Compressive Wideband Spectrum Sensing for Wireless Cognitive Radios

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Outline

- Context: covariance-spectrum estimation and applications
- Wireless cognitive radio (CR) access
  - Compressive cyclic feature detection for wideband sensing
- Foundation: compressive covariance sensing (CCS)
  - Sub-Nyquist-rate compressive sampling for non-sparse signals
- Research outlook
Covariance and spectrum estimation

Cognitive radio (CR) frequency spectrum

Radar Doppler + angular spectra

Radio astronomy spatial spectrum

MRI resonance spectrum

Seismic seismic design response spectrum
Emerging Challenges

(Ultra-)wideband signals

Impulse radio

Very large arrays

Large Arrays

Large-scale networks

Internet backbone network

Cognitive radio (CR)

Massive MIMO

Disease gene network

Sampling rate/size issue → Need for compressive techniques
Goal: overcome (perceived) bandwidth scarcity
- Spectrum hole = opportunities
- Opportunistic spectrum access

Technical challenges
- Find holes in wideband spectrum
- Allocate spectrum resources dynamically
- Adjust transmit-waveforms
Challenge 1: Wideband signal acquisition

- RF circuit choices: *multiple NB or single WB*?

**Multiple narrowband (NB) circuits**
- Multiple, fixed RF chains
- Preset LO filter range
- Simple detection per BPF

**Wideband (WB) circuit**
- Single, flexible RF chain
- Burden on A/D: $f_s \sim$ GHz
- Complex wideband sensing

**Q:** How can we alleviate DSP burden on wideband circuit design?

**A:** Adopt a compressive sampling (CS) framework
## Challenge 2: User hierarchy

"IEEE 802.22 requires CRs to sense PU signals as low as -114dBm"

<table>
<thead>
<tr>
<th>Operating Conditions</th>
<th>Technical Challenges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Protection of primary systems</td>
<td>Sensing at low SNR</td>
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<tr>
<td></td>
<td>Short sensing time</td>
</tr>
<tr>
<td></td>
<td>Modulation classification</td>
</tr>
<tr>
<td>Random sources of interference and noise</td>
<td>Robustness to noise uncertainty</td>
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<tr>
<td></td>
<td>Interference identification</td>
</tr>
</tbody>
</table>

**Q:** How can we alleviate noise uncertainty effects at low SNR?

**A:** Cyclic feature based spectrum sensing
Compressive sampling (CS) [Chen-Donoho-Saunders’98], [Candès et al’04-06]

- Given $z$ and $A$, unknown $x$ can be found when (a1) and (a2) hold

(a1) $x$ is sparse (nonzero entries unknown)

(a2) $A$ fat ($M/N \leq 1$); satisfies restricted isometry property (RIP)

Sparse regression $z = A \cdot x + w$ [Tibshirani’96], [Tipping’01]

- Efficient inverse solution ($CS^{-1}$): $L_1$-norm regularized least-squares (LR-LS)

$$
\hat{x} = \arg \min_{x} \frac{1}{2} \|z - Ax\|_2^2 + \lambda \|x\|_1
$$
Compressive ADC (CS-ADC)

- Periodic analog sampling devices
  [Herley-Wong, 1999], [Venk-Bresler, 2000];
  [Tropp et al, 2010]; [Mishali-Eldar, 2010];
  [Becker, 2011]; [Yoo et al, 2012];
  AIC [Kirolos et al'06], [Hoyos et al’08]

\[
z_i = \int a_i(t)x(t)dt, \quad i = 1 : M
\]

\[
z_l = Ax_l
\]
Example: Fourier measurements

Sampling basis: Time Domain

Sparsity basis: Frequency Domain

Measure $M$ Samples out of $N$ Nyquist samples

$$f(t) = \sum_{i=1}^{S} x_i e^{j\omega_i t} \quad t_1, \ldots, t_K$$

$S$ Nonzero Components

$$F(\omega) = \sum_{i=1}^{S} x_i \delta(\omega - \omega_i)$$
Example: Fourier measurements (cont’d)

- Performance (Prob. of inexact recovery) vs. # measurements $M$

- $(M; N, S)$: depends on *measurement matrix* and *recovery method*
- RIP conditions are sufficient rather than necessary
Early contribution: spectrum hole detection

Cyclostationary modulated signals

- Cyclic features reveal critical signal parameters:
  - carrier frequency
  - symbol rate
  - modulation type
  - timing, phase etc.

- Non-cyclic signals (e.g. noise) do not possess cycle frequencies

\[ R_x(t, \tau) = R_x(t + T_0, \tau), \forall \tau \]

Periodic autocorrelation

\[ S_x(\alpha, f) \]

Cyclic spectrum
Noise suppression in cyclic domain

Energy detection vs. cyclic feature detection
   e.g., [Sahai-Cabric’05]

- power spectrum density (PSD) $(\alpha = 0)$
- spectral correlation density (SCD)

High SNR:

![High SNR spectrum](image)

Low SNR:

![Low SNR spectrum](image)

- Multi-harmonics peaks at $\alpha = kf_m$
- $f$ ✔ no noise components when $\alpha \neq 0$
Source separation in cyclic domain

Spectral correlation density (SCD)
e.g., [Gardner’88]  

(b) BPSK signal alone

(a) BPSK in noise plus five AM interferers  
(c) White noise plus five AM interferences

✔ overlapping in PSD, separable in SCD

- Cyclostationarity-based approach to detection
  ✔ resilient against Gaussian noise  ✔ robust to multipath
  ✔ can differentiate modulation types and separate interference
  ✔ insensitive to unknown signal parameters
Cyclic feature detection for CR?

- **Major challenges**
  - Cyclostationarity typically induced by OVER-sampling → excessive sampling-rate requirements
  - Cyclic statistics converge slowly with finite samples → long sensing time

  Our contribution: Compressive sampling to the rescue …

- **Leverages sparsity in two dimensions**
  - Sparsity in frequency domain \((f)\)
    - low spectrum utilization
  - Sparsity in cyclic-freq. domain \((\alpha)\)
    - modulation-dependent cycles
Wideband cyclic feature detection

**Goal:** to reconstruct $S_x(\alpha, f)$ from samples $z[n]$ at sub-Nyquist rate $\frac{M}{N} f_s$

- Periodic autocorrelation and cyclic spectrum

$$R_x(t, \tau) \xrightarrow{\mathcal{F}_{t \rightarrow \alpha}} R_x(\alpha, \tau) \xrightarrow{\mathcal{F}_{\alpha \rightarrow t}^{-1}} S_x(\alpha, f)$$

Discrete-time

Discrete-time $R_x : [r_x(n, \nu)]_{N \times N}$

$$R_x \approx \frac{1}{L} \sum_{l=1}^{L} x_{t,l} x_{t,l}^T$$

- Compressive samples over $L$ blocks:

$$z_{t,l} = Ax_{t,l}, \quad l = 1 : L$$

**✗** 2D cyclic spectrum is **not linear** in the time-domain samples

$\Rightarrow$ CS framework not immediately applicable
Compressive signal sensing

- Traditional CS applied to spectrum estimation for random processes

- Recovering original signals via CS is over-kill
  - × high computation cost
  - × slow convergence/sensing time
  - × sampling requirements on signal rather than covariance
Compressive covariance sensing

Observation: many applications just require second-order statistics (power spectrum) → bypass recovering signal itself

Key: identify linear relationship among 2nd-order statistics
- Covariance of compressed samples
- Covariance and cyclic spectrum of original signal

\[ \hat{R}_z = \frac{1}{L} \sum_l z_{t,l} z_{t,l}^T \]

Compressed domain

CS^-1 (LR-LS)
or ML, closed-form LS

Covariance or cyclic spectrum
\( R_x, S_x^{(c)} \)

uncompressed domain

✔ reduced computational load
✔ fast convergence
✔ can incorporate structural knowledge of covariance/spectrum

Vector-form relationship (1)

- Linking covariance matrix with cyclic spectrum
  - Covariance matrix: \[ R_x = \mathbb{E}\{x_t x_t^T\} \]
    \[
    R_x = \begin{bmatrix}
    r_x(0,0) & r_x(0,1) & r_x(0,2) & \cdots & r_x(0,N-1) \\
    r_x(0,1) & r_x(1,0) & r_x(1,1) & \cdots & r_x(1,N-2) \\
    r_x(0,2) & r_x(1,1) & r_x(2,0) & \cdots & r_x(2,N-3) \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    r_x(0,N-1) & \cdots & \cdots & \cdots & r_x(N-1,0)
    \end{bmatrix}
    \]
  - Degrees of freedom: \( N(N + 1)/2 \)
    \[ r_x = [r_x(0,0), r_x(1,0), \cdots, r_x(N-1,0), r_x(0,1), r_x(1,1), \cdots, r_x(0,N-1)]^T \in \mathcal{R}^{N(N+1)/2}. \]
  - Vectorized cyclic spectrum
    \[ s_{x(c)} = \text{vec}\{S_{x(c)}\} = \left( \mathbf{I} \otimes \mathbf{F} \right) \sum_{\nu=0}^{N-1} \left( D_{\nu}^T \otimes G_{\nu} \right) \mathbf{B}^T r_x := T. \]
Vector-form relationship (2)

- Linking covariance matrices
  - TV covariance of compressed data: \( \mathbf{R}_z = \mathbb{E}\{\mathbf{z}_t\mathbf{z}_t^T\} \in \mathbb{R}^{M \times M} \)
  - Finite-sample estimate:
    \[
    \hat{\mathbf{R}}_z = \frac{1}{L} \sum_l \mathbf{z}_t,l \mathbf{z}_t,l^T
    \]
  - Degrees of freedom: \( M(M + 1)/2 \)
    \[
    \mathbf{r}_z = [r_z(0, 0), r_z(1, 0), \ldots, r_z(M - 1, 0), r_x(0, 1), r_z(1, 1),
    \ldots, r_z(M - 2, 1), \ldots \ldots, r_z(0, M - 1)]^T.
    \]
  - Relationship: \( \mathbf{z}_t = \mathbf{A}\mathbf{x}_t \rightarrow \mathbf{R}_z = \mathbf{A}\mathbf{R}_x\mathbf{A}^T \)

- Linear representation for compressed covariance
  \[
  \mathbf{r}_z = \mathbf{Q}_M \text{vec}\{\mathbf{A}\mathbf{R}_x\mathbf{A}^T\} = \mathbf{Q}_M (\mathbf{A} \otimes \mathbf{A}) \text{vec}\{\mathbf{R}_x\} = \Phi \mathbf{r}_x
  \]
  \[
  \frac{M(M + 1)}{2} \times 1 \quad \frac{N(N + 1)}{2} \times 1
  \]
Sparse cyclic spectrum recovery

- **Reformulated linear relationship**
  \[ r_z = \Phi r_x \quad s^{(c)}_x = Tr x \]
  - \( \Phi : \frac{M(M+1)}{2} \times \frac{N(N+1)}{2} \) fat matrix

- **Prior Information**
  - \( s^{(c)}_x \) is highly sparse
  - \( R_x \) is positive semi-definite (psd)

- **\( L_1 \)-norm regularized LS (LR-LS)**
  \[
  \min_{r_x} \|Tr_x\|_1 + \lambda \|r_z - \Phi r_x\|^2_2 \\
  s.t. \quad R_x \text{ is psd, with } \text{vec}\{R_x\} = P_N r_x.
  \]
  Convex!

  \[
  \min_{s^{(c)}_x} \|s^{(c)}_x\|_1 + \lambda \|r_z - \Phi T^{-1}s^{(c)}_x\|^2_2
  \]
$x(t) \leftrightarrow x_t$

CS-ADC (sub-Nyquist)

$\{z_{t,l}\}_l$

Sparse cyclic spectrum reconstruction

$S_x(\alpha, f)$

Cyclic feature detection

Cyclic feature classification

Wideband aspect: multiple signal sources

power

frequency
Spectrum occupancy estimation

- Band-by-band estimation

  Is \( f^{(n)} \) occupied or not?

  - Region of relevance to \( f^{(n)} \)

    \[
    \begin{align*}
    \frac{f + \alpha}{2} &= f^{(n)} \\
    |f| + \frac{|\alpha|}{2} &\leq f_{\text{max}} \\
    b_i + \frac{a_i}{2} &= n \\
    \left| b_i - \frac{N-1}{2} \right| + \frac{|a_i|}{2} &\leq \frac{f_{\text{max}} N}{f_s} &\leq \frac{N}{2}
    \end{align*}
    \]

  - Relevant SCD vector for band \( n \)

    \[
    \hat{c}^{(n)} : \left\{ \hat{s}_x^{(c)} (a_i, b_i) \right\}_i
    \]

\[
 f(n) = n - \frac{N-1}{N} f_s \quad f_s \in \left[ -\frac{f_s}{2}, \frac{f_s}{2} \right]
\]

\[
 S_{x^{(c)}}
\]
Multi-cycle GLRT

- Binary hypothesis test on band $n$
  \[ \begin{align*}
  H_1 : \quad & \hat{c}^{(n)} = c^{(n)} + \epsilon \\
  H_0 : \quad & \hat{c}^{(n)} = \epsilon
  \end{align*} \]

  - $c^{(n)} : \left\{ s_x^{(c)}(a_i, b_i) \right\}_i$ : unknown true SCD; multiple cyclic freq.
  - $\epsilon : \mathcal{N}(0, \Sigma_\epsilon)$ : noise statistics determined mainly by finite-sample effects, not ambient noise

- GLRT formulation
  - Test statistics: $T^{(n)} = (\hat{c}^{(n)})^H \Sigma^{-1}_\epsilon \hat{c}^{(n)}$
  - Binary decisions by thresholding

- A single wideband DSP, as opposed to multiple NB filters
Simulation: Robustness to rate reduction

Probability of detection vs. compression ratio ($P_{FA}=0.1$, N=32, L=200 blocks)

- Monitored band $|f_{max}| < 300$ MHz
- 2 sources (noise-free): PU$_1$ - BPSK at 150MHz; PU$_2$ - QPSK at 225MHz; $T_s=0.02667\mu$s
- Cisco 802.11 DSSS Spread spectrum

50% compression
Simulation: Robustness to noise uncertainty

Receiver Operating Characteristic (ROC): $P_D$ vs $P_{FA}$ (SNR=5dB, 50% compression)

- Ø outperforms energy detection
- Ø insensitive to noise uncertainty

- cyclic
- Energy detection (noise uncertainty = 0, 1, 2, 3dB)

- outperforms energy detection
- insensitive to noise uncertainty
Classification using cyclic statistics

- **2D: Spectral Correlation Density (SCD)**
  - BPSK
  - QPSK

- **1D: cyclic-frequency domain profile (CDP)**
  \[ I(\alpha) = \max_f |S_x(\alpha, f)| \]

[Kim et al. 2007]
Simulations: Classification

Confusion matrix (SVM classifier)

<table>
<thead>
<tr>
<th></th>
<th>BPSK</th>
<th>QPSK</th>
<th>DS-BPSK</th>
<th>DS-QPSK</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>95.45%</td>
<td>0%</td>
<td>4.55%</td>
<td>0%</td>
</tr>
<tr>
<td>QPSK</td>
<td>0%</td>
<td>90.9%</td>
<td>9.09%</td>
<td>0%</td>
</tr>
<tr>
<td>DS-BPSK</td>
<td>9.09%</td>
<td>0%</td>
<td>59.09%</td>
<td>31.82%</td>
</tr>
<tr>
<td>DS-QPSK</td>
<td>4.5%</td>
<td>4.5%</td>
<td>36.46%</td>
<td>54.54%</td>
</tr>
</tbody>
</table>

- When compression ratio is adequate for detection, classification accuracy is comparable to non-compression
- Good separation of narrowband from spread spectrum
- Considerable confusion among spread spectrum signals
Summary: Compressive covariance sensing (CCS)

Key is to directly extract useful (2nd-order) statistics, which has less degrees of freedom than the random signal itself:
- ✔ strong compression allowed
- ✔ low computational load, bypassing signal recovery
- ✔ fast convergence, short sensing time

Q: Does the framework generalize? Do we really need sparsity?
- ✔ enables compression for non-sparse signals
- ✔ permits close-form solution; easy to analyze performance
Non-sparse stationary signals

- Stationary processes as a special case of cyclostationary ones
  [Tian et al., JSTSP’2012; Leus et al., SPL’2011, TSP’2012]
- 2D cyclic spectrum reduces to 1D power spectrum

\[
r_x(n, \nu) = \tilde{r}_x(\nu), \quad \forall n
\]

\[
\mathbf{R}_x = \begin{bmatrix}
\tilde{r}_x(0) & \tilde{r}_x(1) & \tilde{r}_x(2) & \cdots & \tilde{r}_x(N-1) \\
\tilde{r}_x(1) & \tilde{r}_x(0) & \tilde{r}_x(1) & \cdots & \tilde{r}_x(N-2) \\
\tilde{r}_x(2) & \tilde{r}_x(1) & \tilde{r}_x(0) & \cdots & \tilde{r}_x(N-3) \\
: & \tilde{r}_x(1) & \tilde{r}_x(0) & \cdots & \tilde{r}_x(N-4) \\
\tilde{r}_x(N-1) & : & : & \cdots & \tilde{r}_x(0)
\end{bmatrix}
\]

Toeplitz

- # measurements \( r_z \) from cross-correlations in \( \mathbf{R}_z \): \( M(M+1)/2 \)
- # unknowns \( \{ r_x(\nu) \} \) in power spectrum from \( \mathbf{R}_x \): \( N \)
Compressive covariance/PSD estimation

\[ \frac{M(M+1)}{2} \times 1 \]

\[ r_z = \Phi \bar{r}_x = \Phi \Phi^{-1} s_f \]

\[ N \]

- Minimum sampling rates for non-sparse signals [Tian et al. JSTSP’12]

**Proposition:** When the compression ratio satisfies \( M(M+1)/2 \geq N \), lossless recovery of power spectrum is possible via closed-form least-squares solution, even in the absence of signal sparsity.

\[
\left( \frac{M}{N} \right)_{\text{min}} = \frac{\sqrt{8N+1} - 1}{2N} \xrightarrow{N \to \infty} \sqrt{\frac{2}{N}} \ll 1
\]

- Stronger compression allowed for sparse signals
Sampler design

- **Periodic sampling devices**

\[ z_i = \int a_i(t) x(t) dt, \quad i = 1 : M \]

- **Multi-coset sampling**

\[ a_i(t) = \delta(t - c_i / f_s) \]

Sampler design for covariance estimation

- Minimal complete sparse rulers
  [Moffet’68] [Ariananda-Leus-Tian’12]

<table>
<thead>
<tr>
<th>$N$</th>
<th>Compression ($M/N$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.667</td>
</tr>
<tr>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td>20</td>
<td>0.4</td>
</tr>
<tr>
<td>50</td>
<td>0.24</td>
</tr>
<tr>
<td>78</td>
<td>0.1923</td>
</tr>
<tr>
<td>128</td>
<td>0.1563</td>
</tr>
</tbody>
</table>

$N \to \infty \quad \sqrt{2/N}$
Simulations: sparse ruler

Theoretical Noisy PSD
Sparse Ruler (460746 samples, M/N=0.5)

Reconstructed PSD
Covariance structures

**Toeplitz**
\[ S = 2N-1 \]

**d-Banded:**
\[ S = 2d-1 \]

**Circulant:**
\[ S = N \]
Compression limits

RS: random (dense) sampler; L/C-SR: linear/circular-sparse ruler

\[(N/M)_{\text{max}} \approx c\sqrt{N}\]

Results for cyclostationary signals

- Sampler design for reconstructing cyclic spectrum
  - Multiple samplers used periodically across $K$ cyclic periods

<table>
<thead>
<tr>
<th>Original signal $x[n]$</th>
<th>Structure of $R_x$</th>
<th>Compressed signal $z[n]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stationary</td>
<td>Toeplitz</td>
<td>Cyclostationary</td>
</tr>
<tr>
<td>Cyclostationary (period $N$)</td>
<td></td>
<td>?</td>
</tr>
</tbody>
</table>

**Proposition:** it is possible to reconstruct non-sparse 2D cyclic spectrum in closed form when

$$\left(\frac{M}{N}\right)_{\text{min}} = \sqrt{\frac{1}{K}} \leq 1$$

Summary

- Exploitation of signal sparsity in 2D cyclic spectrum domain
  - reformulated linear relationship between spectrum and covariance
  - robustness to noise uncertainty
  - capability in signal separation and classification

- Compression and recovery of 2nd-order statistics for (non-sparse) random processes

- Direct reconstruction of test statistics from compressive samples
  - bypass the recovery of the entire cyclic spectrum
  - reduce sensing time and responsiveness

- Sensing techniques for spread spectrum signals
  - utilize fusion techniques to take advantage of all features in the wide spectrum of DSSS signals
Other applications
Compressive Covariance Sensing (CCS)

- Cognitive radio (CR) frequency spectrum
- Radar Doppler + angular spectra
- Radio astronomy spatial spectrum
- MRI resonance spectrum
- Seismic seismic design response spectrum
Outlook

- Super resolution
- Big data
- Combining spatial and temporal domains
- Extensions to Doppler spectrum, imaging, …
- Applications to radar, MRI, seismic, radio astronomy, …
Outlook: angular spectrum estimation

- Array processing
  - Imaging
    - Optical/radar/ultrasound/acoustic
    - Radio astronomy
    - Seismology
  - DoA estimation → localization

- MIMO Radar

- Massive MIMO for 5G

Acquisition and processing hardware prop. to # antenna elements

**Challenge:** reduce number of antennas w/o sacrificing spatial/ angular resolution
Simulations: DoA estimation

Reconstructed spectrum using LS and MUSIC for the minimal sparse ruler array (continuous source from 30 to 40 degrees; for LS S=71; ULA N=36; M=10; SNR = 0dB)
Outlook: spectrum cartography

- **Idea:** CRs collaborate to form a spatial map of the spectrum

given the PSD $\Phi_r(f) = \Phi(f; v_r)$ at position $v_r$, find $\Phi(v, f)$, $\forall v$

- **Goal:** $\Phi(v, f)$, $\forall v$
- **Specifications:** coarse approx. suffices
- **Approach:** basis expansion of $\Phi(v, f)$
- **Compressive Sampling (CS)** possible
to form the PSD data

---

Modeling (space-frequency)

- Transmitters
  \( \text{Tx}_s, \ s = 1, \ldots, N_s \)

- Sensing CRs
  \( \text{CR}_r, \ r = 1 : N_r \)

- Frequency bases
  \( b_{\nu}(f), \ \nu = 1 : N_b \)

- Sensed frequencies
  \( f_k, \ k = 1 : K \)

\[ \Phi_s(f) = \sum_{\nu=1}^{N_b} \theta_{sv} b_{\nu}(f) \]

Sparsity present in space and frequency
Power spectrum cartography

- 5 sources
- \( N_s = 121 \) candidate locations, \( N_r = 50 \) CRs

- Sparsity-unaware NNLS is prone to false alarms
- As a byproduct, LR-LS localizes all sources via variable selection for example, Jammer Localization
Outlook: correlation mining

- Large-scale networks

Sample size issue $\Rightarrow$ sparsity-enforcing regularization
Outlook: monitoring in large-scale networks

- Sparse event detection using large-scale wireless sensor nets
  - Local phenomena induce spatially sparse signals
  - Desiderata: energy efficiency, scalability, robustness

- Distributed optimization
  [Ling-Tian, 2010, 2011]

Structural Health Monitoring

[Ling-Tian et al, 2009]
## Summary: CS vs. CCS

<table>
<thead>
<tr>
<th>Compressed Covariance Sensing</th>
<th>Compressed Sensing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aims at recovering statistics</td>
<td>Aims at recovering the signal</td>
</tr>
<tr>
<td>Lossy</td>
<td>Lossless</td>
</tr>
<tr>
<td>Use sparse sampling</td>
<td>Use random sampling</td>
</tr>
<tr>
<td>No <strong>sparsity</strong> is required</td>
<td><strong>Sparsity</strong> is required</td>
</tr>
<tr>
<td>Linear/non-linear reconstruction</td>
<td>Non-linear reconstruction</td>
</tr>
<tr>
<td>Overdetermined</td>
<td>Underdetermined</td>
</tr>
</tbody>
</table>

Thank you!