Compressive Wideband Spectrum Sensing for Wireless Cognitive Radios

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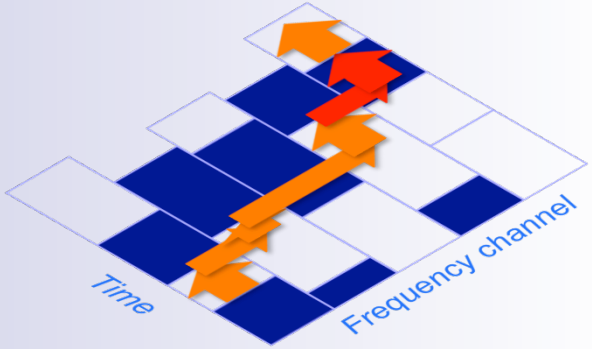
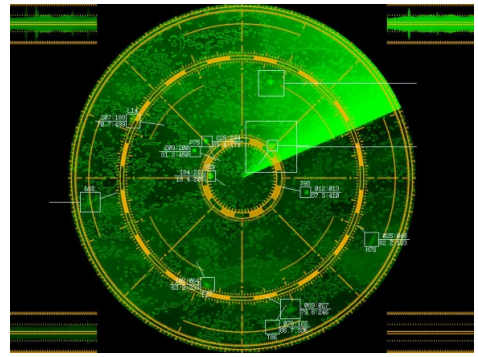
Outline

- Context: covariance-spectrum estimation and applications
- Wireless cognitive radio (CR) access
  - Compressive cyclic feature detection for wideband sensing
- Foundation: compressive covariance sensing (CCS)
  - Sub-Nyquist-rate compressive sampling for non-sparse signals
- Research outlook

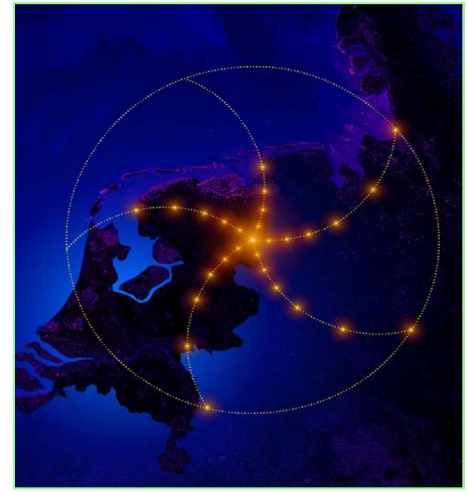
Covariance and spectrum estimation



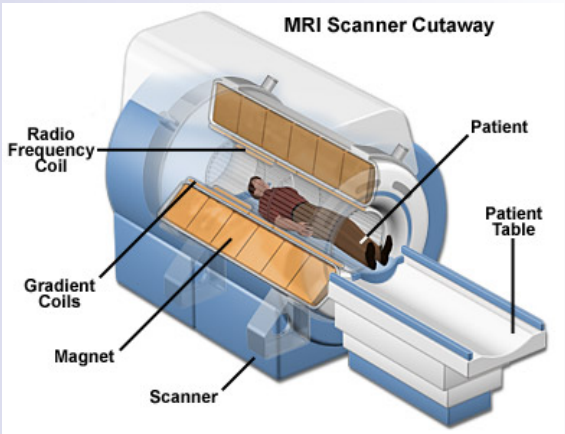
Cognitive radio (CR)
frequency spectrum



Radar
Doppler + angular spectra



Radio astronomy
spatial spectrum



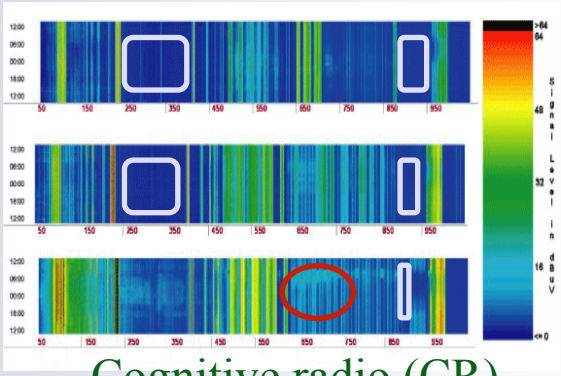
MRI
resonance spectrum

Seismic
seismic design response spectrum

Emerging Challenges

(Ultra-)wideband signals

Impulse radio



Cognitive radio (CR)

Very large arrays

Large Arrays

Massive MIMO

Large-scale networks

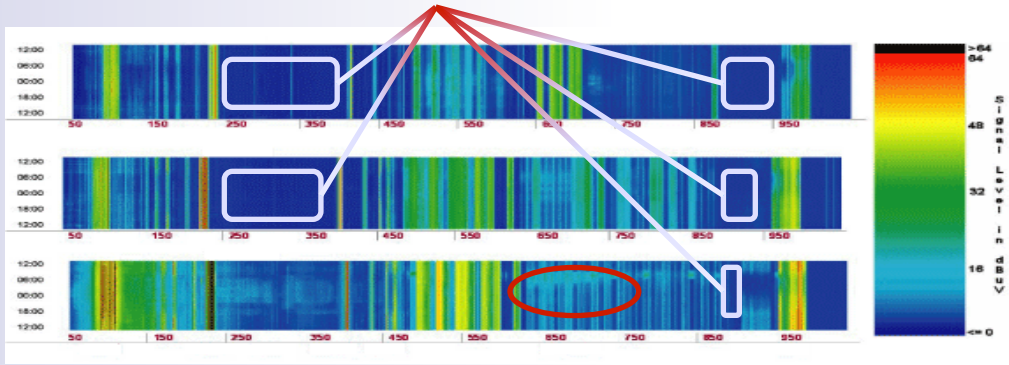
Internet backbone network

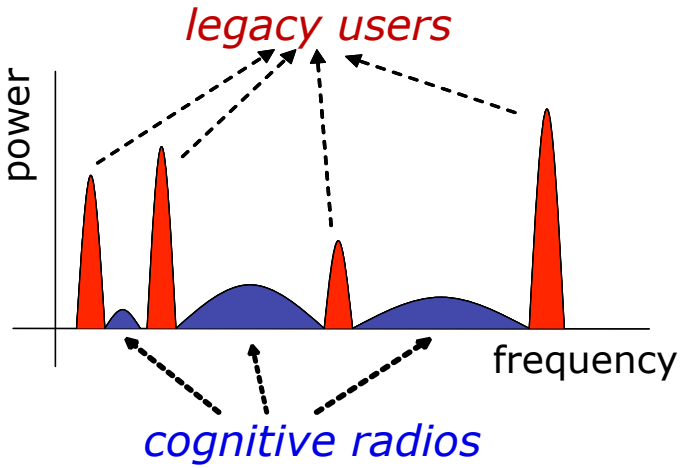
Disease gene network

Sampling rate/size issue → **Need for compressive techniques**

Wireless cognitive radios

- Goal: overcome (perceived) bandwidth scarcity
  - Spectrum hole = opportunities



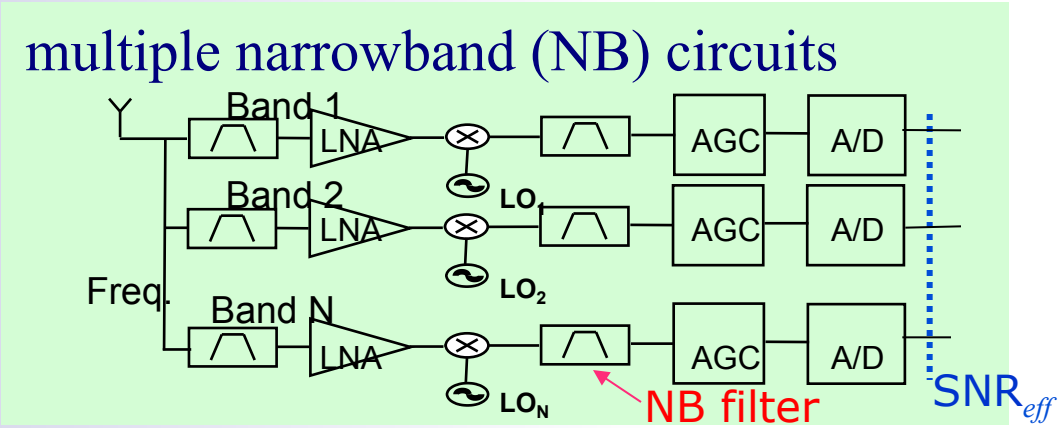


  - Opportunistic spectrum access

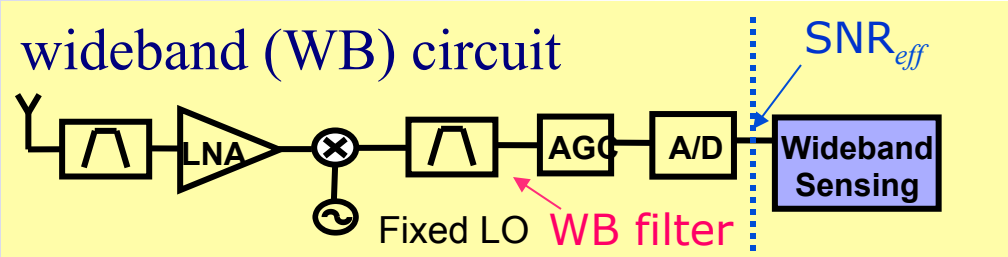
- Technical challenges
  - Find holes in wideband spectrum
  - Allocate spectrum resources dynamically
  - Adjust transmit-waveforms

Challenge 1: Wideband signal acquisition

- RF circuit choices: *multiple NB or single WB ?*



- Multiple, fixed RF chains
- Preset LO filter range
- Simple detection per BPF



- Single, flexible RF chain
- burden on A/D: $f_s \sim$ GHz
- complex wideband sensing

Q: ***How can we alleviate DSP burden on wideband circuit design?***

A: Adopt a compressive sampling (CS) framework

Challenge 2: User hierarchy

"IEEE 802.22 requires CRs to sense PU signals as low as -114dBm"

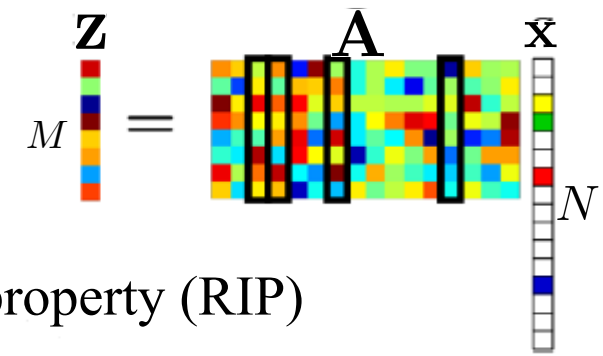
Operating Conditions	Technical Challenges
Protection of primary systems	Sensing at low SNR
	Short sensing time
	Modulation classification
Random sources of interference and noise	Robustness to noise uncertainty
	Interference identification

Q: ***How can we alleviate noise uncertainty effects at low SNR?***

A: Cyclic feature based spectrum sensing

Compressive sampling in a nutshell

$$\mathbf{z} = \mathbf{A}_{M \times N}\ \mathbf{x}$$



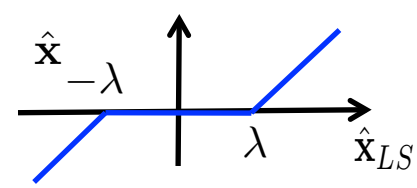
(a1) **x** is sparse (nonzero entries unknown)

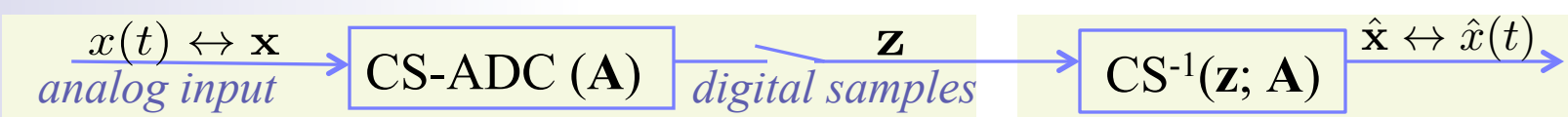
(a2) **A** fat ($M/N \leq 1$); satisfies restricted isometry property (RIP)

- Compressive sampling (CS) [Chen-Donoho-Saunders'98], [Candès et al'04-06]
  - Given **z** and **A**, unknown **x** can be found when (a1) and (a2) hold

- Sparse regression $\mathbf{z} = \mathbf{A}\ \mathbf{x} + \mathbf{w}$ [Tibshirani'96], [Tipping'01]
  - efficient inverse solution (**CS^{-1}**): L_1-norm regularized least-squares (LR-LS)

$$\hat{\mathbf{x}} = \arg\min\ \frac{1}{2}\|\mathbf{z} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda\|\mathbf{x}\|_1$$





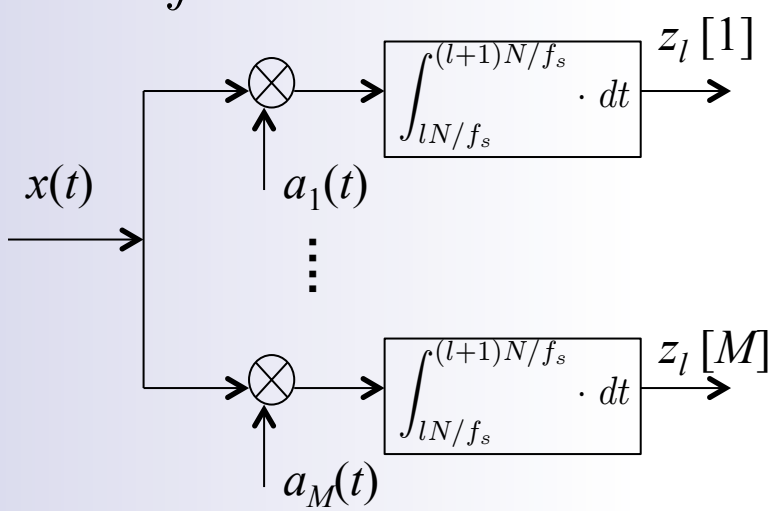
Compressive ADC (CS-ADC)

- Periodic analog sampling devices

  [Herley-Wong,1999], [Venk-Bresler,2000]; [Tropp et al, 2010]; [Mishali-Eldar,2010]; [Becker, 2011]; [Yoo et al, 2012];

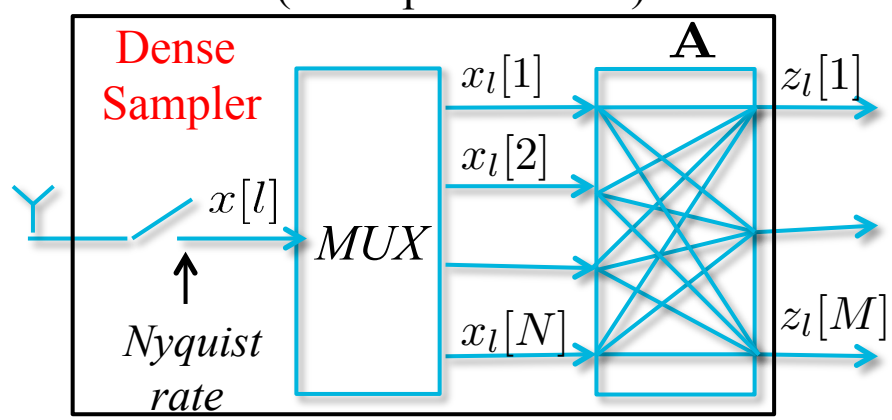
  AIC [Kirolos et al'06], [Hoyos et al'08]

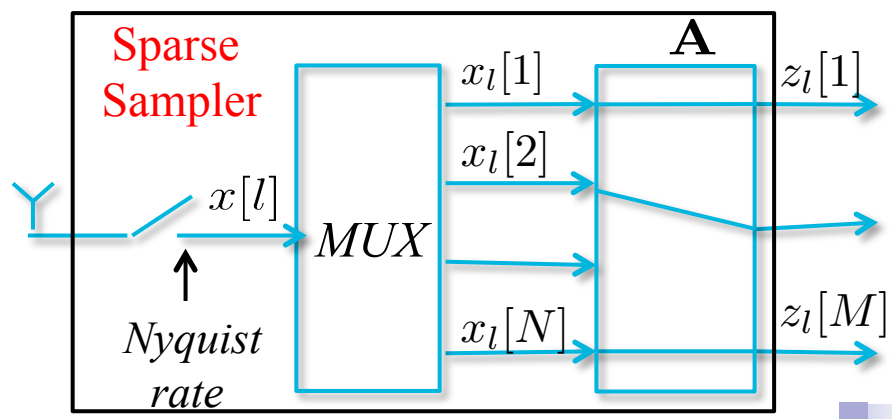
$$z_i = \int a_i(t)x(t)dt, \quad i = 1:M$$



$$\mathbf{z}_l = \mathbf{A}\mathbf{x}_l$$

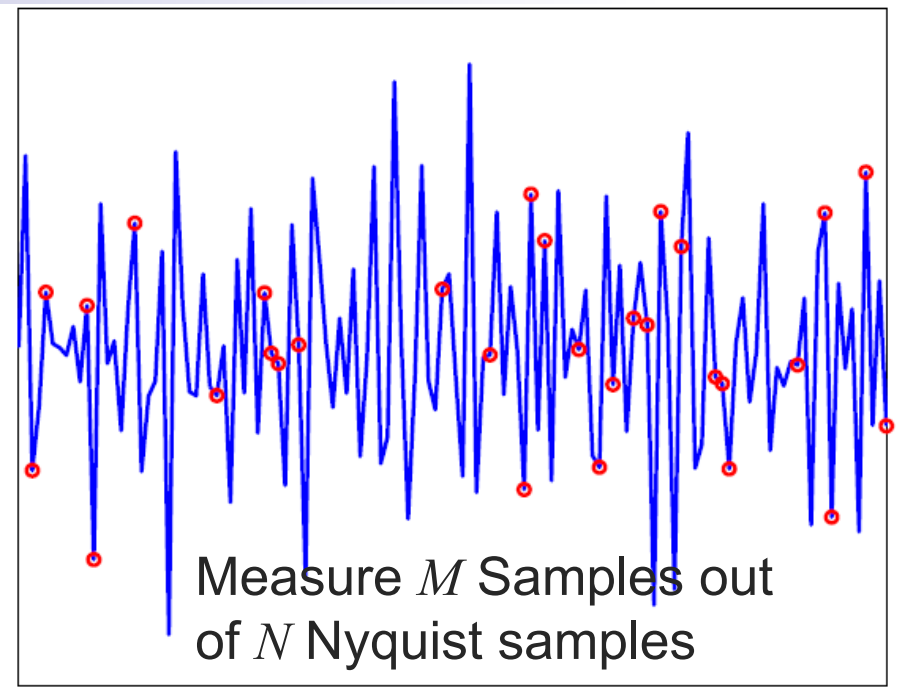
(conceptual model)





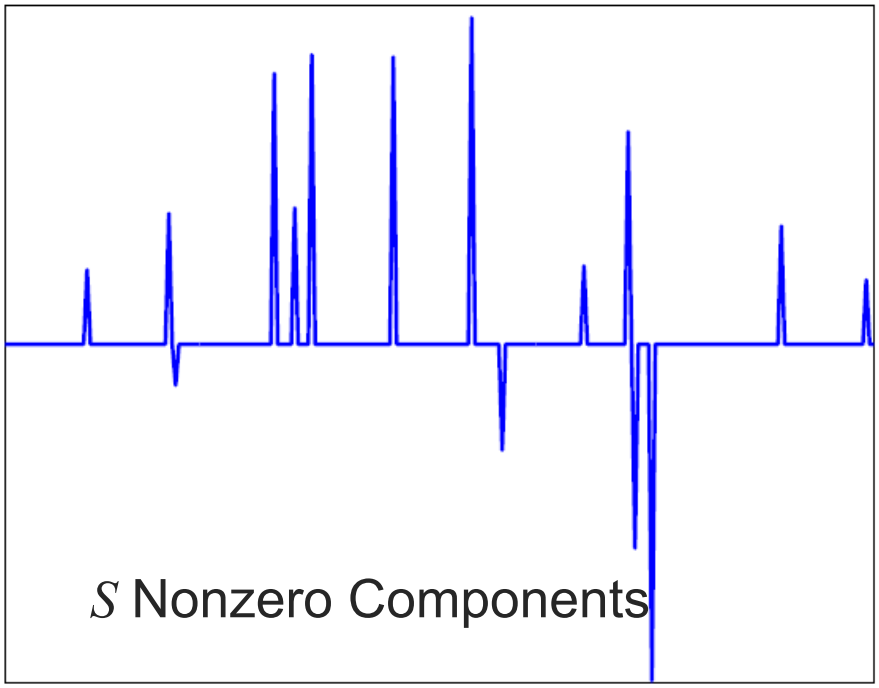
Example: Fourier measurements

Sampling basis: Time Domain



$$f(t) = \sum_{i=1}^{S} x_i e^{j\omega_i t} \qquad t_1, \ldots, t_K$$

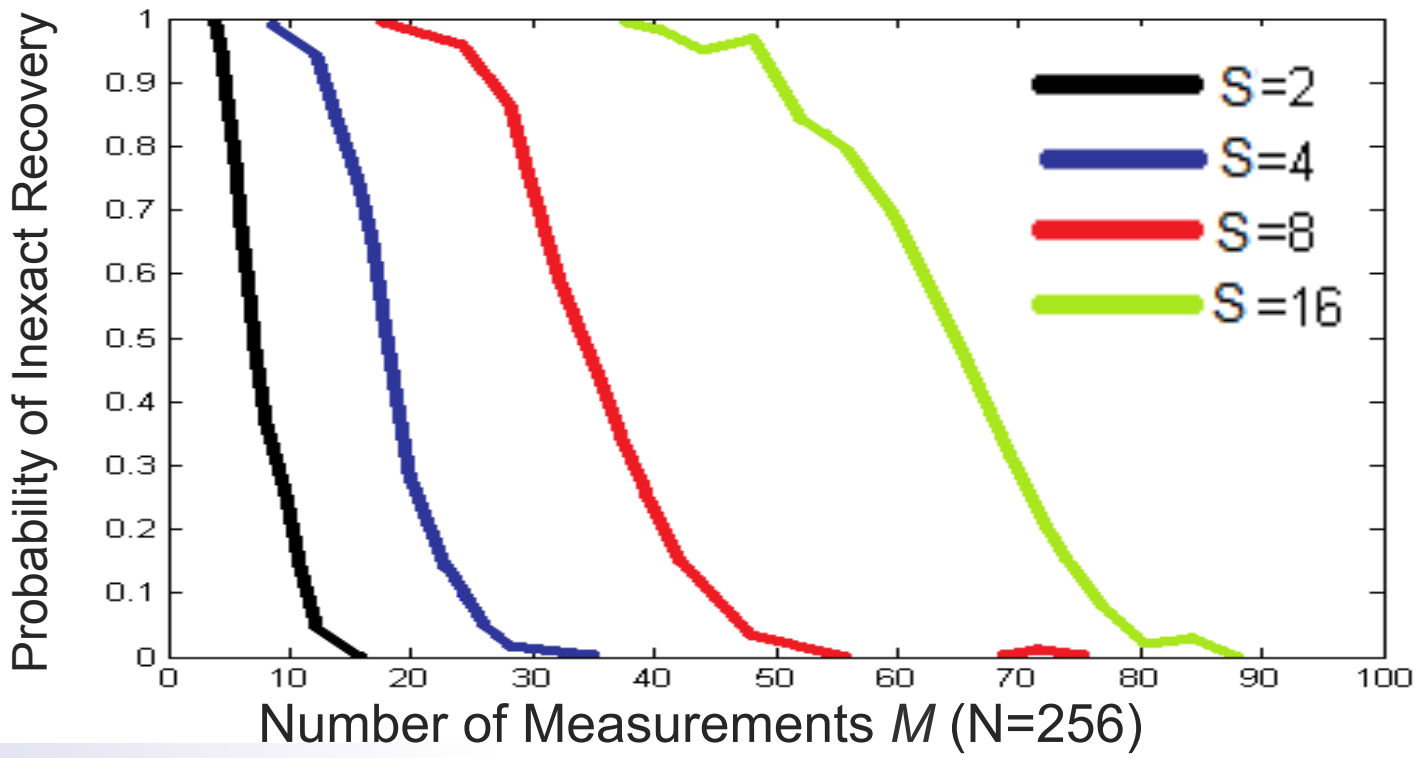
Sparsity basis: Frequency Domain



$$F(\omega) = \sum_{i=1}^{S} x_i \delta(\omega - \omega_i)$$

Example: Fourier measurements (cont' d)

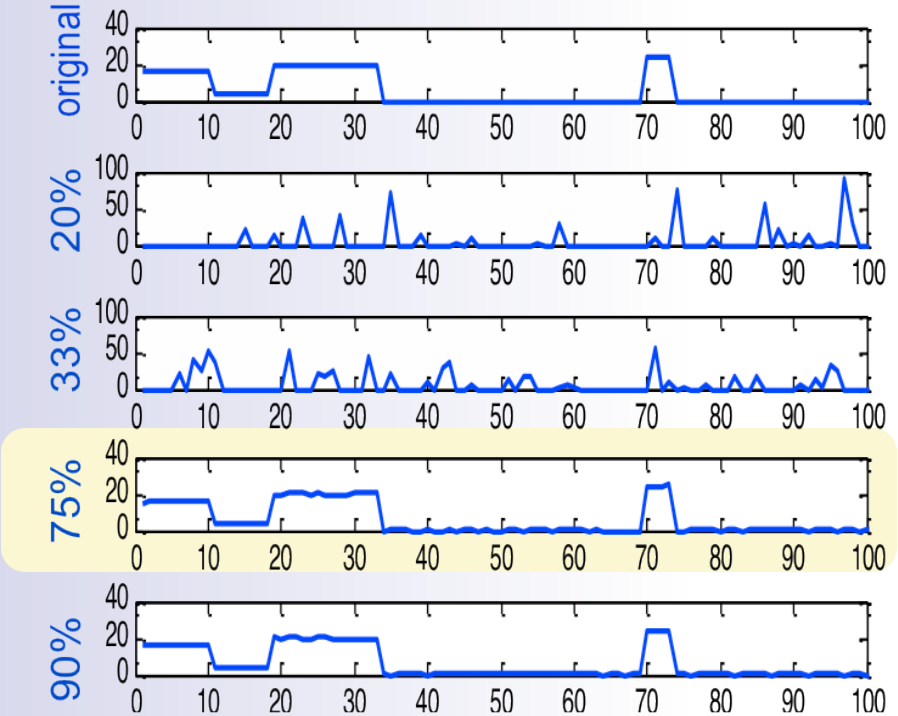
- Performance (Prob. of inexact recovery) vs. # measurements M



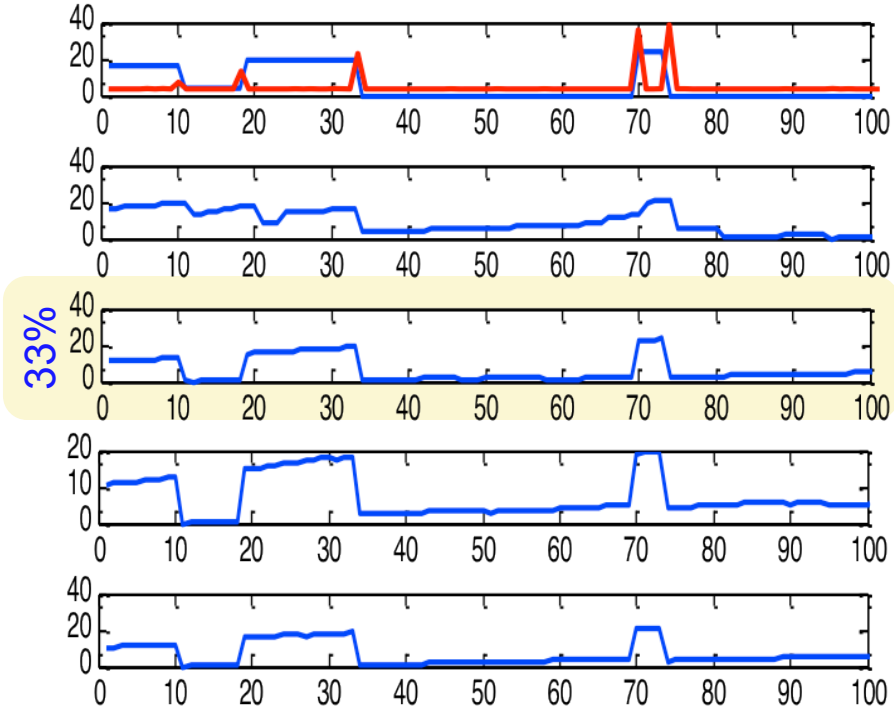
- (M; N, S): depends on *measurement matrix* and *recovery method*
- RIP conditions are sufficient rather than necessary

Early contribution: spectrum hole detection

Spectrum reconstruction



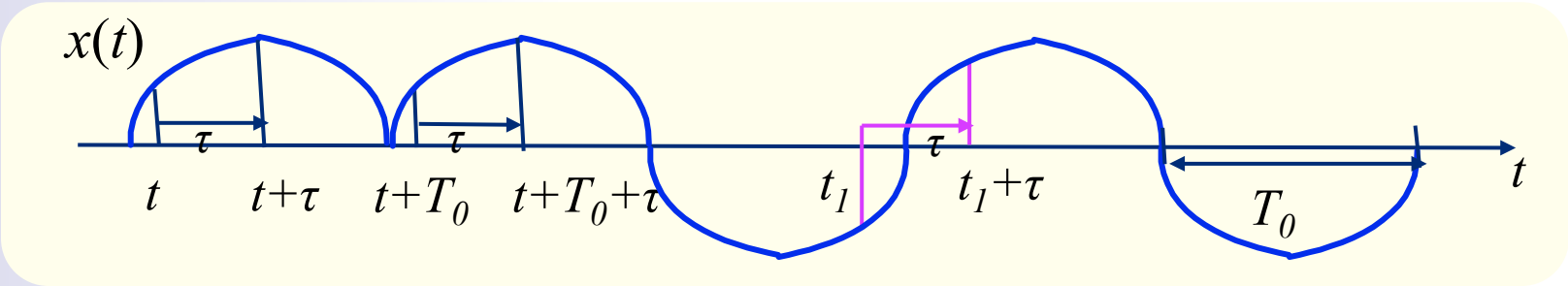
Spectrum hole detection
(edge detection on wavelet basis)



Z. Tian, and G. B. Giannakis, "Compressed sensing for wideband cognitive radios," *ICASSP Conf.*, 2007.

Cyclostationary modulated signals

- Cyclic features reveal critical signal parameters:
  - carrier frequency
  - symbol rate
  - modulation type
  - timing, phase etc.
- Non-cyclic signals (e.g. noise) do not possess cycle frequencies



$$R_x(t,\tau) = R_x(t+T_0,\tau), \quad \forall\tau \quad \xleftrightarrow[t \leftrightarrow \alpha \quad \tau \leftrightarrow f]{\text{2x Fourier Transform}} \quad S_x(\alpha, f)$$

Periodic autocorrelation

Cyclic spectrum

Noise suppression in cyclic domain

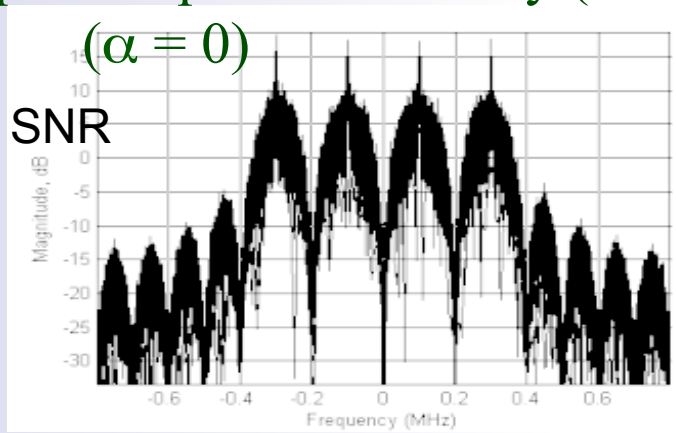
Energy detection vs. cyclic feature detection

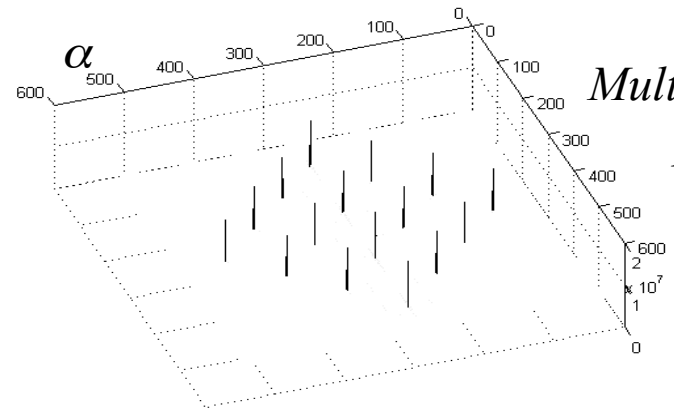
e.g., [Sahai-Cabric'05]

power spectrum density (PSD) ($\alpha = 0$)

spectral correlation density (SCD)

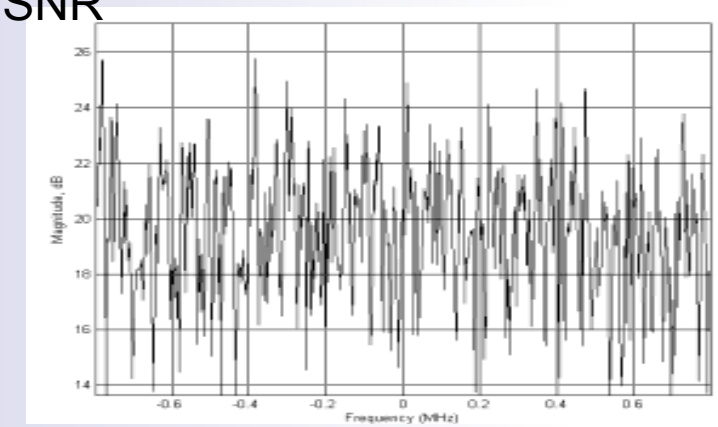
High SNR

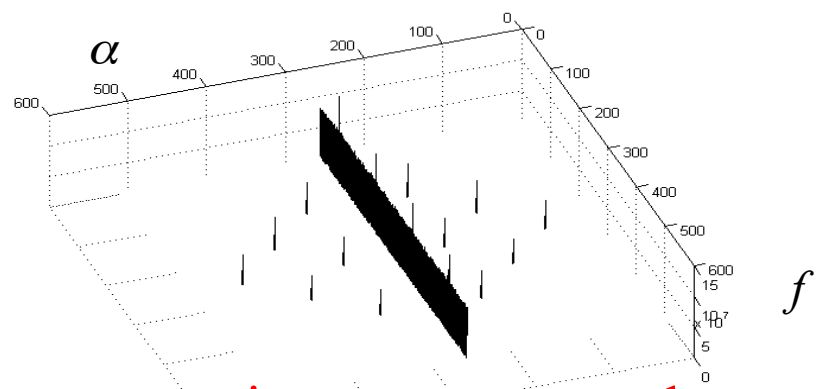




Multi-harmonics peaks at $\alpha = kf_m$

Low SNR



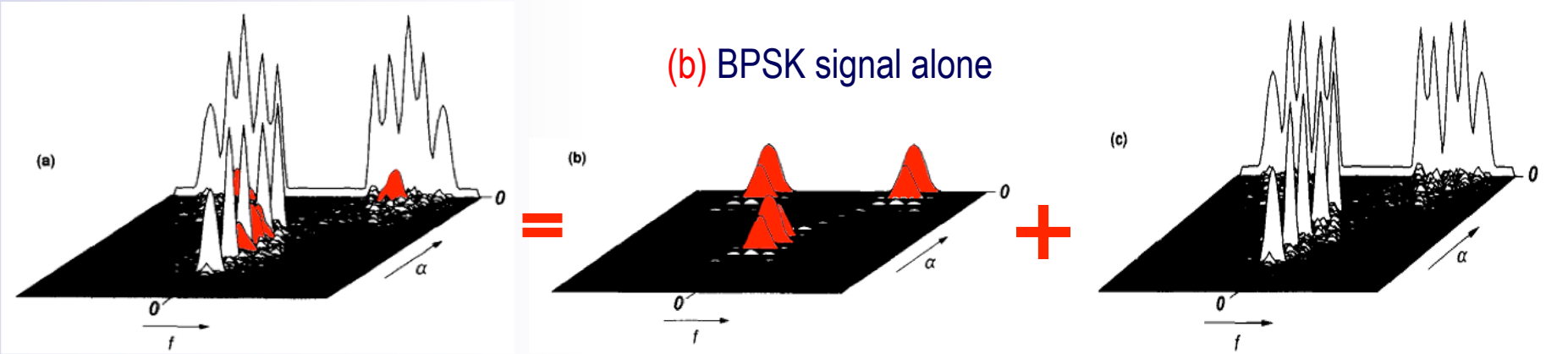


✔ no noise components when $\alpha \neq 0$

Source separation in cyclic domain

Spectral correlation density (SCD)

e.g., [Gardner'88]



(b) BPSK signal alone

(a) BPSK in noise plus five AM interferers

(c) White noise plus five AM interferences

✔ overlapping in PSD, separable in SCD

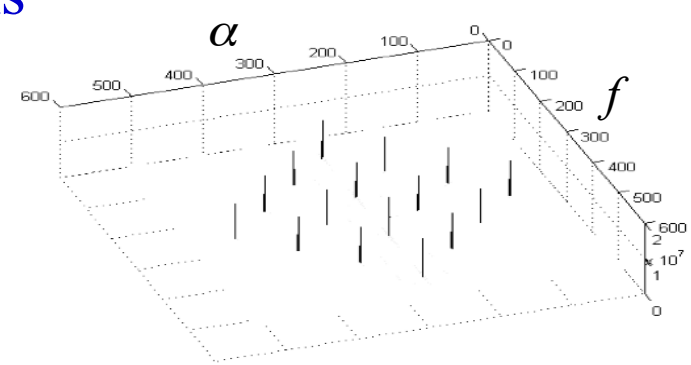
- Cyclostationarity-based approach to detection
  - ✔ resilient against Gaussian noise
  - ✔ robust to multipath
  - ✔ can differentiate modulation types and separate interference
  - ✔ insensitive to unknown signal parameters

Cyclic feature detection for CR?

- Major challenges
  - ✘ Cyclostationarity typically induced by OVER-sampling
    → excessive sampling-rate requirements
  - ✘ Cyclic statistics converge slowly with finite samples
    → long sensing time

Our contribution: Compressive sampling to the rescue …

- Leverages sparsity in two dimensions
  - ➢ Sparsity in frequency domain (f)
    - ✧ low spectrum utilization
  - ➢ Sparsity in cyclic-freq. domain (α)
    - ✧ modulation-dependent cycles



Wideband cyclic feature detection

Goal: to reconstruct $S_x(\alpha,f)$ from samples $z[n]$ at sub-Nyquist rate $\frac{M}{N} f_s$

- Periodic autocorrelation and cyclic spectrum

$$R_x(t,\tau) \underset{\mathcal{F}^{-1}_{\alpha\to t}}{\overset{\mathcal{F}_{t\to\alpha}}{\rightleftarrows}} R_x(\alpha,\tau) \underset{\mathcal{F}^{-1}_{f\to\tau}}{\overset{\mathcal{F}_{\tau\to f}}{\rightleftarrows}} S_x(\alpha,f)$$

Discrete-time $\downarrow (t,\tau)\to(n,\nu)$ $\qquad$ $(\alpha,f)\to(a,b)\downarrow$

$$\mathbf{R}_x : \left[r_x(n,\nu)\right]_{N\times N} \overset{\text{DFT in } n \text{ and } \nu}{\rightleftarrows} \mathbf{S}_x^{(c)} : \left[s_x^{(c)}(a,b)\right]_{N\times N}$$

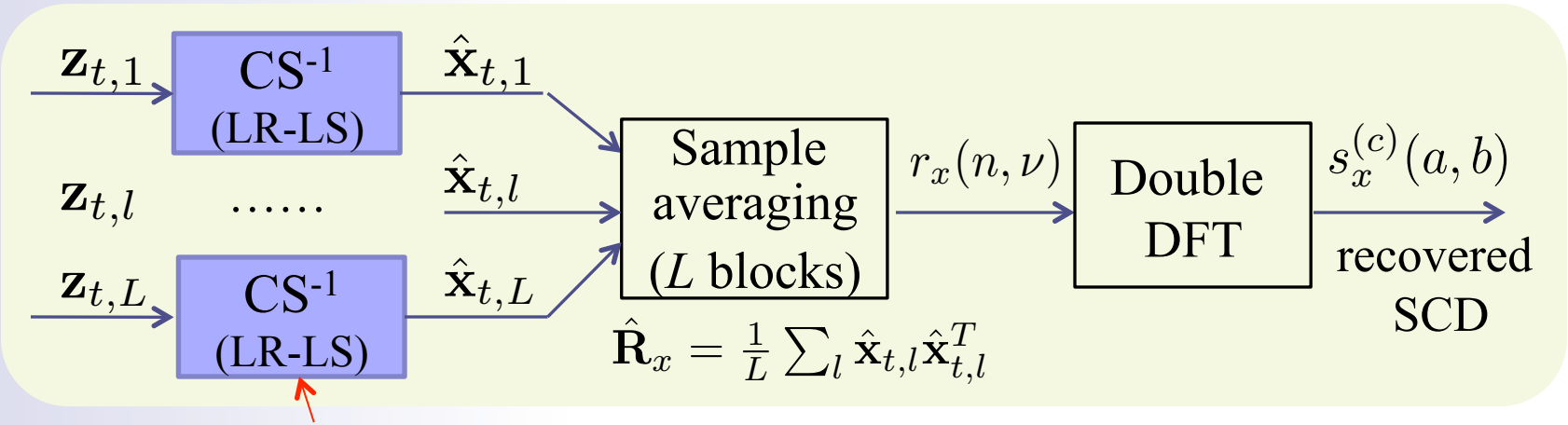
$$\mathbf{R}_x \approx \frac{1}{L}\sum_{l=1}^{L} \mathbf{x}_{t,l}\mathbf{x}_{t,l}^T$$

- Compressive samples over L blocks: $\mathbf{z}_{t,l} = \mathbf{A}\mathbf{x}_{t,l}, \quad l = 1:L$

✘ 2D cyclic spectrum is **not linear** in the time-domain samples
→ CS framework not immediately applicable

Compressive signal sensing

- Traditional CS applied to spectrum estimation for random processes



Sparsity constraint

- Recovering original signals via CS is over-kill

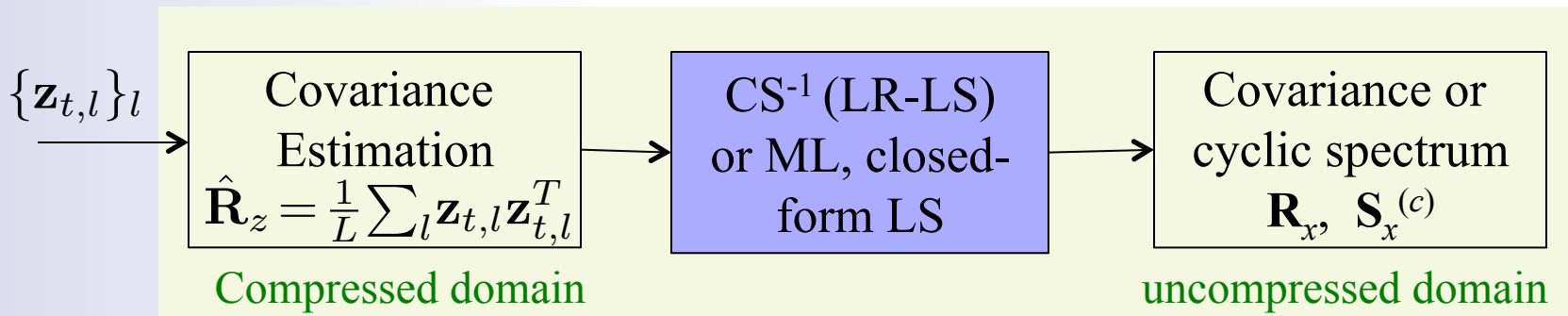
  ✗ high computation cost ✗ slow convergence/sensing time

  ✗ sampling requirements on signal rather than covariance

Compressive covariance sensing

Observation: many applications just require second-order statistics (power spectrum) → bypass recovering signal itself

- Key: identify linear relationship among 2nd-order statistics
  - Covariance of compressed samples
  - Covariance and cyclic spectrum of original signal



✔ reduced computational load ✔ fast convergence

✔ can incorporate structural knowledge of covariance/spectrum

Z. Tian, Y. Tafesse, and B. M. Sadler, "Cyclic feature detection from sub-Nyquist samplers for wideband spectrum sensing," *IEEE Journal of Selected Topics in Signal Processing* (JSTSP), Feb. 2012.

Vector-form relationship (1)

- Linking covariance matrix with cyclic spectrum
  - Covariance matrix: $\mathbf{R}_x = \mathbf{E}\{\mathbf{x}_t \mathbf{x}_t^T\}$

$$\mathbf{R}_x = \begin{bmatrix} r_x(0,0) & r_x(0,1) & r_x(0,2) & \cdots & r_x(0,N-1) \\ r_x(0,1) & r_x(1,0) & r_x(1,1) & \cdots & r_x(1,N-2) \\ r_x(0,2) & r_x(1,1) & r_x(2,0) & \cdots & r_x(2,N-3) \\ \vdots & & & \ddots & \vdots \\ r_x(0,N-1) & \cdots & \cdots & \cdots & r_x(N-1,0) \end{bmatrix}$$

  - Degrees of freedom: $N(N+1)/2$

$$\mathbf{r}_x = [r_x(0,0), r_x(1,0), \cdots, r_x(N-1,0), r_x(0,1), r_x(1,1), \cdots, r_x(N-2,1), \cdots\cdots, r_x(0,N-1)]^T \in \mathcal{R}^{\frac{N(N+1)}{2}}.$$

  - Vectorized cyclic spectrum

$$\mathbf{s}_x^{(c)} = \mathrm{vec}\{\mathbf{S}_x^{(c)}\} = \underbrace{(\mathbf{I} \otimes \mathbf{F}) \sum_{\nu=0}^{N-1} (\mathbf{D}_\nu^T \otimes \mathbf{G}_\nu) \mathbf{B}^T}_{:=\mathbf{T}} \mathbf{r}_x$$

Vector-form relationship (2)

- Linking covariance matrices
  - TV covariance of compressed data $\mathbf{R}_z = \mathrm{E}\{\mathbf{z}_t\mathbf{z}_t^T\} \in \mathcal{R}^{M\times M}$
    - Finite-sample estimate: $\hat{\mathbf{R}}_z = \frac{1}{L}\sum_l \mathbf{z}_{t,l}\mathbf{z}_{t,l}^T$
  - Degrees of freedom: $M(M+1)/2$

$$\mathbf{r}_z = [r_z(0,0), r_z(1,0), \cdots, r_z(M-1,0), r_x(0,1), r_z(1,1), \cdots, r_z(M-2,1), \cdots\cdots, r_z(0,M-1)]^T.$$

  - Relationship: $\mathbf{z}_t = \mathbf{A}\mathbf{x}_t \longrightarrow \mathbf{R}_z = \mathbf{A}\mathbf{R}_x\mathbf{A}^T$
- Linear representation for compressed covariance

$$\mathbf{r}_z = \mathbf{Q}_M \mathrm{vec}\{\mathbf{A}\mathbf{R}_x\mathbf{A}^T\} = \mathbf{Q}_M(\mathbf{A}\otimes\mathbf{A})\mathrm{vec}\{\mathbf{R}_x\} = \mathbf{\Phi}\mathbf{r}_x$$

$\frac{M(M+1)}{2}\times 1$ $\qquad$ $\frac{N(N+1)}{2}\times 1$

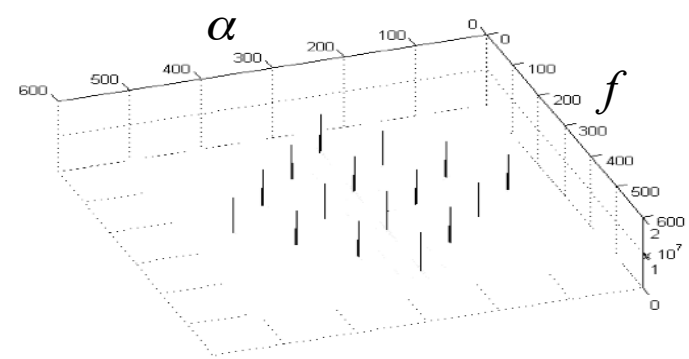
Sparse cyclic spectrum recovery

- Reformulated linear relationship

$$\mathbf{r}_z = \mathbf{\Phi}\mathbf{r}_x \qquad\qquad \mathbf{s}_x^{(c)} = \mathbf{T}\mathbf{r}_x$$

  - $\mathbf{\Phi} : \frac{M(M+1)}{2} \times \frac{N(N+1)}{2}$ fat matrix

- Prior Information
  - $\mathbf{s}_x^{(c)}$ is highly sparse
  - $\mathbf{R}_x$ is positive semi-definite (psd)

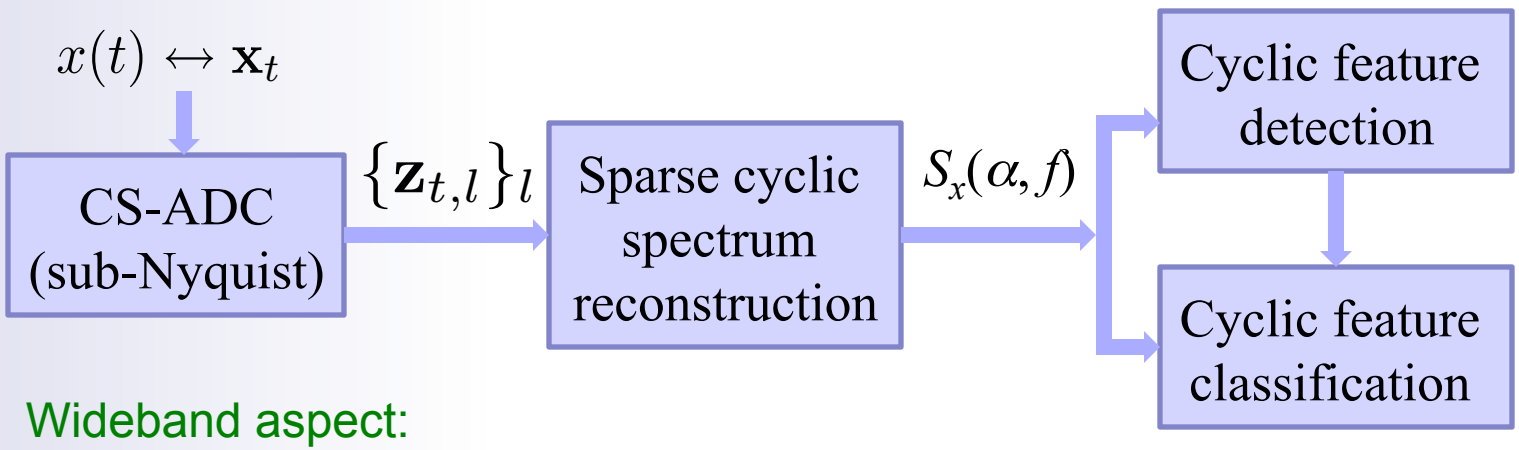


- L_1-norm regularized LS (LR-LS)

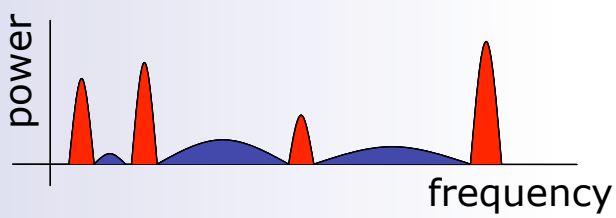
$$\boxed{\begin{aligned} \min_{\mathbf{r}_x} \quad & \|\mathbf{T}\mathbf{r}_x\|_1 + \lambda \|\mathbf{r}_z - \mathbf{\Phi}\mathbf{r}_x\|_2^2 \\ s.t. \quad & \mathbf{R}_x \text{ is psd, with } \mathrm{vec}\{\mathbf{R}_x\} = \mathbf{P}_N \mathbf{r}_x. \end{aligned}}$$

Convex!

$$\boxed{\min_{\mathbf{s}_x^{(c)}} \left\|\mathbf{s}_x^{(c)}\right\|_1 + \lambda \left\|\mathbf{r}_z - \mathbf{\Phi}\mathbf{T}^{-1}\mathbf{s}_x^{(c)}\right\|_2^2}$$



Wideband aspect:
multiple signal sources



Spectrum occupancy estimation

- Band-by-band estimation

Is $f^{(n)}$ occupied or not?

$$f^{(n)} = \frac{n - \frac{N-1}{2}}{N} f_s \in \left[-\frac{f_s}{2}, \frac{f_s}{2}\right]$$

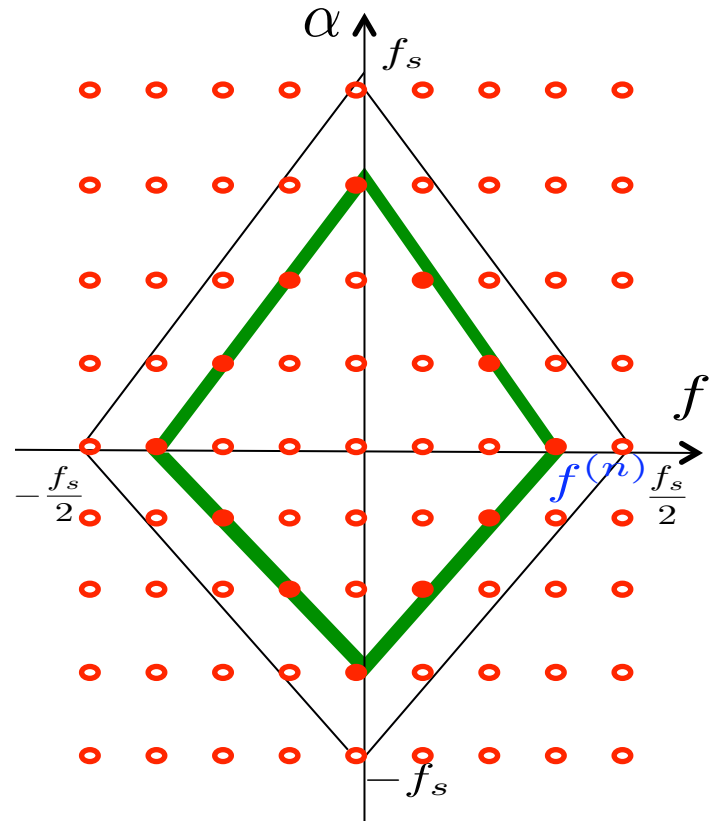
- Region of relevance to $f^{(n)}$

$$(\alpha, f) : \begin{cases} f + \dfrac{\alpha}{2} = f^{(n)} \\ |f| + \dfrac{|\alpha|}{2} \leq f_{\max} \end{cases}$$

$$(a_i, b_i) : \begin{cases} b_i + \dfrac{a_i}{2} = n \\ \left|b_i - \frac{N-1}{2}\right| + \dfrac{|a_i|}{2} \leq \dfrac{f_{\max} N}{f_s} \leq \dfrac{N}{2} \end{cases}$$

- Relevant SCD vector for band n

$$\hat{\mathbf{c}}^{(n)} : \left\{ \hat{s}_x^{(c)}(a_i, b_i) \right\}_i$$



SCD: $\mathbf{S}_x^{(c)}$

Multi-cycle GLRT

- Binary hypothesis test on band *n*

$$\begin{cases} H_1: & \hat{\mathbf{c}}^{(n)} = \mathbf{c}^{(n)} + \epsilon \\ H_0: & \hat{\mathbf{c}}^{(n)} = \epsilon \end{cases}$$

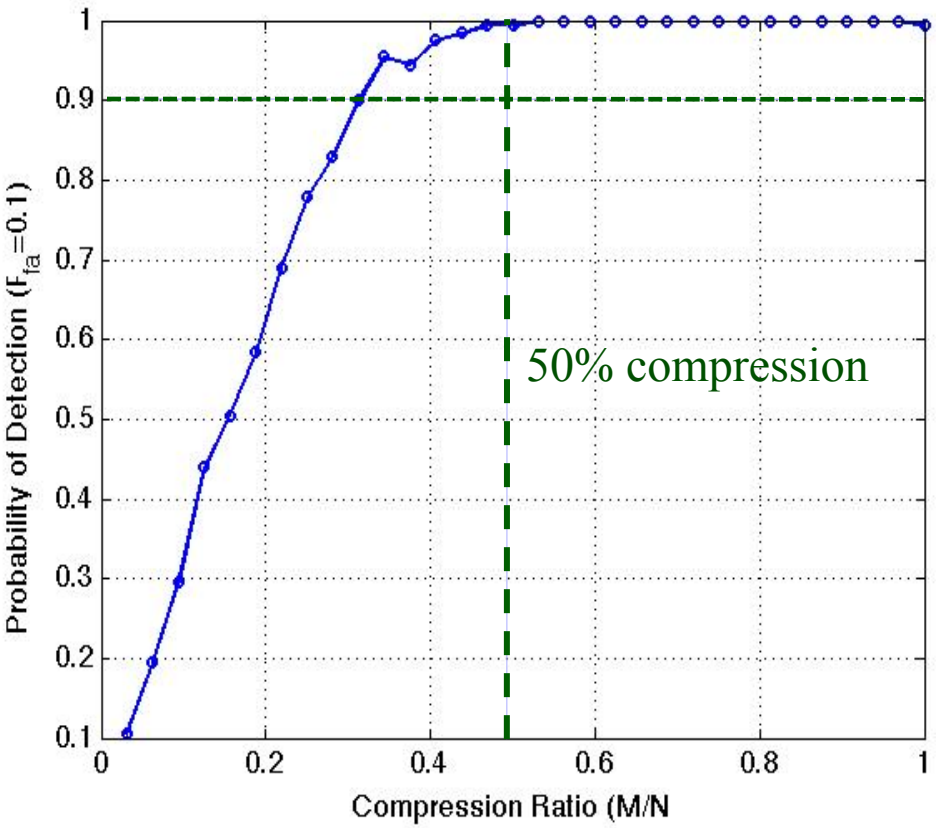
  - $\mathbf{c}^{(n)}$: $\left\{s_x^{(c)}(a_i, b_i)\right\}_i$: unknown true SCD; multiple cyclic freq.
  - $\epsilon : \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_\epsilon)$: noise statistics determined mainly by finite-sample effects, not ambient noise

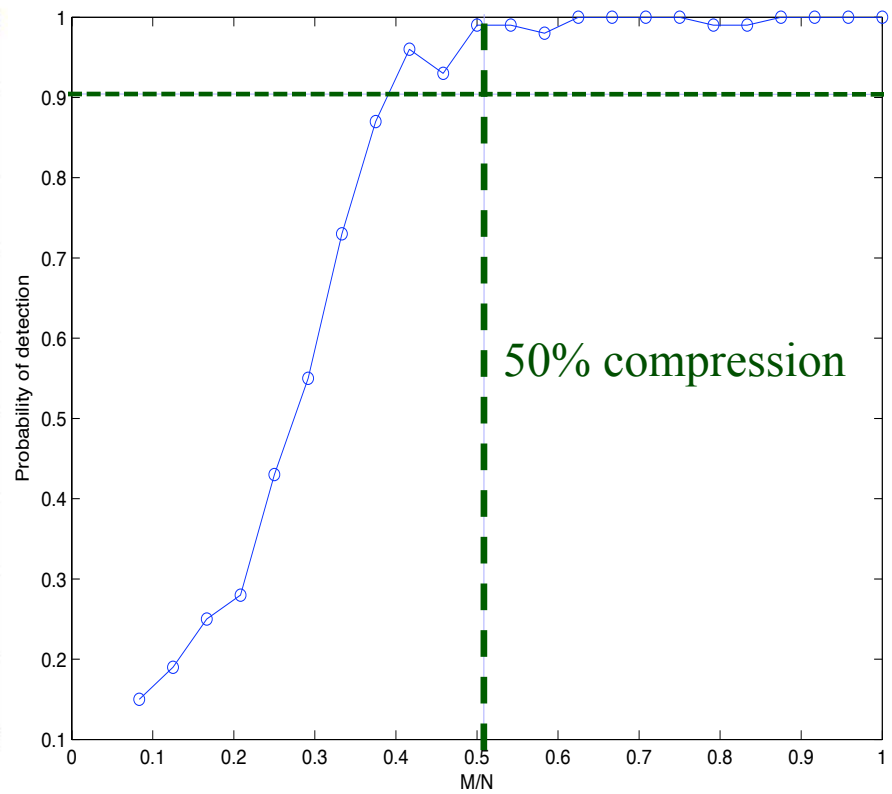
- GLRT formulation
  - Test statistics: $\mathcal{T}^{(n)} = (\hat{\mathbf{c}}^{(n)})^H \mathbf{\Sigma}_\epsilon{}^{-1} \hat{\mathbf{c}}^{(n)}$
  - Binary decisions by thresholding

- A single wideband DSP, as opposed to multiple NB filters

Simulation: Robustness to rate reduction

Probability of detection vs. compression ratio (P_{FA}= 0.1, N=32, L=200 blocks)

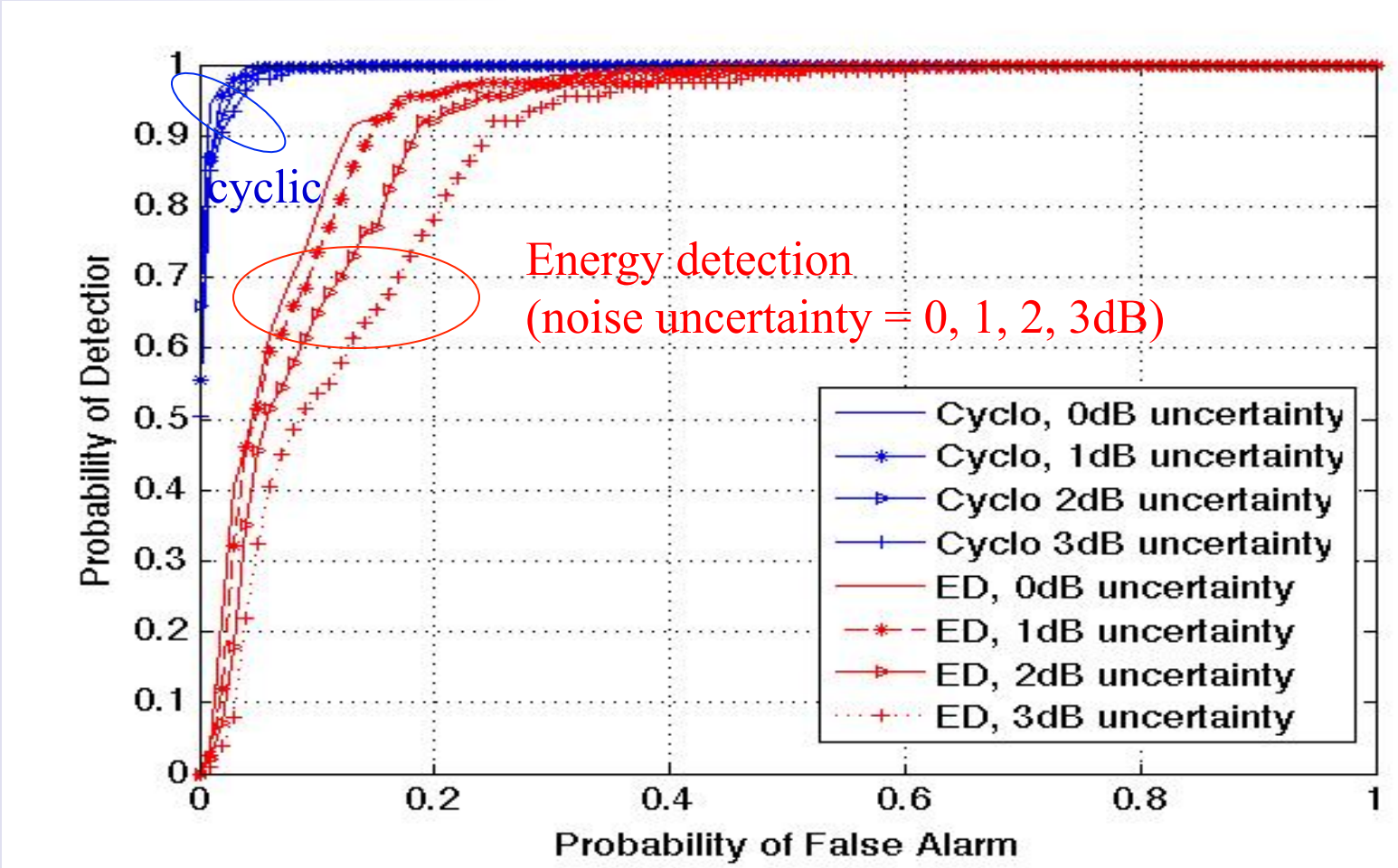




- Monitored band $|f_{max}| < 300$ MHz
- 2 sources (noise-free): PU_1 - BPSK at 150MHz; PU_2 - QPSK at 225MHz; Ts=0.02667μs
- Cisco 802.11 DSSS Spread spectrum

Simulation: Robustness to noise uncertainty

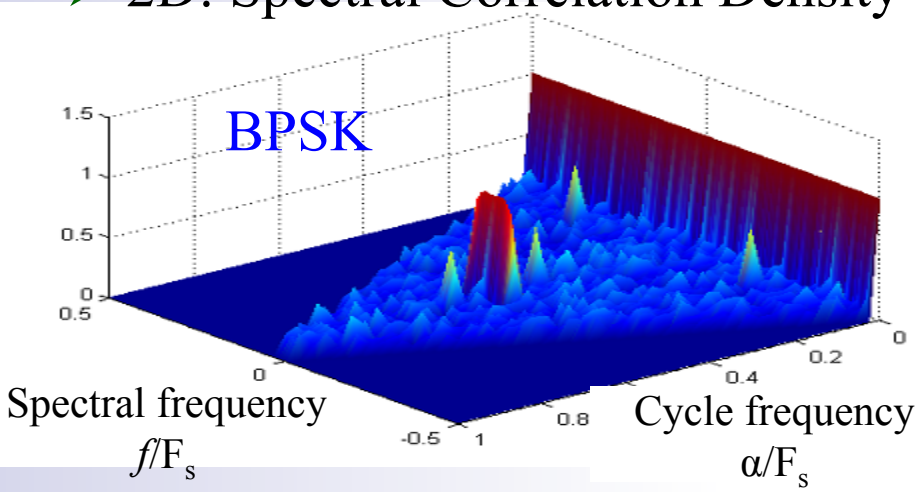
Receiver Operating Characteristic (ROC): P_D vs P_{FA} (SNR=5dB, 50% compression)

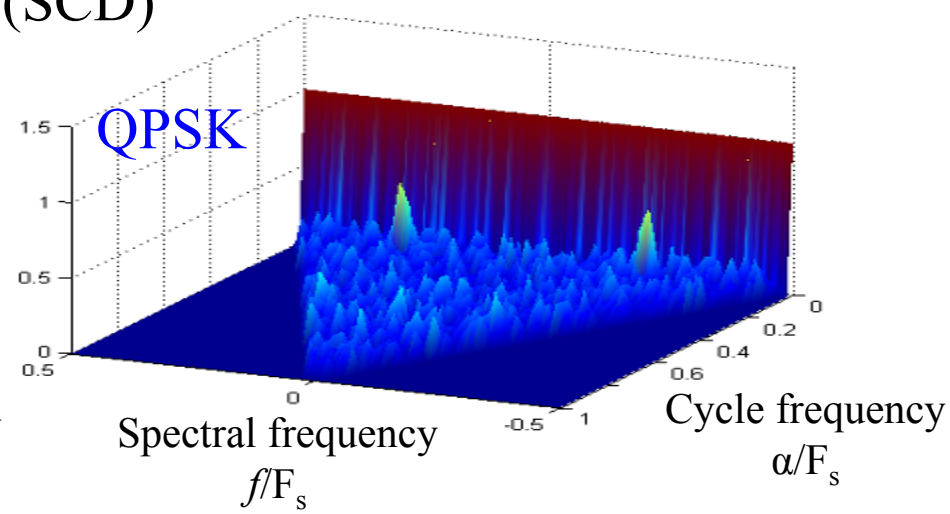


- outperforms energy detection
- insensitive to noise uncertainty

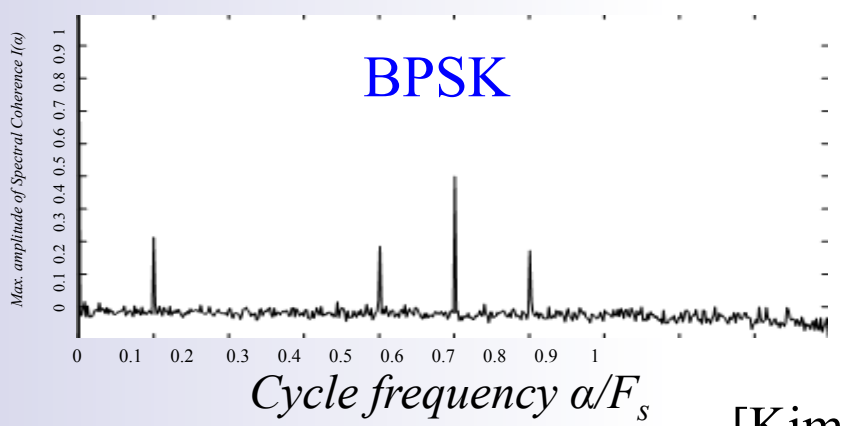
Classification using cyclic statistics

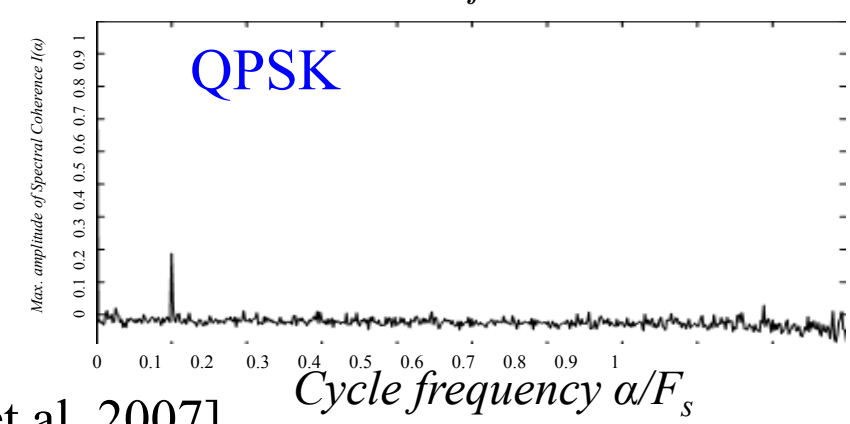
- 2D: Spectral Correlation Density (SCD)





- 1D: cyclic-frequency domain profile (CDP) $I(\alpha) \doteq \max_f |S_x(\alpha, f)|$





[Kim et al. 2007]

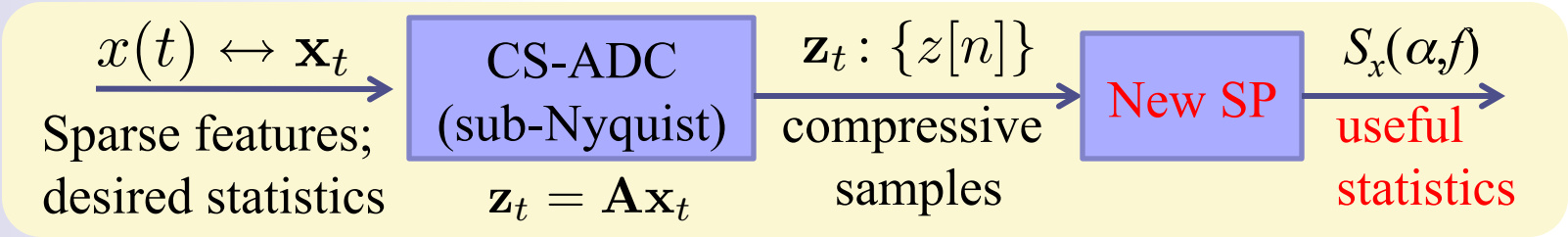
Simulations: Classification

Confusion matrix (SVM classifier)

	BPSK	QPSK	DS-BPSK	DS-QPSK
BPSK	95.45%	0%	4.55%	0%
QPSK	0%	90.9%	9.09%	0%
DS-BPSK	9.09%	0%	59.09%	31.82%
DS-QPSK	4.5%	4.5%	36.46%	54.54%

- When compression ratio is adequate for detection, classification accuracy is comparable to non-compression
- Good separation of narrowband from spread spectrum
- Considerable confusion among spread spectrum signals

Summary: Compressive covariance sensing (CCS)



- Key is to directly extract useful (2nd-order) statistics, which has less degrees of freedom than the random signal itself
  - ✔ strong compression allowed
  - ✔ low computational load, bypassing signal recovery
  - ✔ fast convergence, short sensing time

Q: Does the framework generalize? Do we really need sparsity?

- ✔ enables compression for non-sparse signals
- ✔ permits close-form solution; easy to analyze performance

Non-sparse stationary signals

- Stationary processes as a special case of cyclostationary ones

  [Tian et al., JSTSP'2012; Leus et al., SPL'2011, TSP'2012]

  - 2D cyclic spectrum reduces to 1D power spectrum

$$r_x(n,\nu) = \bar{r}_x(\nu), \quad \forall n \qquad \bar{\mathbf{r}}_x \xleftrightarrow{\mathcal{F}} \bar{\mathbf{s}}_f \text{ (PSD)}$$

$$\mathbf{R}_x = \begin{bmatrix} \bar{r}_x(0) & \bar{r}_x(1) & \bar{r}_x(2) & \cdots & \bar{r}_x(N-1) \\ \bar{r}_x(1) & \bar{r}_x(0) & \bar{r}_x(1) & \cdots & \bar{r}_x(N-2) \\ \bar{r}_x(2) & \bar{r}_x(1) & \bar{r}_x(0) & \cdots & \bar{r}_x(N-3) \\ \vdots & & & \ddots & \vdots \\ \bar{r}_x(N-1) & \vdots & \vdots & \vdots & \bar{r}_x(0) \end{bmatrix}$$

Toeplitz

  - # measurements $\mathbf{r}_z$ from cross-correlations in $\mathbf{R}_z$: $M(M+1)/2$
  - # unknowns $\{r_x(\nu)\}$ in power spectrum from $\mathbf{R}_x$: N

Compressive covariance/PSD estimation

$$\frac{M(M+1)}{2} \times 1 \quad \boxed{\mathbf{r}_z = \bar{\mathbf{\Phi}}\bar{\mathbf{r}}_x = \bar{\mathbf{\Phi}}\mathbf{F}^{-1}\bar{\mathbf{s}}_f} \quad N$$

- Minimum sampling rates for non-sparse signals [Tian et al. JSTSP'12]

> Proposition: When the compression ratio satisfies $M(M+1)/2 \geq N$, lossless recovery of power spectrum is possible via closed-form least-squares solution, even in the absence of signal sparsity.

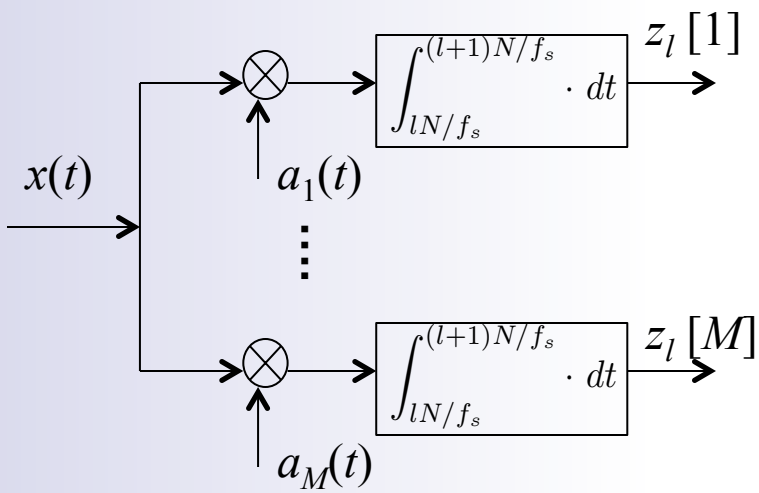
$$\left(\frac{M}{N}\right)_{\min} = \frac{\sqrt{8N+1}-1}{2N} \xrightarrow{N\to\infty} \sqrt{\frac{2}{N}} \ll 1$$

- Stronger compression allowed for sparse signals

Sampler design

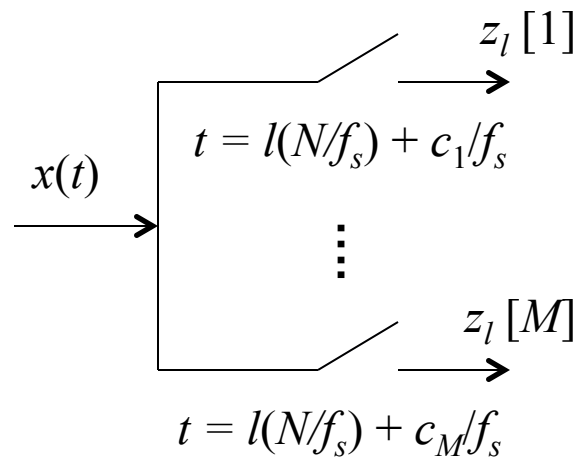
- Periodic sampling devices

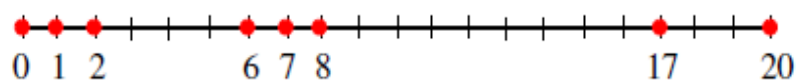
$$z_i = \int a_i(t)x(t)dt, \quad i = 1:M$$



- Multi-coset sampling

$$a_i(t) = \delta(t - c_i/f_s)$$

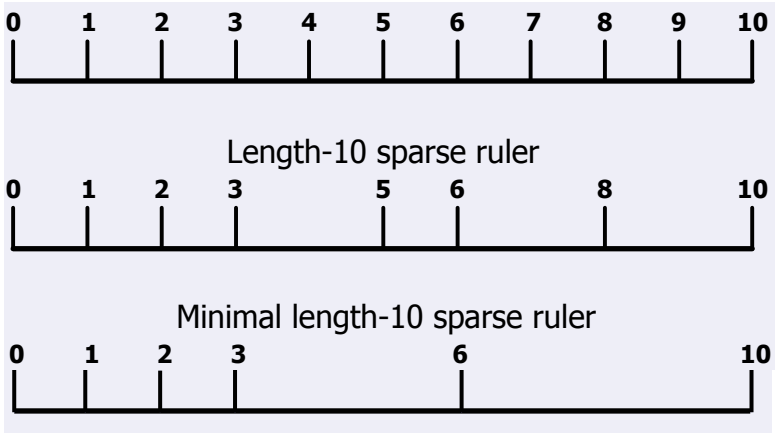


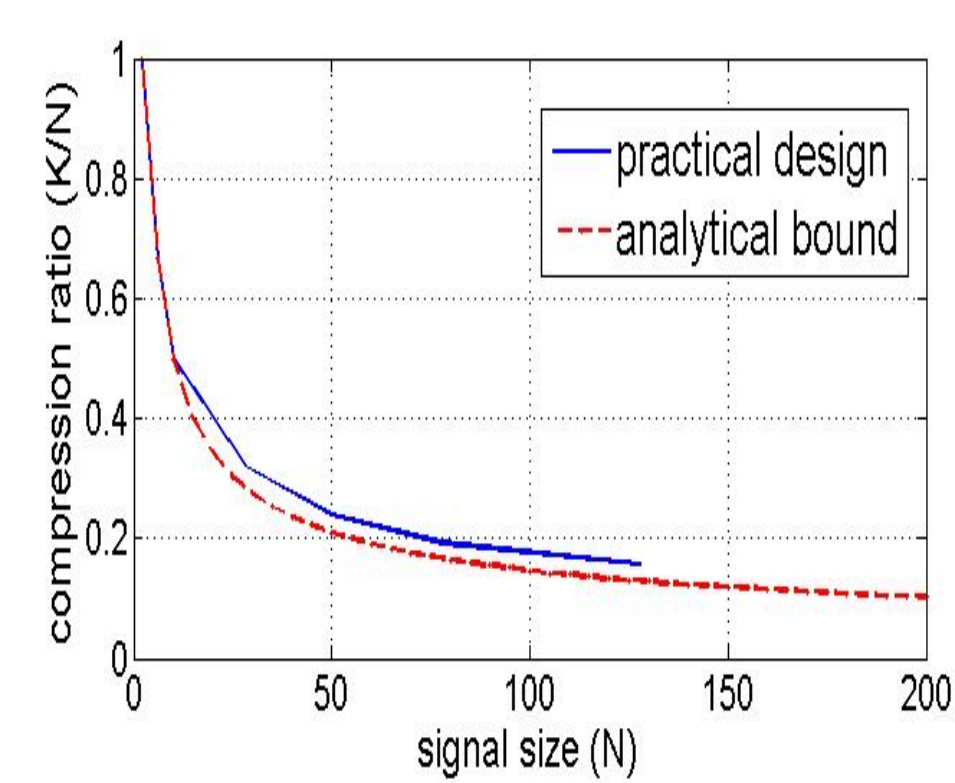


Ariananda, Leus, Tian, "Multi-coset sampling for power spectrum blind sensing," *DSP Conf.*, July 2011.

Sampler design for covariance estimation

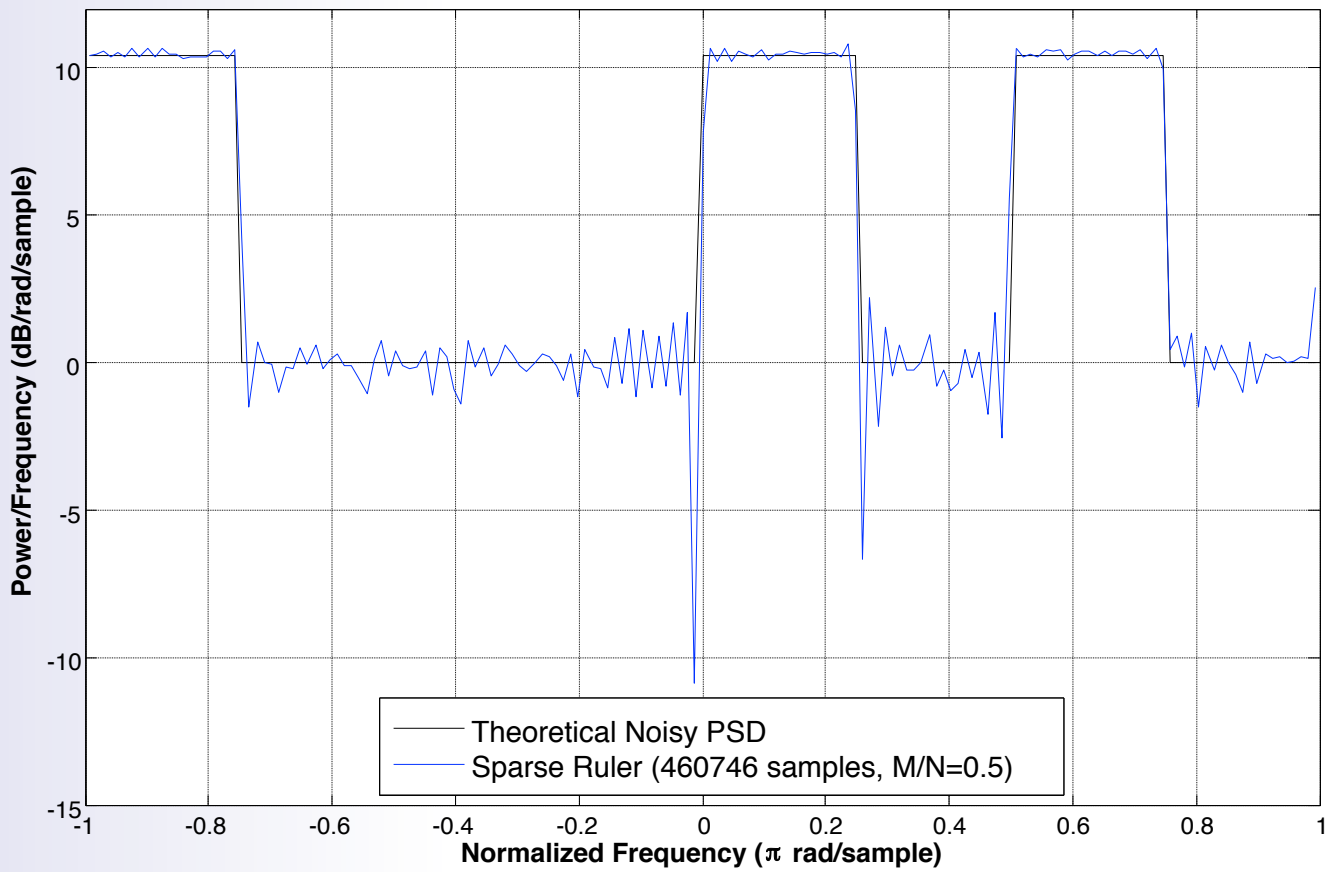
- Minimal complete sparse rulers
  [Moffet'68] [Ariananda-Leus-Tian'12]





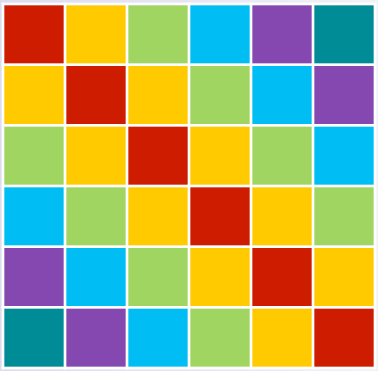
N	Compression (M/N)
6	0.667
10	0.5
20	0.4
50	0.24
78	0.1923
128	0.1563
$N \rightarrow \infty$	$\sqrt{2/N}$

Simulations: sparse ruler



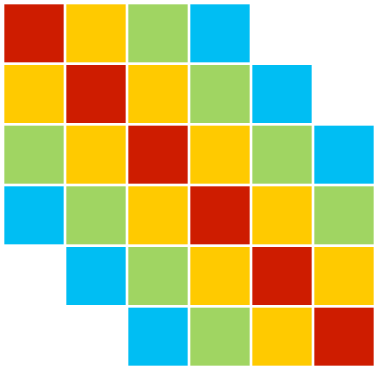
Reconstructed PSD

Covariance structures



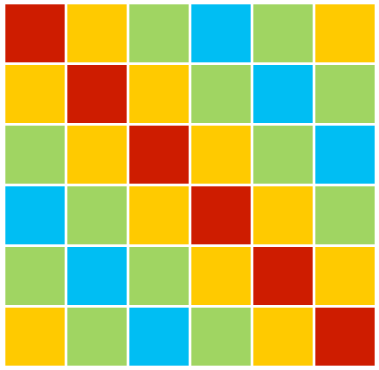
Toeplitz

$S = 2N-1$



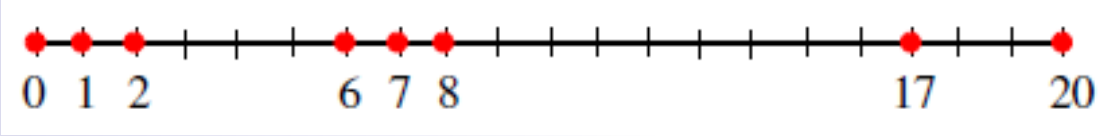
d-Banded:

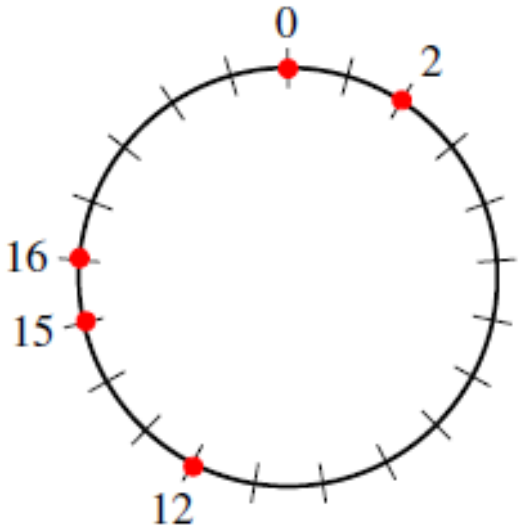
$S = 2d-1$



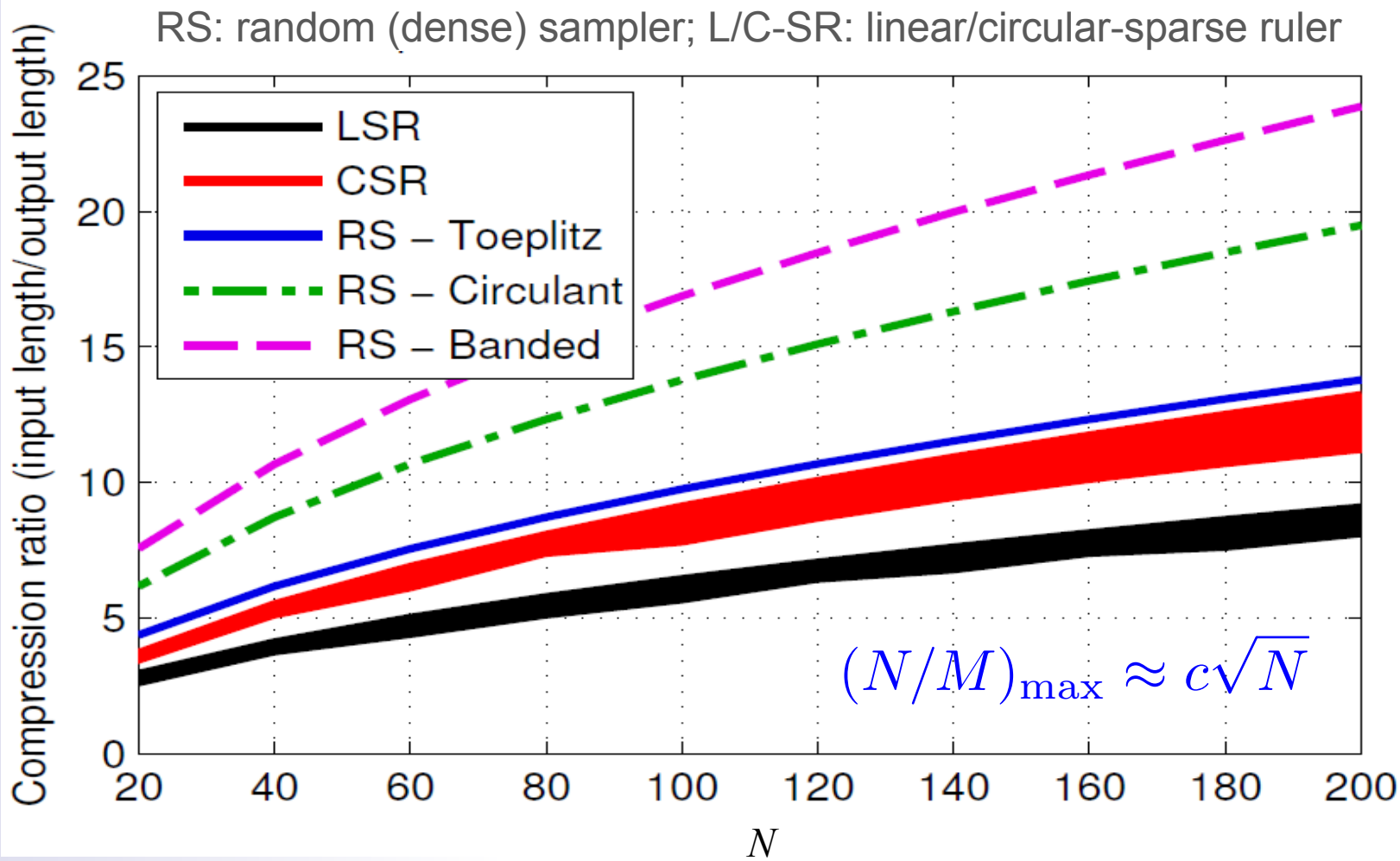
Circulant:

$S = N$





Compression limits



D. Romero, D. Ariananda, Z. Tian, and G. Leus, "Compressive covariance sensing", *IEEE Signal Processing Magazine*, 2015. (upcoming)

Results for cyclostationary signals

- Sampler design for reconstructing cyclic spectrum
  - Multiple samplers used periodically across K cyclic periods

Original signal $x[n]$	Structure of R_x	Compressed signal $z[n]$
Stationary	Toeplitz	Cyclostationary
Cyclostationary (period N)	?	

Proposition: it is possible to reconstruct non-sparse 2D cyclic spectrum in closed form when

$$\left(\frac{M}{N}\right)_{\min} = \sqrt{\frac{1}{K}} \leq 1$$

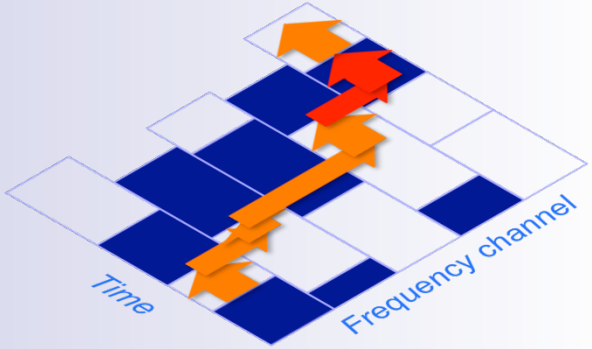
G. Leus and Z. Tian, "Recovering 2nd-order statistics from compressive measurements," *CAMSAP Conf.*, 2011.

Summary

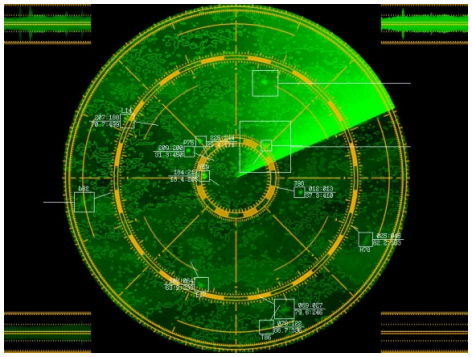
- Exploitation of signal sparsity in 2D cyclic spectrum domain
  - reformulated linear relationship between spectrum and covariance
  - robustness to noise uncertainty
  - capability in signal separation and classification
- Compression and recovery of 2nd-order statistics for (non-sparse) random processes
- Direct reconstruction of test statistics from compressive samples
  - bypass the recovery of the entire cyclic spectrum
  - reduce sensing time and responsiveness
- Sensing techniques for spread spectrum signals
  - utilize fusion techniques to take advantage of all features in the wide spectrum of DSSS signals

Other applications

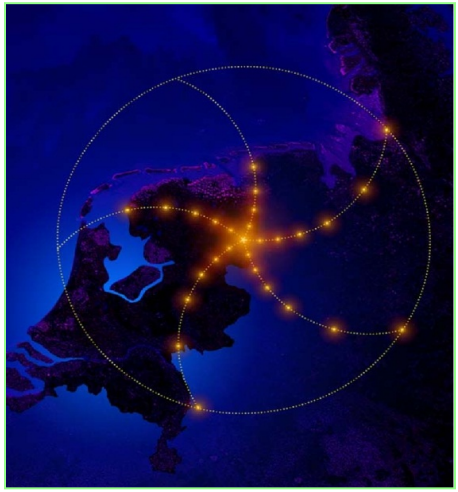
Compressive Covariance Sensing (CCS)



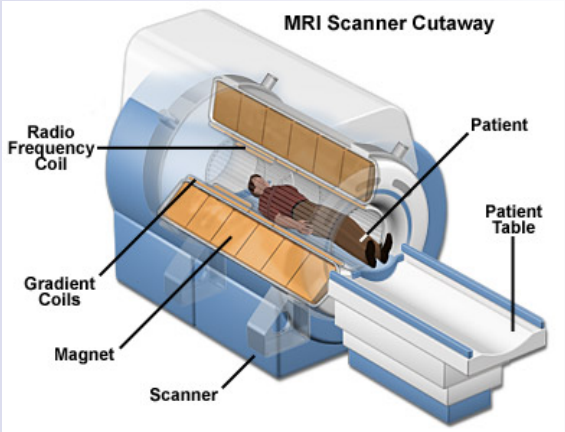
Cognitive radio (CR)
frequency spectrum



Radar
Doppler + angular spectra



Radio astronomy
spatial spectrum



MRI
resonance spectrum

Seismic
seismic design response spectrum

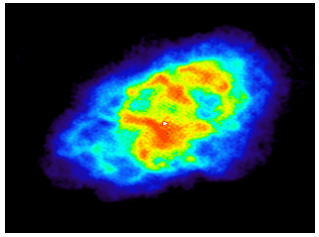
Outlook

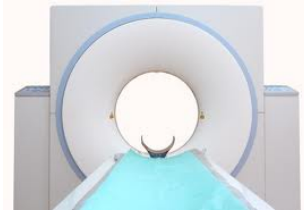
- Super resolution
- Big data
- Combining spatial and temporal domains
- Extensions to Doppler spectrum, imaging, …
- Applications to radar, MRI, seismic, radio astronomy, …

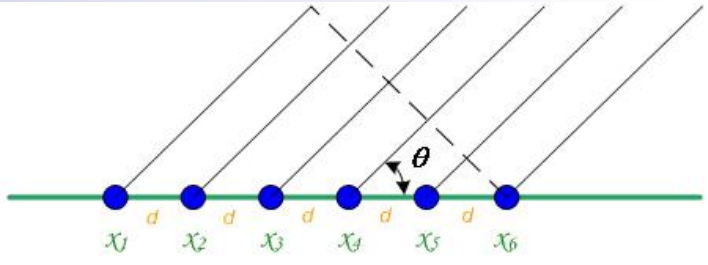
Outlook: angular spectrum estimation

- Array processing
  - Imaging
    - Optical/radar/ultrasound/acoustic
    - Radio astronomy
    - Seismology
  - DoA estimation → localization
- MIMO Radar
- Massive MIMO for 5G

Source: IAI Inc.



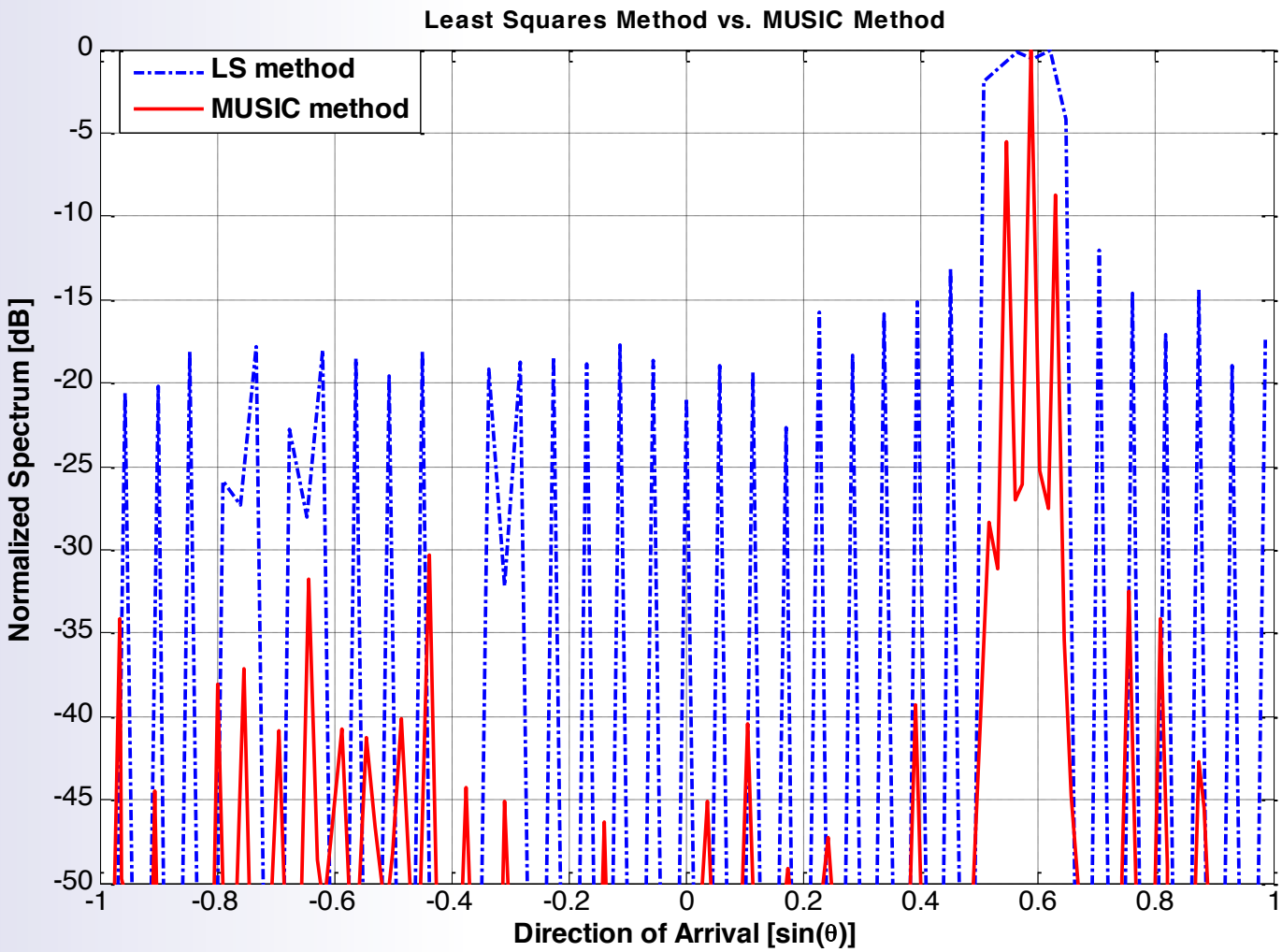




Acquisition and processing hardware prop. to # antenna elements

Challenge: reduce number of antennas w/o sacrificing spatial/angular resolution

Simulations: DoA estimation



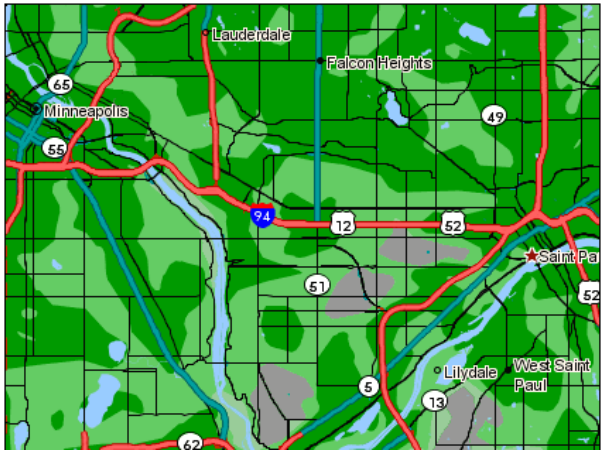
Reconstructed spectrum using LS and MUSIC for the minimal sparse ruler array (continuous source from 30 to 40 degrees; for LS S=71; ULA N=36; M=10; SNR = 0dB)

Outlook: spectrum cartography

- Idea: CRs collaborate to form a spatial map of the spectrum

given the PSD $\Phi_r(f) = \Phi(f; v_r)$ at position v_r, find $\Phi(v, f)$, $\forall v$

- Goal: $\Phi(v, f), \quad \forall v$
- Specifications: coarse approx. suffices
- Approach: basis expansion of $\Phi(v, f)$
- Compressive Sampling (CS) possible to form the PSD data



J.-A. Bazerque, and G. B. Giannakis, ``Distributed Spectrum Sensing for Cognitive Radio Networks by Exploiting Sparsity," *IEEE Trans. on Signal Proc.*, March 2010.

Modeling (space-frequency)

- Transmitters

  $\mathsf{Tx}_s, \ s = 1, \ldots, N_s$

- Sensing CRs

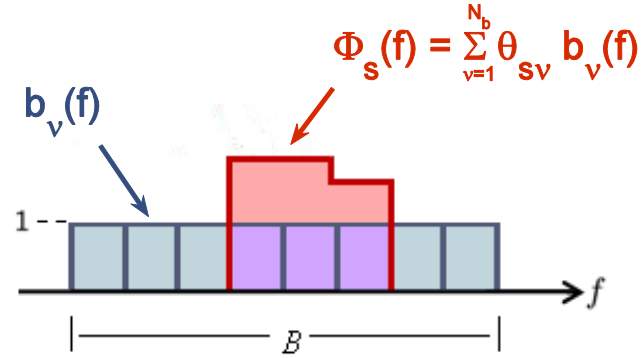
  $\mathsf{CR}_r, \ r = 1 : N_r$

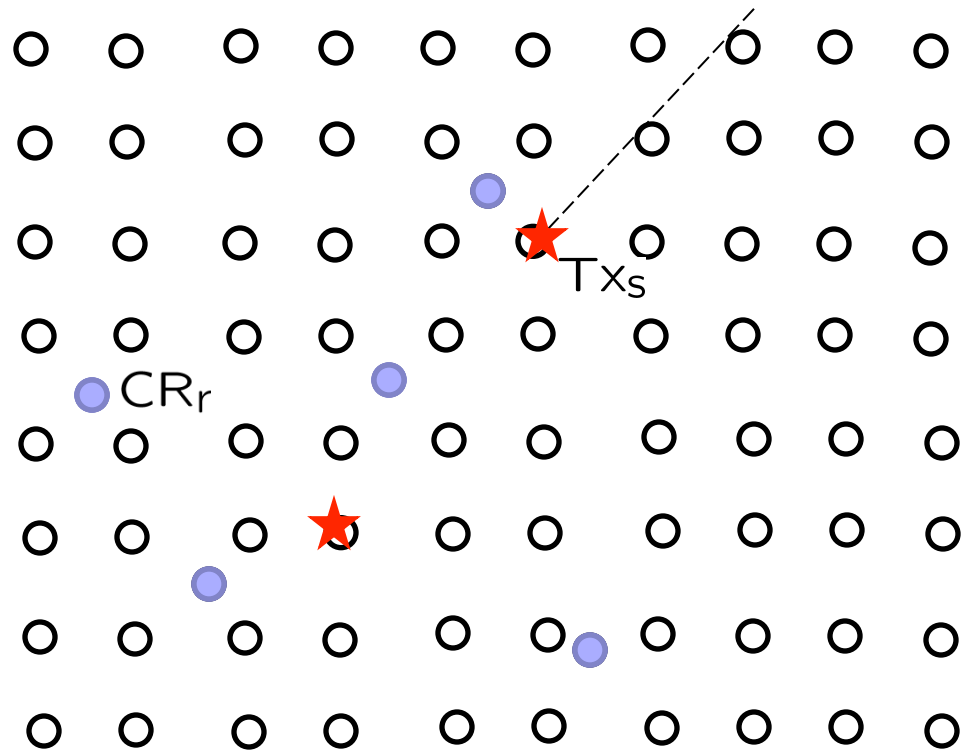
- Frequency bases

  $b_\nu(f), \ \nu = 1 : N_b$

- Sensed frequencies

  $f_k, \ k = 1 : K$

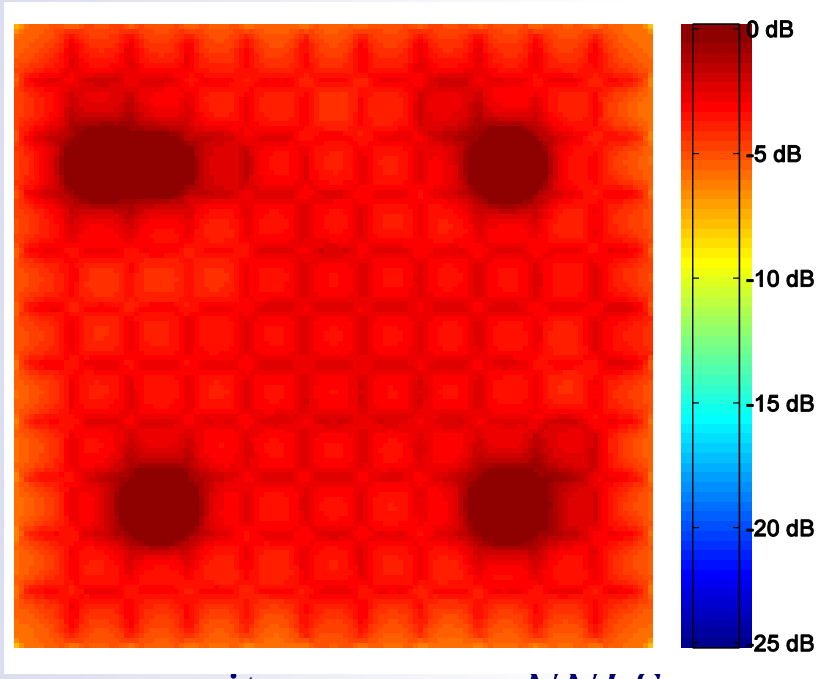




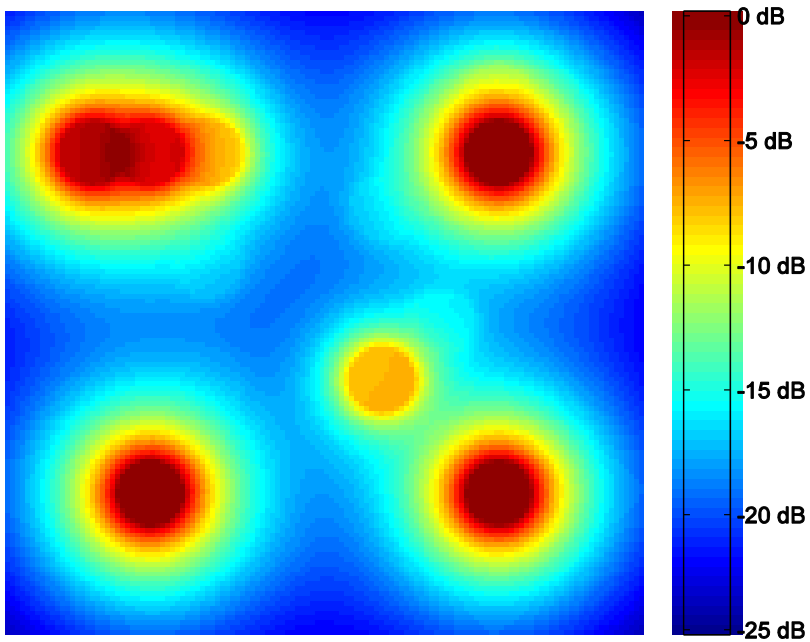
Sparsity present in space and frequency

Power spectrum cartography

- 5 sources
- N_s = 121 candidate locations, N_r = 50 CRs



sparsity-unaware NNLS



sparsity-aware LR-LS

- Sparsity-unaware NNLS is prone to false alarms
- As a byproduct, LR-LS localizes all sources via variable selection for example, **Jammer Localization**

Outlook: correlation mining

- Large-scale networks

Internet backbone network (Abilene)

Disease gene network

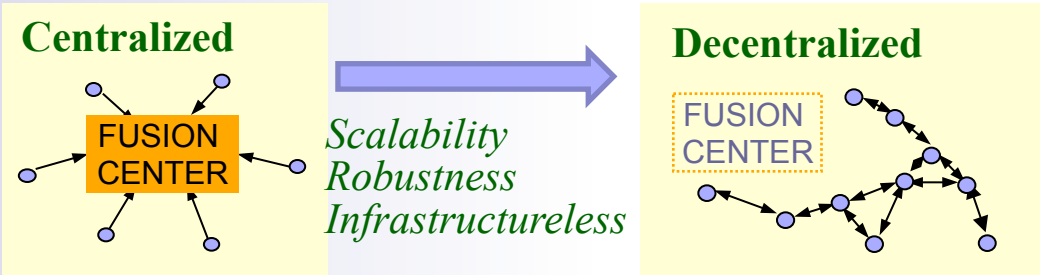
Sample size issue → sparsity-enforcing regularization

Outlook: monitoring in large-scale networks

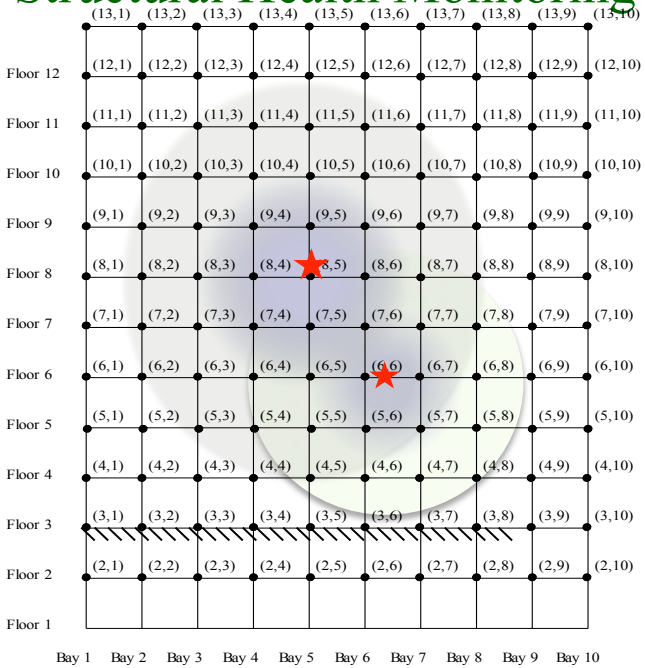
- Sparse event detection using large-scale wireless sensor nets
  - Local phenomena induce spatially sparse signals
  - Desiderata: energy efficiency, scalability, robustness

- Distributed optimization

  [Ling-Tian, 2010, 2011]



Structural Health Monitoring



[Ling-Tian et al, 2009]

Summary: CS vs. CCS

Compressed Covariance Sensing	Compressed Sensing
Aims at recovering statistics	Aims at recovering the signal
Lossy	Lossless
Use sparse sampling	
Use random sampling	
No **sparsity** is required	**Sparsity** is required
Linear/non-linear reconstruction	Non-linear reconstruction
Overdetermined	Underdetermined

Thank you!