scenario, is a matrix of size $N \times N$.

$$\mathbf{p}_{x,x'} = \mathbf{U}\mathbf{V}_{x,x'} \tag{13}$$

By calculating all $\mathbf{p}_{x,x'}$ values, for $0 \le x \le X$ and $x - D \le x' \le x + A$, it is obtained the system's complete transition matrix.

Considering that the states of the Markov Chain are recurrent and aperiodic [19], the probabilities of the system to be found in each possible state are determined by the vector $\boldsymbol{\pi} = \begin{bmatrix} \pi_0 & \pi_1 & \cdots & \pi_i & \cdots \end{bmatrix}$. This vector can be obtained from the transition matrix of the Markov Chain by solving the following system of linear equations:

$$\boldsymbol{\pi} = \boldsymbol{\pi} \mathbf{P} \tag{14}$$

$$\sum_{i \in S} \pi_i = 1 \tag{15}$$

V. QOS PARAMETERS ESTIMATION

The QoS parameters are calculated from the steady-state probabilities of the system to be found in each possible state.

For each of the X possible queue states, the model considers N different states for the round-robin scheduling. Thus, the steady-state probability of the user queue to contain x packets (where $0 \le x \le X$) can be calculated as:

$$\pi\left(x\right) = \sum_{i=(x\times N)+1}^{(x+1)\times N} [\boldsymbol{\pi}]_{i}$$
(16)

Based on these values, the average number of packets in the queue (or backlog) can be obtained by:

$$b = \sum_{x=0}^{X} x\pi(x) \tag{17}$$

The average delay of a packet, defined as the average time that a packet waits in the queue from its arrival until its transmission, is given by Little's law:

$$d = \frac{b}{\lambda} \tag{18}$$

where λ is the average number of incoming packets per time slot, obtained from the average of the considered traffic series.

From the law of total probability, the probability of the queue to be in state X is obtained by the sum of the probabilities of the buffer to contain X packets given that the buffer has overflowed and of the buffer to contain X packets given that the buffer has not overflowed.

$$\pi \left(X \right) = P\left(X|O\right) P\left(O\right) + P\left(X|\bar{O} \right) P\left(\bar{O} \right) \tag{19}$$

Once the user buffer gets completely filled every time there is an overflow, then P(X|O) = 1. Therefore:

$$\pi\left(X\right) = P\left(O\right) + P\left(X|\bar{O}\right)P\left(\bar{O}\right)$$
(20)

It is verified that, when the average incoming flow is lower than the average service flow, the probability of the buffer to contain X packets when there is no overflow decreases while the buffer size is increased, since the queue tends to occupy a smaller space within the buffer considering the system in steady-state. In this case, the probability that there is no overflow and, therefore, the queue has X packets, is far superior to the probability of the queue to contain X packets when there is no overflow, ie, $P(O) \gg P(X|\bar{O})P(\bar{O})$. Thus, the probability of buffer overflow can be approximated as:

$$P_{over} \approx \pi(X) \tag{21}$$

VI. SIMULATION AND RESULTS

In the experiments, we analyzed the performance of an user queue in an OFDM-TDMA system with 5 active connections. The incoming traffic process of the users was modeled based on real TCP/IP network traffic traces.

Real TCP/IP network traffic presents bursty features in many scales and long range dependence among samples. The Poisson model is inefficient to describe such characteristics, as shown by Paxson and Floyd [15], what makes this work's proposal interesting for real applications.

Each sample of the series records the number of bytes that form the transmitted packet. In this experiment, it was assumed that the samples were collected every millisecond. The first 1500 values of the considered series are shown in figure 3.

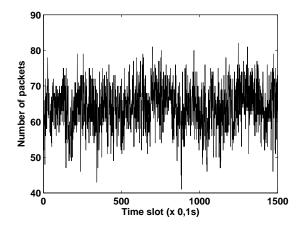


Fig. 3. First 1500 samples of the user's incoming traffic series. Each sample describes the number of packets transmitted in a frame

Further, the Kernel method was applied and the probability density function that models the user's incoming traffic was obtained. The function is shown in figure 4.

The channel is composed of 512 subcarriers and has bandwidth of 13.5 MHz (thus, $\Delta f = 26367, 19$ Hz). Various scenarios were considered in order to study the quality of the channel by varying the average signal-to-noise ratio (SNR). From the developed model of the OFDM channel (assuming $P_{ber} = 10^{-6}, C = 5, T = 100$ ms and L = 329 bytes), were obtained the probability mass functions of the channel's total packet transmission rate (equation 8) shown in figure 5

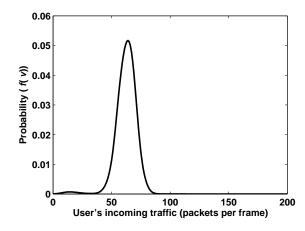


Fig. 4. Probability density function that models the user's incoming traffic

for SNR values ranging from 9.5 dB to 15 dB. As expected, increasing the SNR reflects an improvement in the quality of the channel, since the average transmission rate is increased.

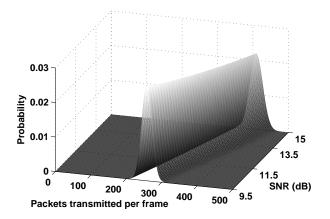


Fig. 5. Probability mass functions of the channel's total packet transmission rate for different SNR values

Once determined the functions that describe the traffic arrival process and the transmission in the OFDM channel for the observed user, the implemented model based on Markov Chains to analyze the queue in OFDM-TDMA system was applied. Assuming the user buffer with capacity for 400 packets and a limit of 200 packets that can arrive during a time slot (X = 400 and V = 200), the steady-state probabilities (shown in figure 6) of the user queue to be found in each possible state were calculated.

In order to validate the numerical results obtained with the analytical model, we developed a simulator of the considered system using the Matlab. The probabilities, obtained from the simulation, of the user queue to have each possible length can be seen in figure 7.

The results show the accuracy of the model in representing the behavior of the queue in a real OFDM-TDMA system. It is observed that, due to the cyclic scheduling, the user's packet queue tends to increase in iterations in which the user

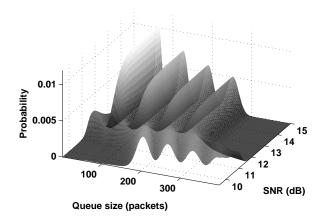


Fig. 6. Steady-state probabilities, obtained through the model, of the user queue to be found in each possible state

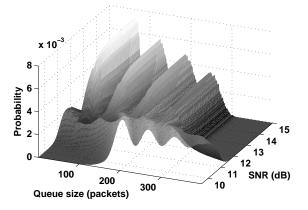


Fig. 7. Steady-state probabilities, obtained by the simulation, of the user queue to be found in each possible state

is not served, creating different higher probability regions. Furthermore, the implemented model, due to the nonparametric estimation of the incoming traffic, is able to reproduce irregularities that a model based on parametric estimation can not.

The graphs in figures 6 and 7 do not include the probabilities $\pi(0)$ and $\pi(X)$, the probabilities of buffer to be empty and full, respectively. In the analyzed SNR range, the values $\pi(0)$ and $\pi(X)$ are much higher than the values $\pi(x)$, for 0 < x < X, making it difficult to visualize the graphs if these values were included. The high value of the probability of the buffer to be empty is explained by the presence of idle capacity. Every time the transmission rate within the frame is higher than the number of packets in the queue plus the number of packets arriving, the buffer to be full is explained by the occurrence of overflows in the buffer. Every time there is an overflow, the buffer remains full at the end of the transmission of the frame.

The figure 8 shows the average number of packets in the buffer, or the queue average length, obtained by the equation

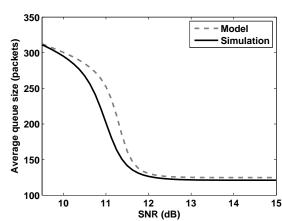


Fig. 8. Queue average length versus SNR

The average delay of a packet, calculated from the equation 18, can be visualized in figure 9 and the probability of buffer overflow in the figure 10.

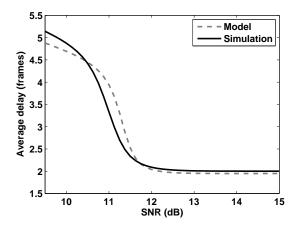


Fig. 9. Average delay of a packet versus SNR

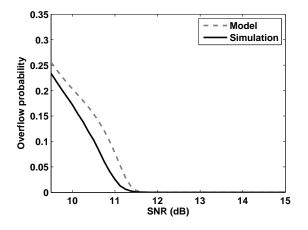


Fig. 10. Probability of buffer overflow versus SNR

It is observed that, as the SNR is increased, there is a decrease in the queue average length, in the average delay of the transmission and in the probability of the user buffer to overflow. This effect was expected, since the improvement of the channel conditions makes the OFDM-TDMA system, through adaptive modulation and coding, provide higher transmission rates.

The achieved results, compared to the simulation, confirm that the model is effective in describing the considered performance indicators. The Kernel method (for nonparametric estimation) applied to model the packet arrival process improves the estimates of QoS parameters, making their values very close to simulation results. Thus, it can be said that the developed model represents well the behavior of an user queue in an OFDM-TDMA system.

VII. CONCLUSIONS

The OFDM is a multiplexing technique widely deployed in actual wireless communications systems. It is characterized by its high resistance against multipath and inter symbolic interferences, besides its high spectral efficiency.

In this paper, the OFDM-TDMA transmission system is studied and a model based on Queueing Theory for the system's queue behavior is proposed. For this purpose, models were established for the system's transmission process and for the user's packet arrival process. The first was built assuming the channel undergoes fast Rayleigh fading and it considers the use of Adaptive Modulation and Coding. The second model was built by applying the Kernel method for estimating the probability density function of the packet arrival process.

Different papers, in the reviewed literature, that study the behavior of an user queue in OFDM systems model the traffic arrival as a Poisson process. However, real traffic often does not follow the behavior defined by the Poisson model. This study employs the Kernel method in the definition of traffic arrival process, resulting in a model more approximate of real systems.

Through the use of Markov Chains, it was possible to define the steady-state behavior of the user queue by calculating the probabilities of the queue to be found in each possible state. From these probabilities, some indicators, useful when analyzing the performance of the considered system, were estimated: the average number of packets in the transmission queue, the average delay for a packet to be transmitted and the probability of the user buffer to overflow.

The performance indicators obtained were compared to those given by the simulations of the OFDM-TDMA system. It was verified that the model is improved by using the Kernel method for nonparametric estimation. The resulting estimates for the QoS parameters were very close to the simulation results, confirming the efficiency of the proposed model.

References

- [1] Prasad, R. *Ofdm for Wireless Communications Systems*. Artech House, 2004. (Artech House universal personal communications series).
- [2] IEEE. Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications. New York, NY, 2012.

- [3] IEEE. IEEE Standard for Local and Metropolitan Area Networks Part 16: Air Interface for Fixed Broadband Wireless Access Systems. New York, NY, USA, 2004.
- [4] 3GPP. Evolved Universal Terrestrial Radio Access (E-UTRA) and Evolved Universal Terrestrial Radio Access Network (E-UTRAN); Overall description. Valbonne, FR, 2013.
- [5] Tarokh, V. New Directions in Wireless Communications Research. Springer, 2009.
- [6] Niyato, D.; Hossain, E. "Queueing analysis of ofdm/tdma systems". *IEEE Global Telecommunications Conference*, vol. 6, p. 5 pp. 3712–3716. December, 2005.
- [7] Webb, Andrew R.; Copsey, Keith D. "Density Estimation Nonparametric". *Statistical Pattern Recognition*. 3rd. ed. John Wiley & Sons, Ltd, 2011. pp. 150–220.
- [8] Ying Jun Zhang; Letaief, K.B. "Adaptive resource allocation and scheduling for multiuser packet-based OFDM networks". 2004 IEEE International Conference on Communications, vol. 5, pp. 2949–2953. June, 2004.
- [9] Dongsheng Liu; Weidong Kou; Guanghui Yu; Liujun Hu. "A New Resource Allocation Scheme for Downlink OFDM Systems". *International Conference on Wireless Communications, Networking and Mobile Computing*, pp. 1–3. September, 2006.
- [10] Das, S.S.; de Carvalho, E.; Prasad, R. "Dynamically Adaptive Bandwidth for Sub Carriers in OFDM Based Wireless Systems". *IEEE Wireless Communications and Networking Conference*, pp. 1378–1383. March, 2007.
- [11] Niyato, D.; Hossain, E. "Connection admission control algorithms for OFDM wireless networks". *IEEE Global Telecommunications Conference*, vol. 5, pp. 2455–2459. December, 2005.
- [12] Jui-Chi Chen; Wen-Shyen E. Chen. "Call blocking probability and bandwidth utilization of OFDM subcarrier allocation in next-generation wireless networks". *IEEE Communications Letters*, vol. 10, no. 2, pp. 82–84. February, 2006.
- [13] Wunder, G.; Chan Zhou. "Queueing analysis for the OFDMA downlink: Throughput regions, delay and exponential backlog bounds". *IEEE Transactions on Wireless Communications*, vol. 8, no. 2, pp. 871–881. February, 2009.
- [14] El Bouchti, A.; El Kafhali, S.; Haqiq, A. "Queueing performance analysis of CAC scheme in OFDMA based WiMAX system". 3rd International Conference on Next Generation Networks and Services (NGNS), pp. 62– 67. December, 2011.
- [15] Paxson, V.; Floyd, S. "Wide area traffic: the failure of Poisson modeling". *IEEE/ACM Transactions on Networking*, vol. 3, no. 3, pp. 226–244. June, 1995.
- [16] Coleri, S.; Ergen, M.; Puri, A.; Bahai, A. "Channel estimation techniques based on pilot arrangement in OFDM systems". *IEEE Transactions on Broadcasting*, vol. 48, no. 3, pp. 223–229. September, 2002.
- [17] Xiaoxin Qiu; Chawla, K. "On the performance of adaptive modulation in cellular systems". *IEEE Transactions on Communications*, vol. 47, no. 6, pp. 884–895. June, 1999.
- [18] Leeuwaarden, J.; Squillante, M.; Winands, E. "Quasi-Birth-And-Death Processes, Lattice Path Counting, and Hypergeometric Functions". *Journal of Applied Probability*, vol. 46, no. 2, pp. 507–520. June, 2009.
- [19] Papoulis, A.; Pillai, S.U. Probability, random variables, and stochastic processes. McGraw–Hill, 2002. (McGraw–Hill electrical and electronic engineering series).