Reflector Antennas

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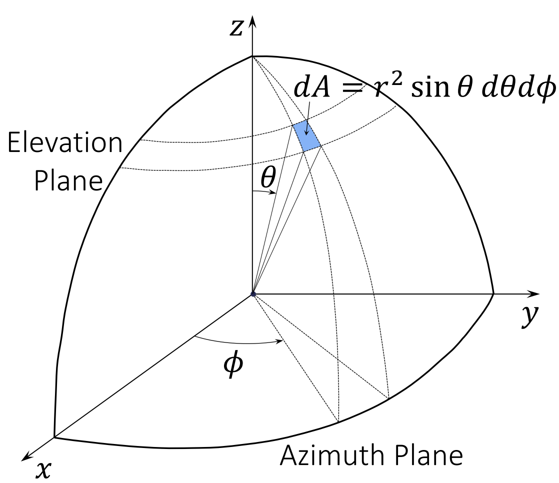
Summary

1. Basic Concepts of Antennas
2. Radiation by Aperture Antennas
3. Reflector Antennas
4. Design and Analysis Methods

Basic Concepts of Antennas

Coordinate System

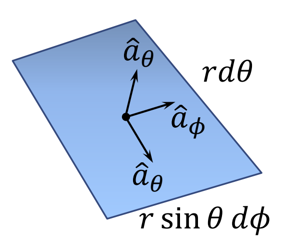


θ : elevation

$0 \leq \theta \leq 2\pi$

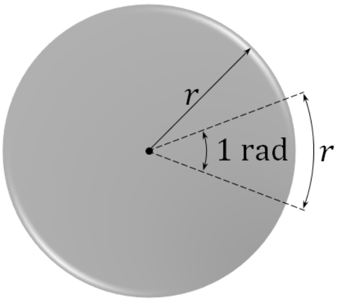
ϕ : azimuth

$0 \leq \theta \leq 2\pi$



Radian

One radian (1 rad) is defined as the plane angle with its vertex at the center of a circle of radius *r* that is subtended by an arc whose length is *r*.

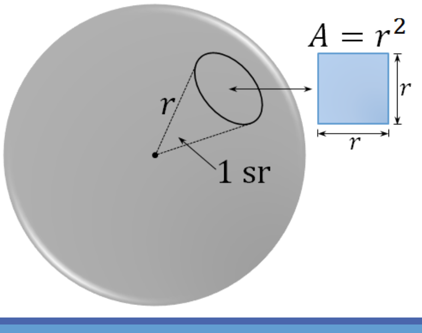


$$C = 2\pi r$$

$$1 \text{ rad} \approx 57{,}295779°$$

Steradian

One steradian (1 sr) is defined as the solid angle with its vertex at the center of a sphere of radius *r* that is subtended by a spherical surface area ....



equal to that of a square with each side *r*.

$$A = 4\pi r^2$$

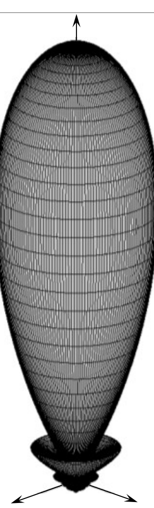
Radiation Pattern

A mathematical function or a graphical representation of the radiation properties of the antenna, such as :

- Field or Power
- Phase
- Polarization, etc.

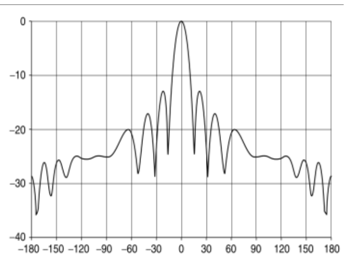
as a function of angular space coordinates θ, ϕ.

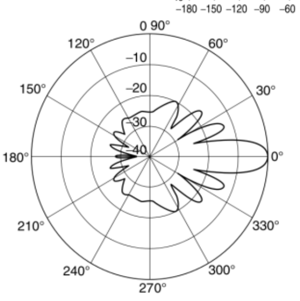
Radiation Pattern



three-dimensional

Rectangular

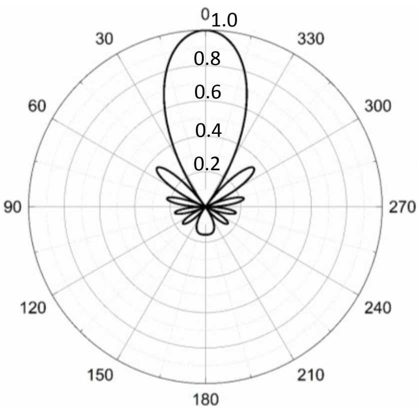




Polar

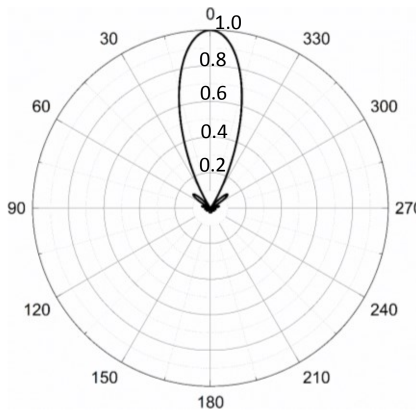
Radiation Pattern

Field Pattern



$|E| = |E(\theta,\phi)/E_{\max}|$

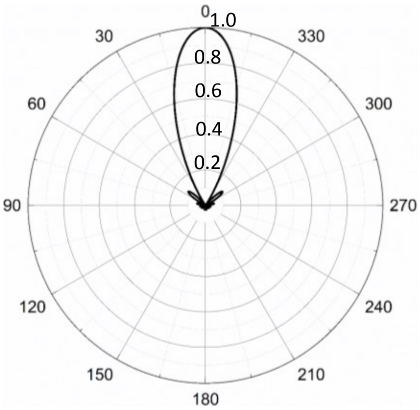
Power Pattern



$P(\theta,\phi) \propto |E(\theta,\phi)/E_{\max}|^2$

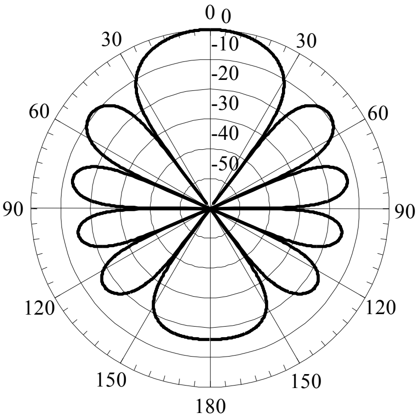
Radiation Pattern

Power Pattern



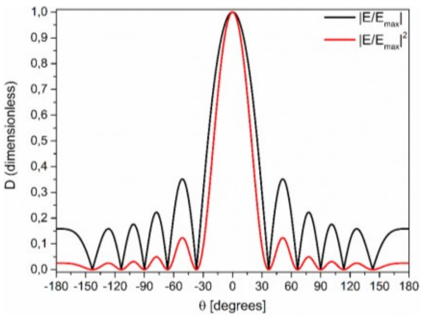
$U(\theta,\phi) \propto |E(\theta,\phi)/E_{\max}|^2$

Power Pattern (dB)



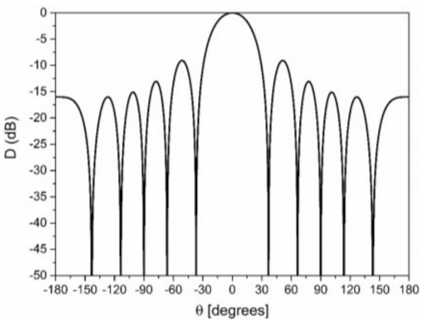
$U_{\mathrm{dB}}(\theta,\phi) = 10\log_{10} U(\theta,\phi)$

Radiation Pattern

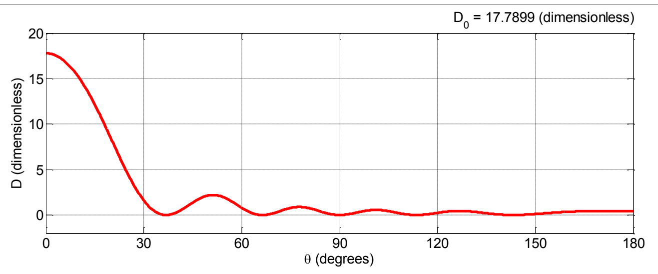


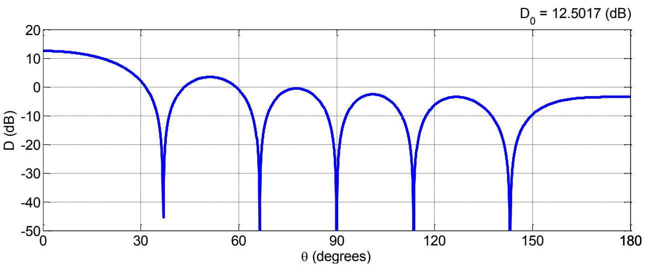
$$P(\theta, \phi) \propto |E(\theta, \phi)/E_{\max}|^2$$

$$P_{\mathrm{dB}}(\theta, \phi) = 10\log_{10} P(\theta, \phi)$$



Radiation Pattern





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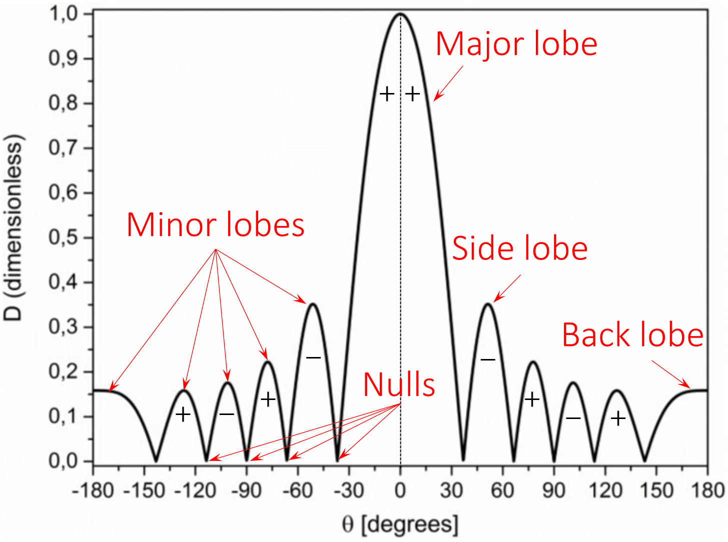
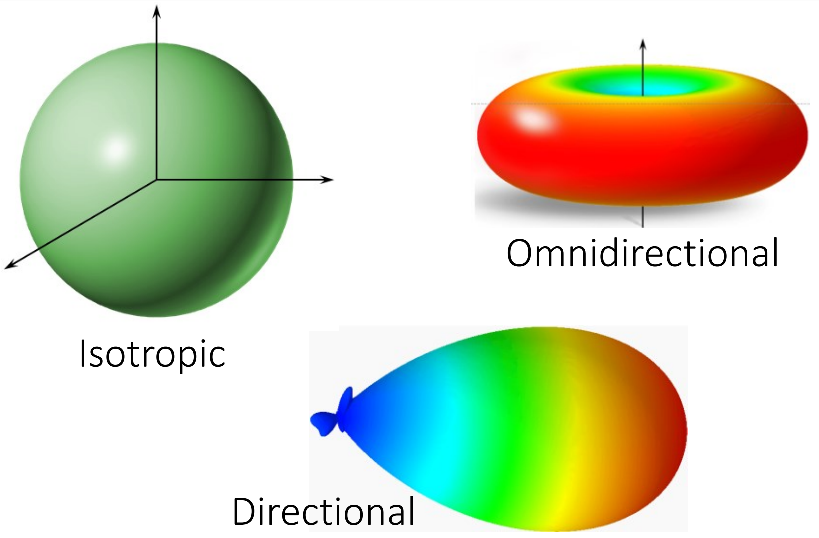
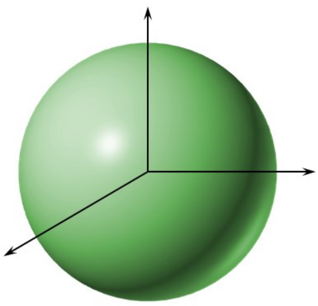


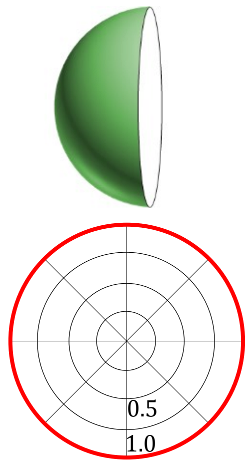
Radiation Pattern



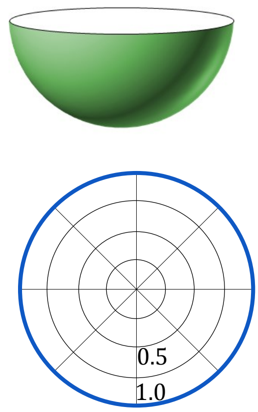
Radiation Pattern



Isotropic

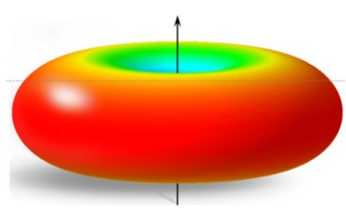


Vertical

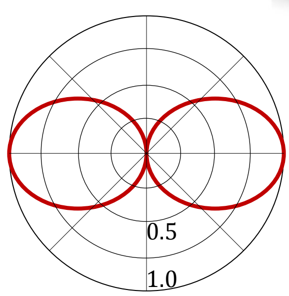


Horizontal

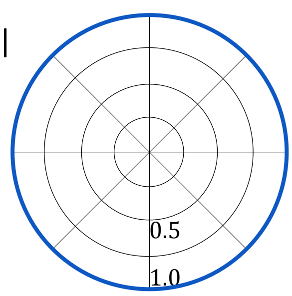
Radiation Pattern



Omnidirectional

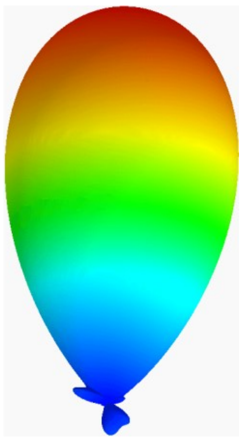


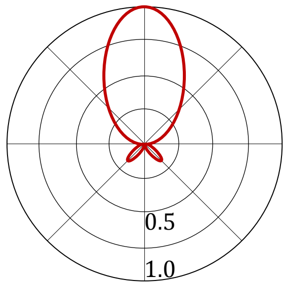
Vertical



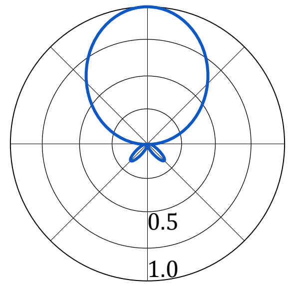
Horizontal

Radiation Pattern





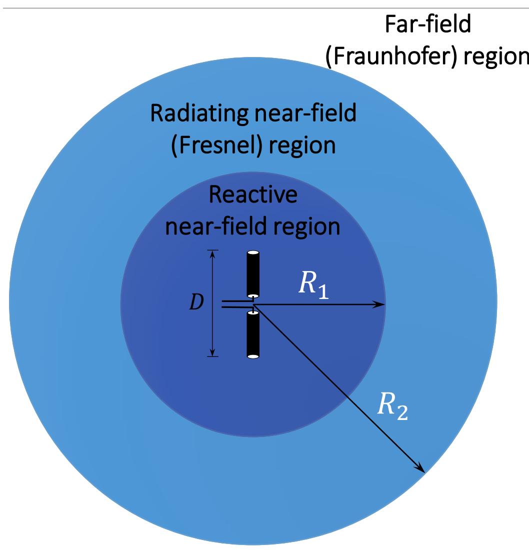
Vertical



Horizontal

Directional

Field Regions



$$R_1 = 0.62\sqrt{D^3/\lambda}$$

$$R_2 = 2D^2/\lambda$$

Field Regions

Reactive Near-Field

- Phases of Electric and Magnetic fields are often near quadrature
- Wave impedance highly reactive (imaginary);
- High content of non-propagating stored energy;

Radiating Near-Field

- Fields are predominantly in phase;
- Fields do not yet display a spherical wavefront;
- Region where near-field measurements are made;
- Wave impedance is active (real) and reactive (imaginary);

Field Regions

Far-Field

- Fields exhibit spherical wavefront : $\frac{e^{-jkr}}{r}$

$$E_r = H_r \approx 0 \qquad E_\theta \approx \eta H_\phi$$

$$\eta = \sqrt{\mu/\varepsilon} \qquad E_\phi \approx -\eta H_\theta$$

TEM

- So, the phase is constant for any θ, ϕ. Ideally, the pattern does not change with distance r;
- Ideally, the wave impedance is strictly active (real);
- Power predominantly real; Propagating energy.

Radiation Power Density

The quantity used to describe the **power associated with an electromagnetic wave** is the average Poynting vector:

$$\vec{W}_{\mathrm{rad}} = \frac{1}{2}\mathrm{Re}\{\vec{E} \times \vec{H}^*\} \qquad \mathrm{W/m^2}$$

$$P_{\mathrm{rad}} = \frac{1}{2}\oiint \mathrm{Re}\{\vec{E} \times \vec{H}^*\} \cdot ds \qquad \mathrm{W}$$

Isotropic Radiator

An isotropic radiator is an **ideal source that radiates equally in all directions**. Although it does not exist in practice, it provides a convenient isotropic reference.

$$P_{\mathrm{rad}} = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi} W_0\, r^2 \sin\theta\, d\theta d\phi = W_0 4\pi r^2$$

$$W_0 = P_{\mathrm{rad}}/4\pi r^2 \qquad \mathrm{W/m^2}$$

Radiation Intensity

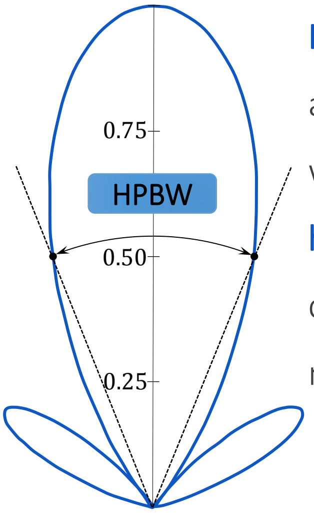
It is the power radiated from an antenna per unit solid angle. This is a far-field parameter mathematical expressed as:

$$U = r^2 \mathrm{W}_{\mathrm{rad}} \qquad \mathrm{W/sr}$$

$$U = \frac{r^2}{2\eta} \left| \vec{\mathrm{E}}(r, \theta, \phi) \right|^2 \qquad \mathrm{W/sr}$$

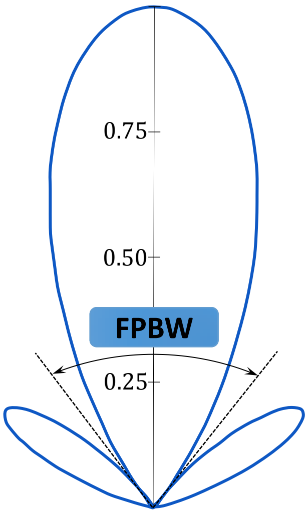
$$U_0 = P_{\mathrm{rad}} / 4\pi$$

Beamwidth



HPBW (Half-Power Beamwidth) the angle between the two directions in which the radiation intensity is one-half value of the beam in the plane containing the direction of the maximum of a beam.

Beamwidth



FNBW (First-Null Beamwidth) is the angular separation between the first nulls of the pattern. Other beamwidths are those where the pattern is −10 dB from the maximum.

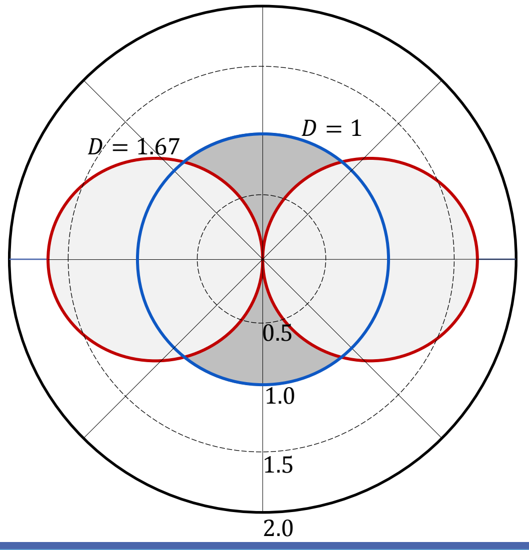
Directivity

The ratio of the radiation intensity in a given direction to the radiation intensity averaged over all directions. The average radiation intensity is equal to the total power radiated by the antenna divided by 4π.

$$D(\theta,\phi) = \frac{U}{U_0} = \frac{4\pi}{P_{\text{rad}}} U(\theta,\phi)$$

$$U(\theta,\phi) \approx \frac{1}{2\eta}\left[|E_\theta|^2 + \left|E_\phi\right|^2\right]$$

Directivity

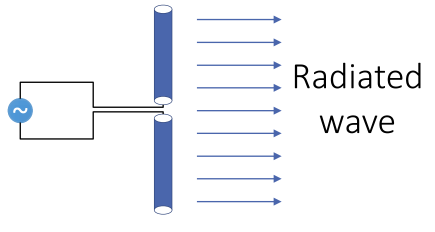


Isotropic
$D = 1$

Dipole λ/2
$D = 1.67$

Input Impedance

The impedance presented by an antenna at its terminals or the **ratio of the voltage to current at a pair of terminals** or the ratio of components of the electric to magnetic fields at a point.



$$Z_A = R_A + jX_A$$

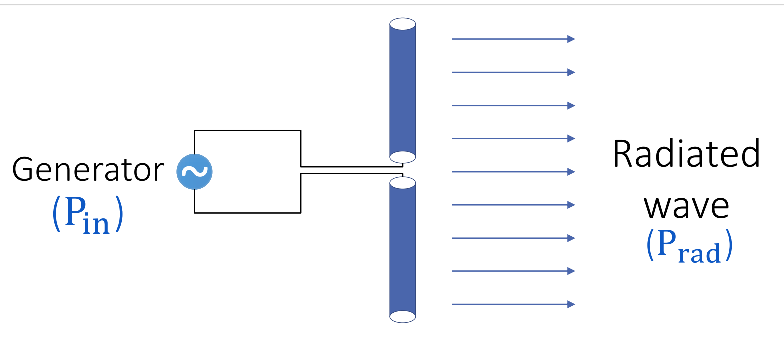
Antenna Efficiency

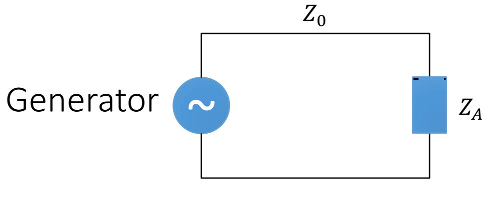
The **ratio of the total power radiated (P_{rad}) by an antenna to the net power accepted by the antenna (P_{in})** from the connected transmitter. It takes into account losses at the input terminals and within the structure of the antenna.

$$e_0 = e_r e_c e_d$$

- Reflection Efficiency (e_r)
- Conduction Efficiency (e_c)
- Dielectric Efficiency (e_d)

Antenna Efficiency





$$e_0 = e_{cd}(1 - \Gamma)$$

$$\Gamma = \frac{Z_A - Z_0}{Z_A + Z_0}$$

Gain

The ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted (P_{in}) by the antenna were radiated isotropically.

$$G(\theta, \phi) = \frac{4\pi}{P_{\text{in}}} U(\theta, \phi)$$

$$P_{\text{rad}} = e_{cd} P_{\text{in}}$$

Gain

The gain of the antenna is closely related to the directivity, it is a measure that takes into account the efficiency of the antenna as well as its directional capabilities.

$$G(\theta, \phi) = e_{cd} \left[\frac{4\pi}{P_{\text{rad}}} U(\theta, \phi) \right]$$

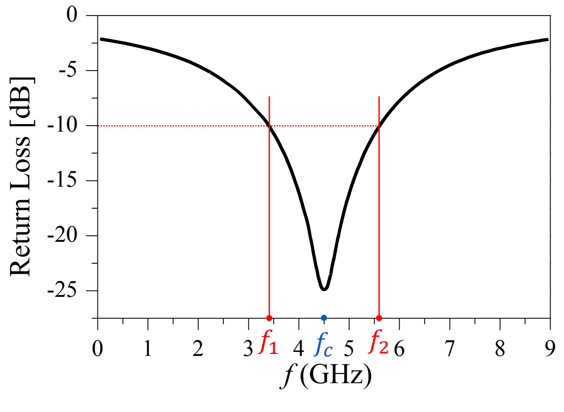
$$G(\theta, \phi) = e_{cd} D(\theta, \phi)$$

Bandwidth

The **range of frequencies** where the antenna characteristics are within an **acceptable value of those at the center frequency**.

1. Input impedance
2. Return Loss
3. Beamwidth
4. Gain, etc.

Bandwidth



$f_c = 4.5$ GHz

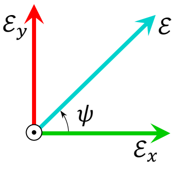
$RL < -10$dB

$BW \approx 2.25$GHz

$BW \approx \pm 25\%$

Polarization

The property of an electromagnetic wave describing the **time-varying direction and relative magnitude of the electric-field vector**.



$$\vec{\mathcal{E}}(z;t) = \mathcal{E}_x(z;t)\hat{a}_x + \mathcal{E}_y(z;t)\hat{a}_y$$

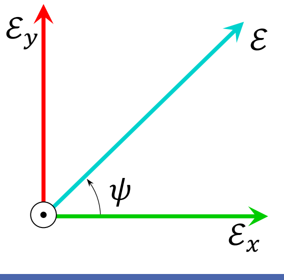
$$\mathcal{E}_x(z;t) = \mathrm{E}_{xo}e^{j(\omega t+kz+\phi_x)} \longrightarrow \mathcal{E}_x(z;t) = \mathrm{E}_{xo}\cos(\omega t+kz+\phi_x)$$

$$\mathcal{E}_y(z;t) = \mathrm{E}_{yo}e^{j(\omega t+kz+\phi_y)} \longrightarrow \mathcal{E}_y(z;t) = \mathrm{E}_{yo}\cos(\omega t+kz+\phi_y)$$

Linear Polarization

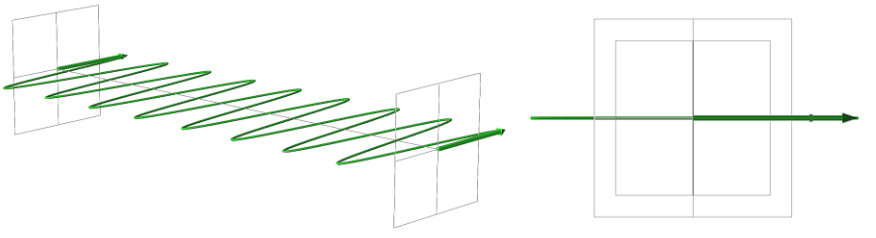
The field vector (electric or magnetic) possesses:

1. Only one component ($\mathcal{E}_x$ or $\mathcal{E}_y$);
2. Two orthogonal linear components that are in time phase or π out-of-phase.



$$\psi = \tan^{-1}\left(\frac{\mathcal{E}_y}{\mathcal{E}_x}\right)$$

Linear Polarization



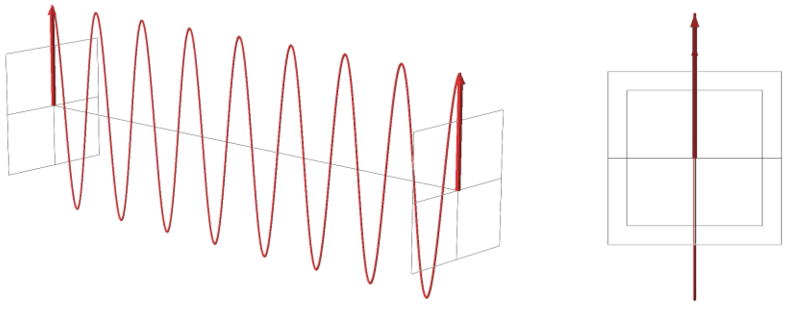
$\mathrm{E}_{xo} \neq 0, \ \ \mathrm{E}_{yo} = 0$

$\psi = \pi/2$

$\Delta\phi = \phi_y - \phi_x = \pm n\pi$

$n = 0, 1, 2, 3, \ldots$

Linear Polarization



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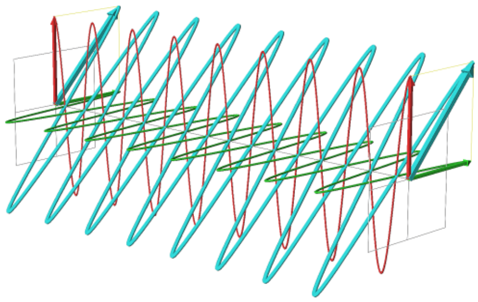
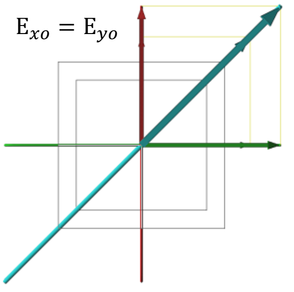
Linear Polarization





$$E_{xo} \neq 0, \quad E_{yo} \neq 0$$

$$\Delta\phi = \phi_y - \phi_x = \pm n\pi$$

$$n = 0, 1, 2, 3, \dots$$

$$\psi = \tan^{-1}\left(\frac{\varepsilon_y}{\varepsilon_x}\right)$$

$$\psi = \pi/4$$

Circular Polarization

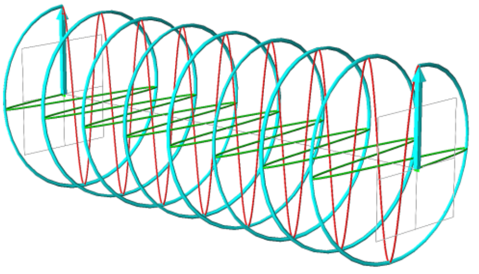
The field vector electric possesses all of the following:

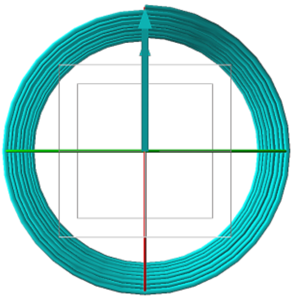
1. Two orthogonal linear components the same magnitude ($\mathcal{E}_x = \mathcal{E}_y$) ;
2. The two components must have a time-phase difference of odd multiples of $90°$;

   a. If positive multiples of $90°$ clockwise (CW);

   b. If negative multiples of $90°$ counterclockwise (CCW);

Circular Polarization





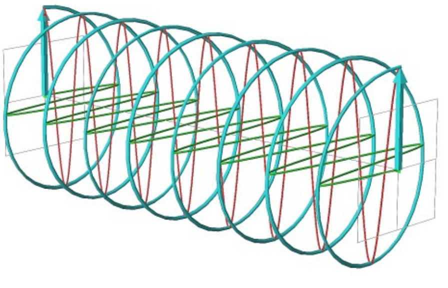
$$E_{xo} = E_{yo} \neq 0$$

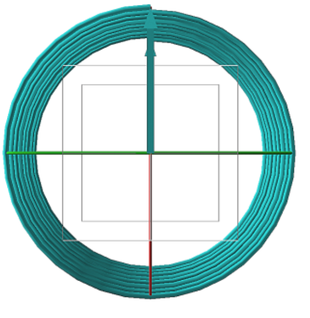
$$\Delta\phi = \phi_y - \phi_x = +\left(\frac{1}{2} + 2n\right)\pi$$

$$n = 0, 1, 2, 3, \ldots$$

Clockwise (CW)

Circular Polarization





$$E_{xo} = E_{yo} \neq 0$$

$$\Delta\phi = \phi_y - \phi_x = -\left(\frac{1}{2} + 2n\right)\pi$$

$$n = 0, 1, 2, 3, \ldots$$

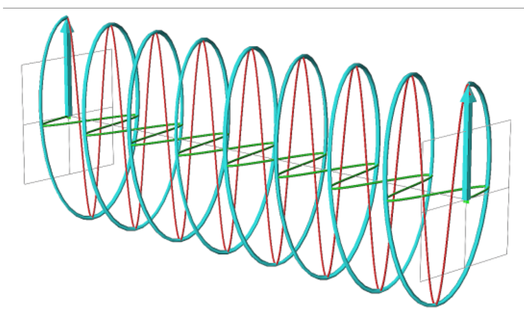
Counterclockwise (CCW)

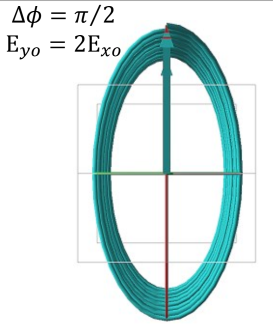
Elliptical Polarization

Two orthogonal linear components:

1. If the two components are of different magnitude, the time-phase difference between the two components must not be 0∘ or multiples of $180°$ (or it will then be linear);
2. If the two components are of the same magnitude, the time-phase difference between the two components must not be odd multiples of $90°$ (or it will then be circular);

Elliptical Polarization





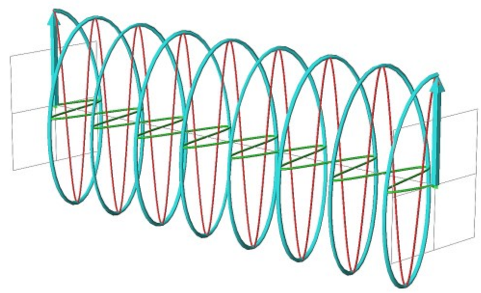
$\mathrm{E}_{xo} \neq \mathrm{E}_{yo} \neq 0$

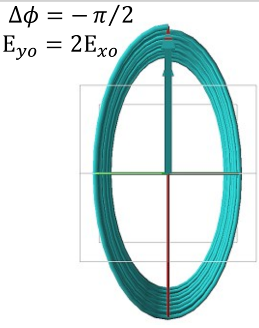
$\Delta\phi = \phi_y - \phi_x \neq n\pi, > 0$

Clockwise (CW)

$n = 0, 1, 2, 3, \ldots$

Elliptical Polarization





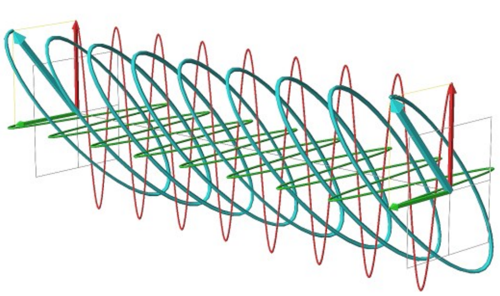
$E_{xo} \neq E_{yo} \neq 0$

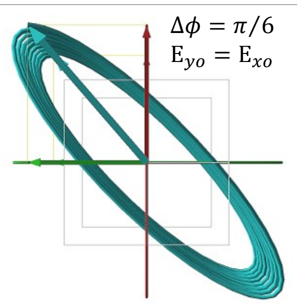
$\Delta\phi = \phi_y - \phi_x \neq n\pi, < 0$

$n = 0, 1, 2, 3, \ldots$

Counterclockwise (CCW)

Elliptical Polarization





$E_{xo} = E_{yo} \neq 0$

$\Delta\phi = \phi_y - \phi_x \neq \frac{n}{2}\pi, > 0$

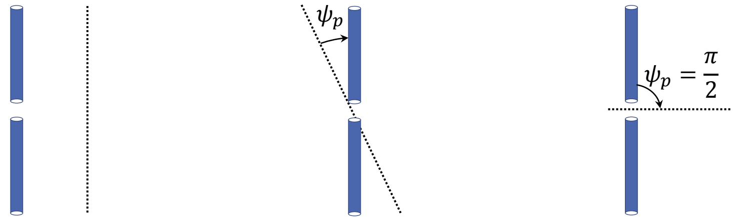
$n = 0, 1, 2, 3, \ldots$

Clockwise (CW)

Polarization Loss Factor (PLF)

The amount of power extracted by the antenna from the incoming signal can be measured by PLF define as:

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left|\cos\psi_p\right|^2$$



$\text{PLF} = 1$ $\qquad$ $\text{PLF} = \cos^2\psi_p$ $\qquad$ $\text{PLF} = 0$

Effective Area (or Aperture)

The ratio of the available power (P_T) at the terminals of a receiving antenna to the power flux density (W_i) of a plane wave incident on the antenna from that direction, the wave being polarization-matched to the antenna.

$$A_e = \frac{P_T}{W_i} = \frac{\lambda^2}{4\pi} D(\theta,\phi)$$

Capture $\qquad$ $A_e = A_c - A_s - A_L$ $\qquad$ Loss

Scattering

Maximum Effective Aperture

The maximum effective aperture antenna (A_{em}) is related to its maximum directivity (D_o) by:

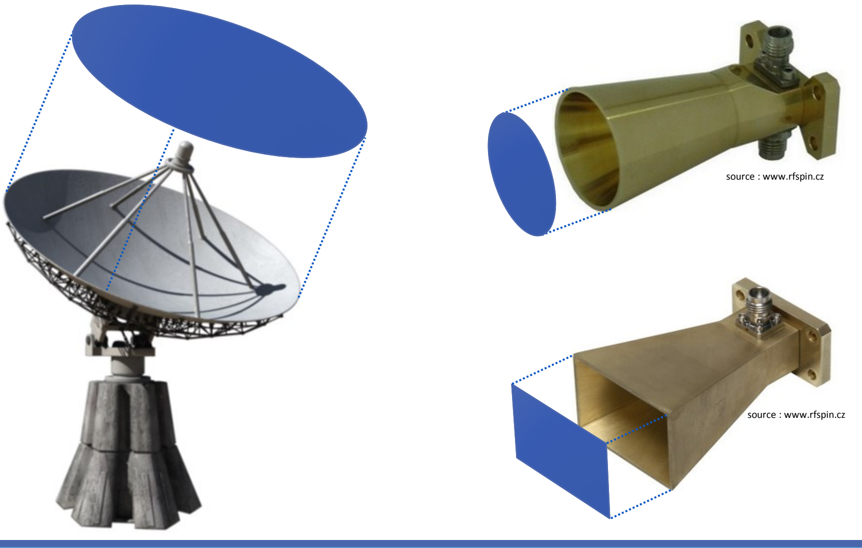
$$A_{em} = \frac{\lambda^2}{4\pi} D_o$$

Aperture efficiency (e_{ap}) is defined as the ratio of the maximum effective area (A_{em}) to its physical area (A_p):

$$e_{ap} = \frac{A_{em}}{A_p} \qquad 0 \leq e_{ap} \leq 1$$

Radiation Characteristics of Aperture Antennas

Introduction



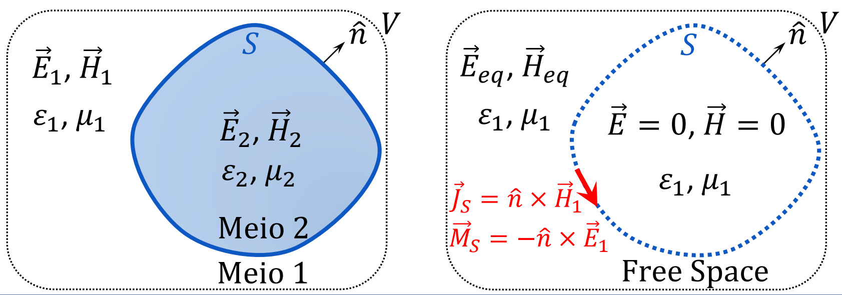
Introduction

The mathematical tools developed to study the radiation characteristics of apertures can be use in analyze o many kind of aperture antennas:

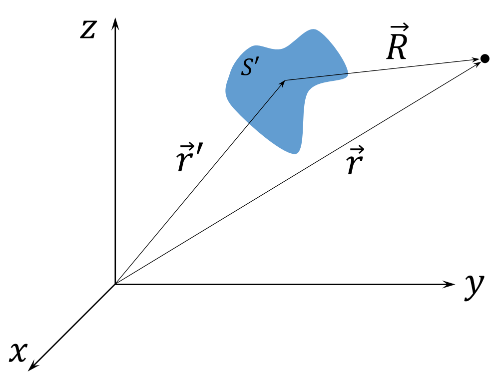
- Waveguide or Horns antennas (square, rectangular, circular, elliptical, or any other configuration)
- Reflector antennas

Field Equivalence Principle

The principle by which sources, such as an antenna, are replaced by equivalent sources. The **fictitious sources are said to be equivalent within a region because they produce the same fields within that region.**



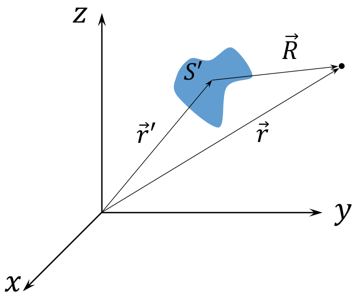
Far-Field Approximations

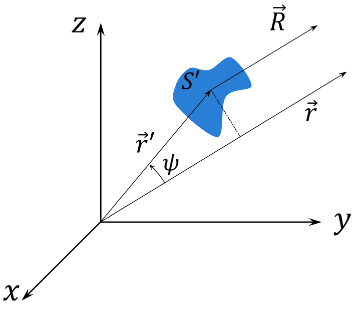


$$\vec{A} = \frac{\mu}{4\pi} \iint \vec{J}_s \frac{e^{-jkR}}{R} \cdot d\vec{s}'$$

$$\vec{F} = \frac{\varepsilon}{4\pi} \iint \vec{M}_s \frac{e^{-jkR}}{R} \cdot d\vec{s}'$$

Far-Field Approximations





Green's Function

$$G(R) = \frac{e^{-jkR}}{R}$$

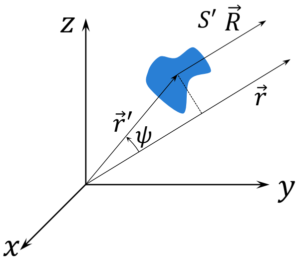
Phase $R \simeq r - r' \cos\psi$

Amplitude $R \simeq r$

Far-Field Approximations

$$\vec{A} \simeq \frac{\varepsilon}{4\pi}\frac{e^{-jkr}}{r}\iint \vec{M}_s e^{-jkr' \cos\psi} \cdot d\vec{s}'$$

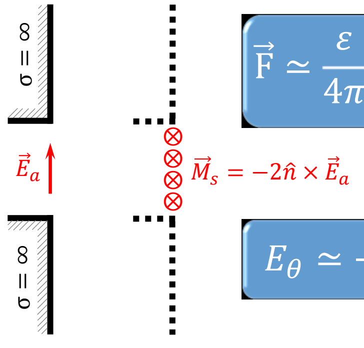
$$\vec{F} \simeq \frac{\varepsilon}{4\pi}\frac{e^{-jkr}}{r}\iint \vec{M}_s e^{-jkr' \cos\psi} \cdot d\vec{s}'$$



$$E_\theta \simeq -j\omega\eta F_\phi$$

Aperture Equivalence

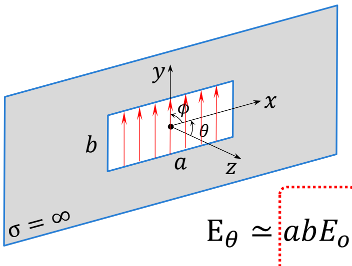
A waveguide aperture is mounted on an infinite ground plane, the tangential components of the electric field over the aperture are known ($\vec{E}_a$).



$$\vec{F} \simeq \frac{\varepsilon}{4\pi} \frac{e^{-jkr}}{r} \iint \vec{M}_s e^{-jkr' \cos\psi} \cdot d\vec{s}'$$

$$E_\theta \simeq -j\omega\eta F_\phi \qquad E_\phi \simeq +j\omega\eta F_\theta$$

Rectangular Aperture



$$\vec{E}_A = E_o\hat{y} \qquad \vec{M}_S = +2E_o\hat{x}$$

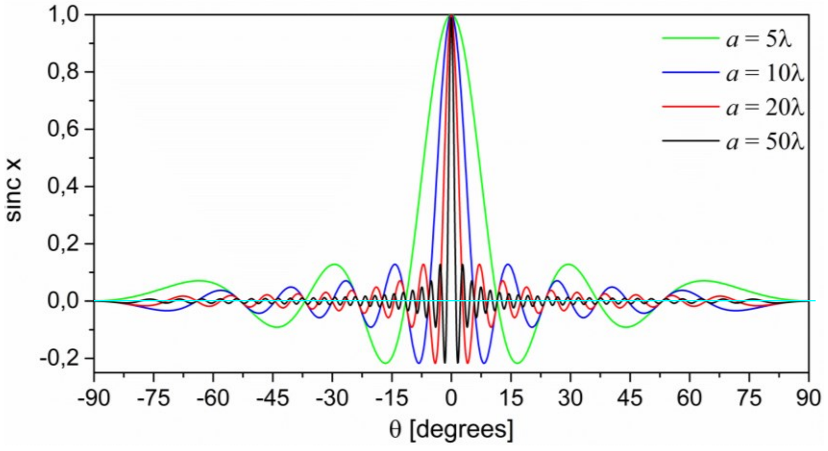
$$E_\theta \simeq abE_o \left(\frac{jk}{2\pi}\right) \frac{e^{-jkr}}{r} \left[\sin\phi \left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right]$$

$$E_\phi \simeq abE_o \left(\frac{jk}{2\pi}\right) \frac{e^{-jkr}}{r} \left[\cos\theta\cos\phi \left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right]$$

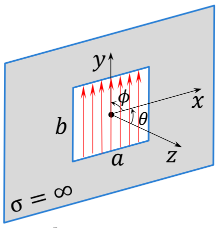
$$X = \frac{ka}{2}\sin\theta\sin\phi \qquad Y = \frac{kb}{2}\sin\theta\cos\phi$$

Sinc

$$\mathrm{sinc}(X) = \frac{\sin X}{X} \qquad X = \frac{ka}{2}\sin\theta\sin\phi$$

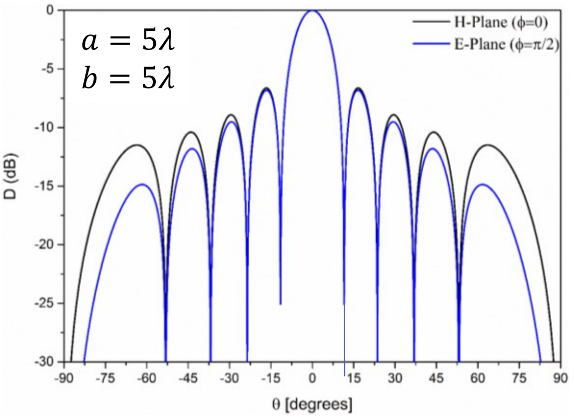


Rectangular Aperture



$$X = \frac{ka}{2}\sin\theta\sin\phi$$

$$Y = \frac{kb}{2}\sin\theta\cos\phi$$



$$\mathrm{E}_\phi = 0; \;\; \mathrm{E}_\theta \propto \left[\left(\frac{\sin Y}{Y}\right)\right]$$

$$\mathrm{E}_\theta = 0; \;\; \mathrm{E}_\phi \propto \left[\cos\theta\left(\frac{\sin X}{X}\right)\right]$$

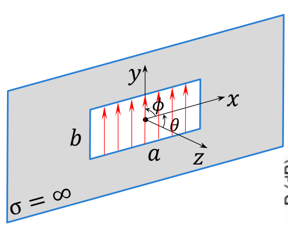
Uniform Aperture

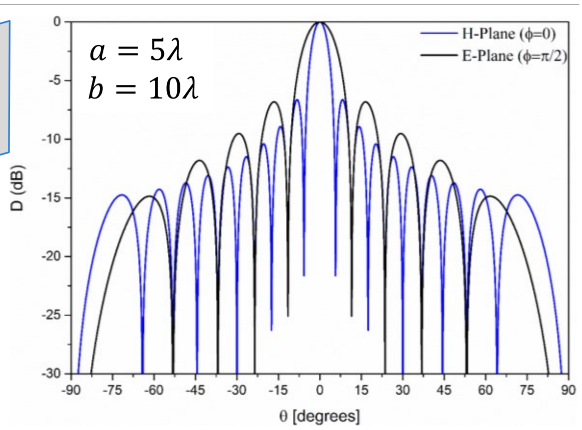
If the phase and polarization of the aperture electric field are uniform, then, the maximum radiation occurs in the normal direction of the aperture plane, regardless of its geometric shape.

$$D_o(\theta = 0) = \frac{4\pi}{P_{\text{rad}}} U(\theta, \phi)$$

$$U(\theta, \phi) \approx \frac{1}{2\eta} \left[|E_\theta|^2 + |E_\phi|^2 \right]$$

Rectangular Aperture





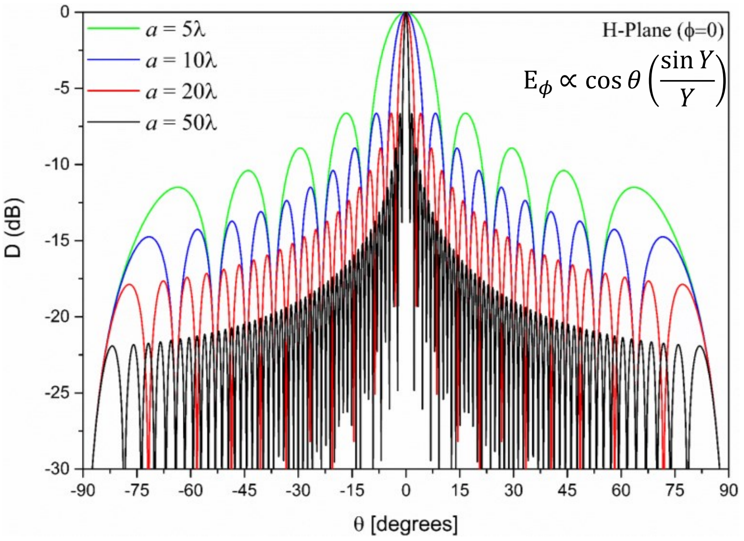
$$X = \frac{ka}{2} \sin\theta \sin\phi$$

$$Y = \frac{kb}{2} \sin\theta \cos\phi$$

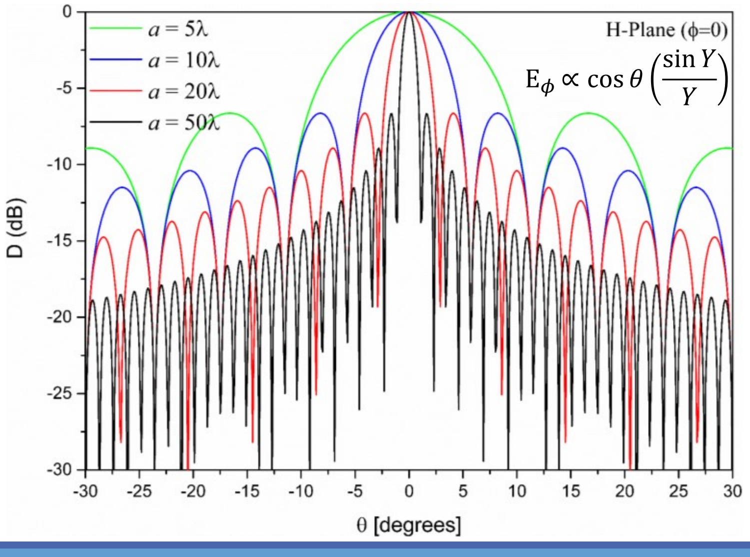
$$E_\phi = 0; \quad E_\theta \propto \left[\left(\frac{\sin Y}{Y} \right) \right]$$

$$E_\theta = 0; \quad E_\phi \propto \left[\cos\theta \left(\frac{\sin X}{X} \right) \right]$$

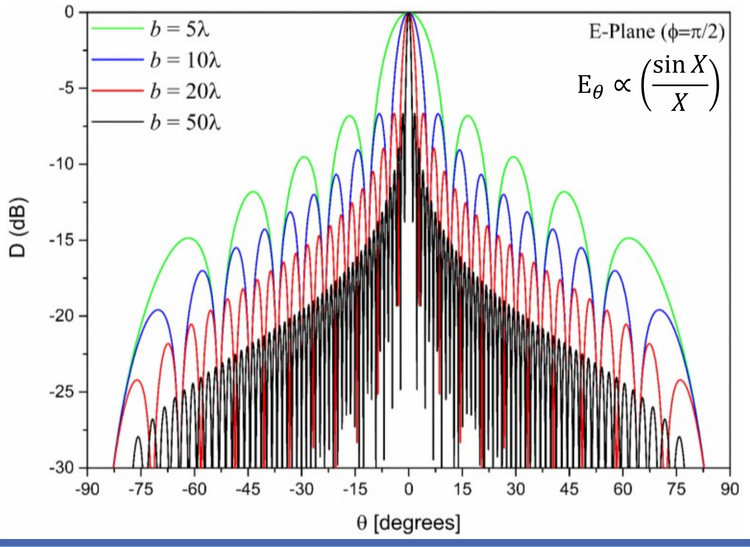
Rectangular Aperture



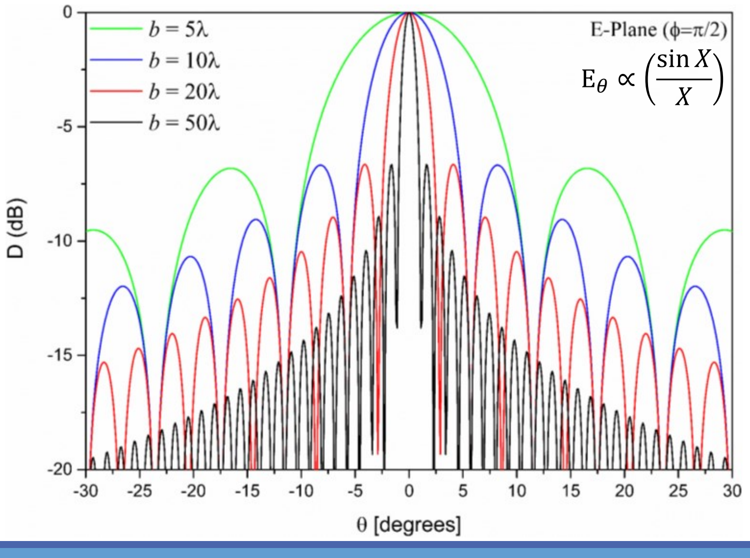
Rectangular Aperture



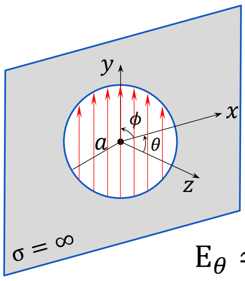
Rectangular Aperture



Rectangular Aperture



Circular Aperture



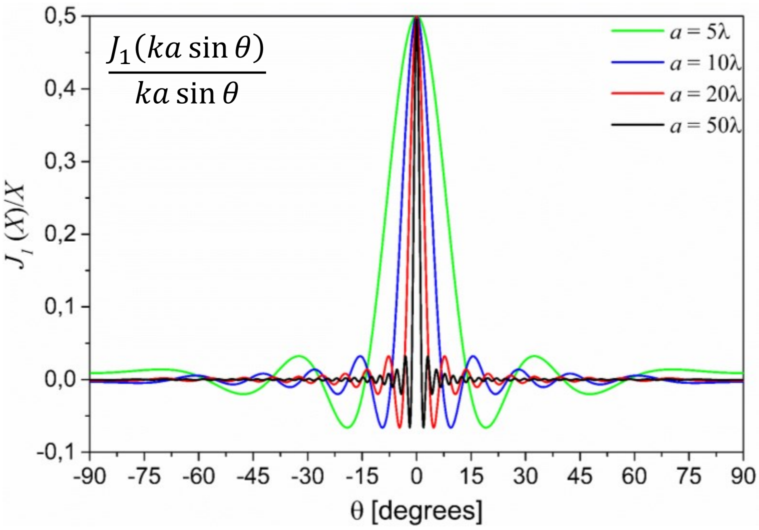
$$\vec{E}_A = E_o\hat{y} \qquad \vec{M}_S = -2E_o\hat{y}$$

$$E_\theta \simeq 2\pi a^2 E_o\left(\frac{jk}{2\pi}\right)\frac{e^{-jkr}}{r}\left\{\sin\phi\left[\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right]\right\}$$

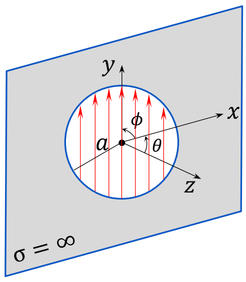
$$E_\phi \simeq 2\pi a^2 E_o\left(\frac{jk}{2\pi}\right)\frac{e^{-jkr}}{r}\left\{\cos\theta\cos\phi\left[\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right]\right\}$$

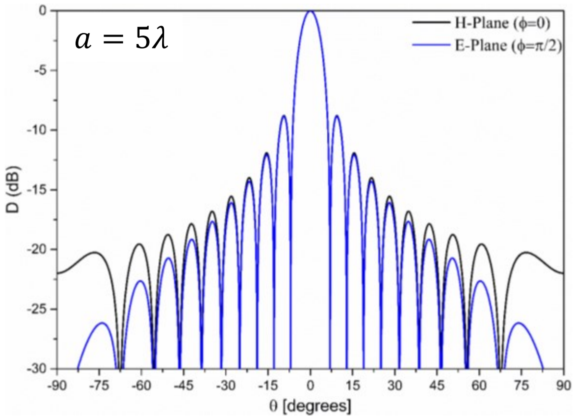
$J_1 \rightarrow$ Bessel function of first order

Bessel Function



Circular Aperture

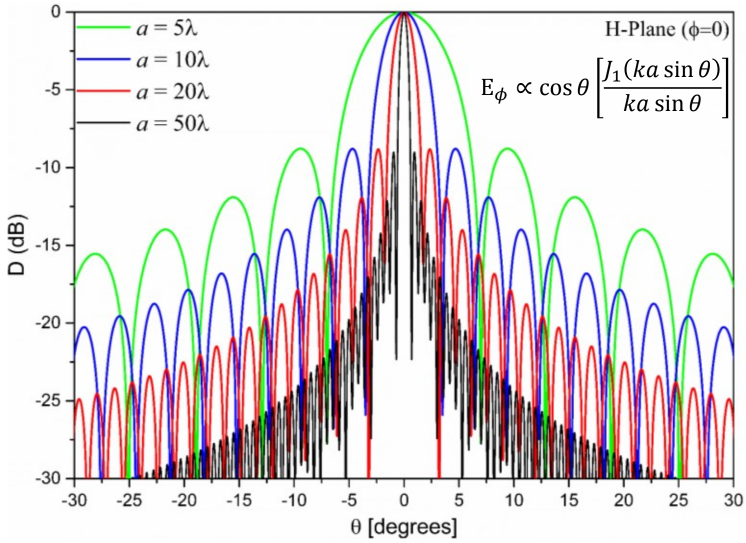




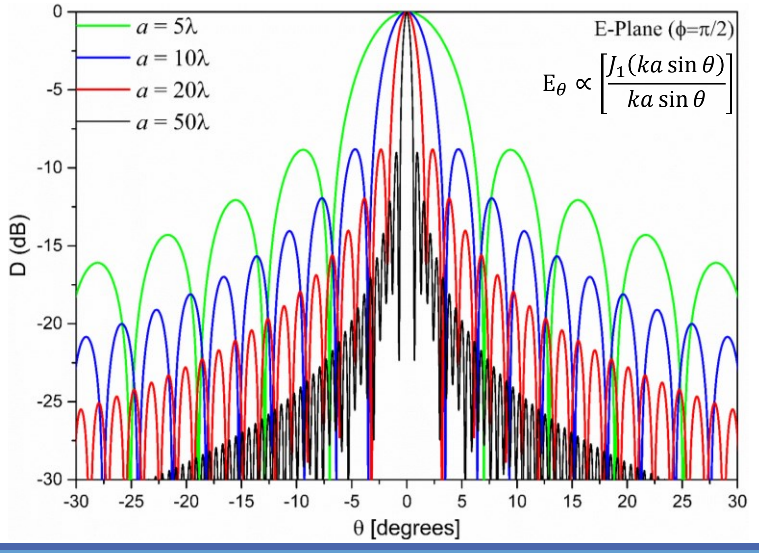
$$\mathrm{E}_{\phi} = 0; \quad \mathrm{E}_{\theta} \propto \left[\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right]$$

$$\mathrm{E}_{\theta} = 0; \quad \mathrm{E}_{\phi} \propto \cos\theta\left[\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right]$$

Circular Aperture



Circular Aperture



Uniform Aperture

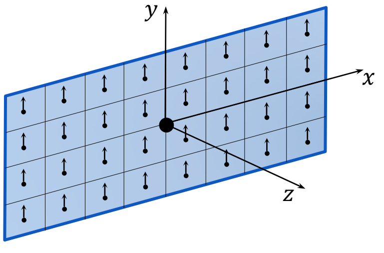
For any planar aperture the Directivity $D(\theta, \phi)$ is a function of:

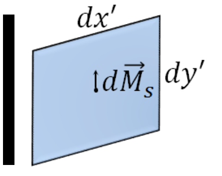
1. Phase and Amplitude of aperture fields ($\vec{E}_A$);
2. Polarization of aperture fields;
3. Area of aperture (a, b);

What are the conditions for maximum directivity ??

Uniform Aperture

The first and second conditions for maximum directivity are uniform polarization and phase of aperture electric field $\vec{E}_A$ (and, consequently, $\vec{M}_s$), so that, the sum of all contributions generates a vector with maximum amplitude.





$$|\vec{E}| \propto \sum dM_s dx' dy'$$

Uniform Aperture

If the phase and polarization of the aperture electric field are uniform, then, the maximum radiation occur in the normal direction to aperture plane, regardless of its geometric cross shape.

The last condition for maximum directivity is uniform amplitude of the aperture electric field .

Uniform Aperture

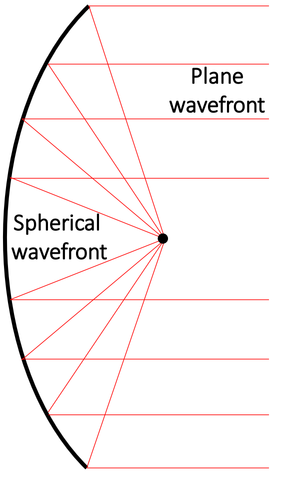
If the phase and polarization of the aperture electric field are uniform, then, the maximum radiation occurs in the normal direction of the aperture plane, regardless of its geometric shape.

The last condition for maximum directivity is uniform amplitude of the aperture electric field.

A planar aperture with amplitude, phase and polarization uniform is said Uniform Aperture.

Reflector Antennas

Introduction



Introduction

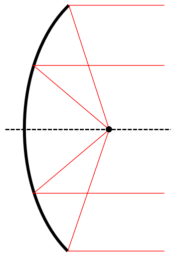
The operating principle of most reflector antennas is the same as lighthouse, lanterns, etc.: collimation of energy.

The parabolic reflector antenna transforms an incoming plane wave into a spherical wave converging toward the focus.

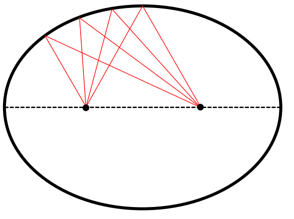
From another point of view, a spherical wave generated by a point source placed in the focus is reflected into a plane wave as a collimated beam.

Confocal Conics

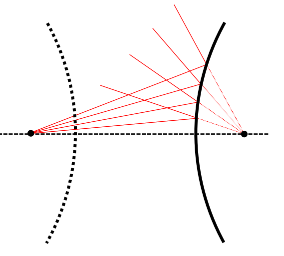
Confocal conics are conic sharing a common focus. The classical reflective surfaces (paraboloid, ellipsoid and hyperboloid) are generated by the rotation of conics around their.



Parable



Ellipse

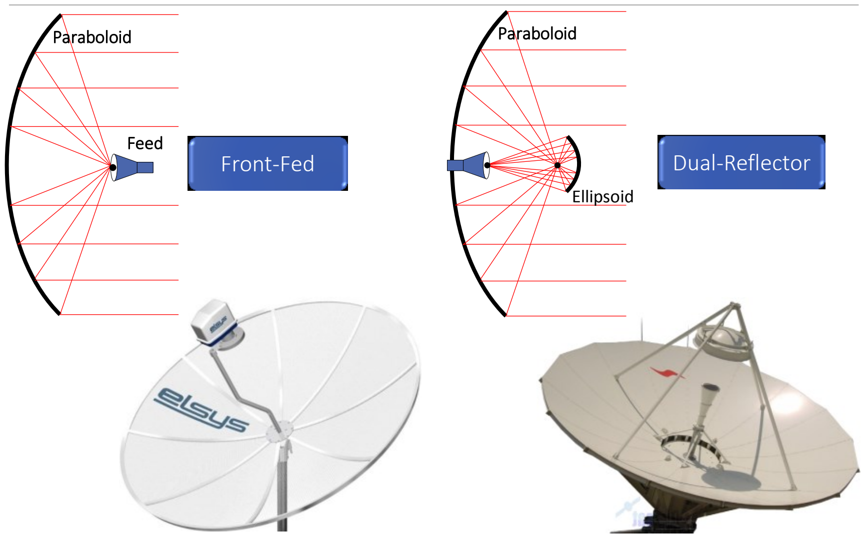


Hyperbola

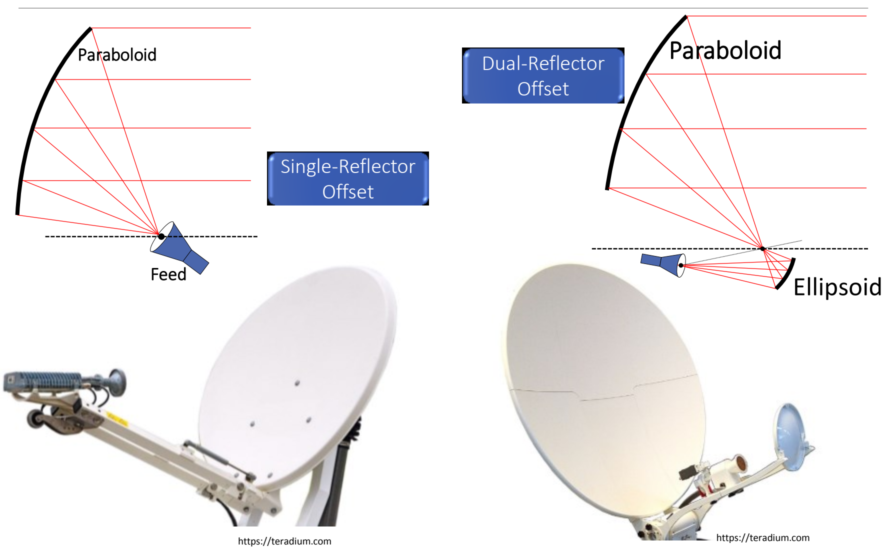
Introduction

- Directive Systems
  - Single Reflector
    - Front-Fed
    - Off-set Reflector (Classical or Shaped)
  - Dual Reflector Circularly Symmetric
    - Classical Reflector
    - Shaped Reflector
  - Dual Reflector Offset
    - Classical Reflector
    - Shaped Reflector
- Omnidirectional Systems
  - Single or Dual Classical Reflectors
  - Shaped Reflector(s)

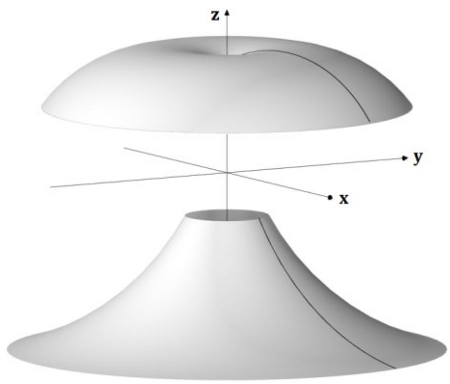
Circularly Symmetric



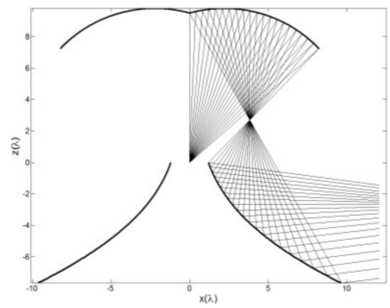
Offset Reflector Antennas

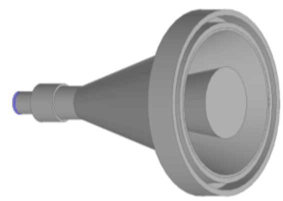


Omnidirecional Dual-Reflector



Dual-Reflector Omnidirectional





Feeders

Single Reflector Feeder

LNB C Band

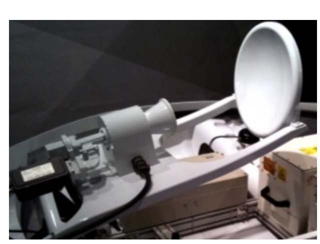
LNB Ku Band

Dual Reflector Feeder

http://www.ara-inc.com

Ku Band

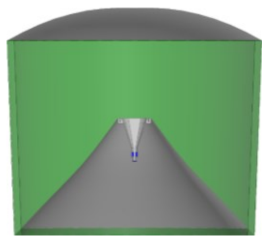
Ku Band



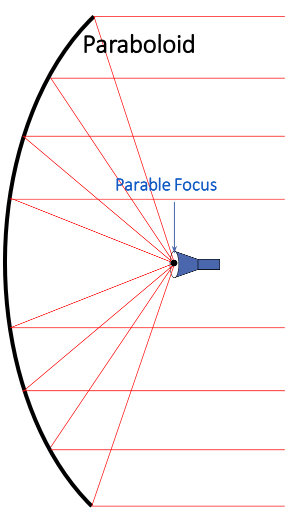
https://teradium.com

Radome

The Radome is a structural weatherproof made to protect the antenna surfaces from weather. They are constructed of dielectric material to be transparent to in antenna’s frequency operation.

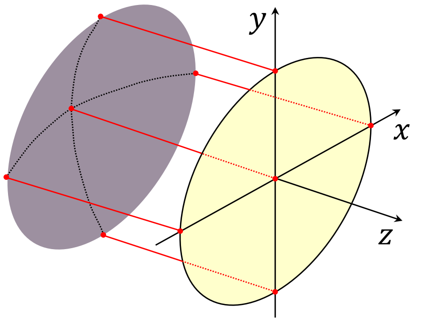


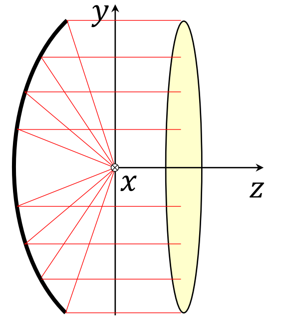
Front-Fed



- Simple Geometry;
- Aperture blockage by feeder and its supports;
- Low manufacturing cost;
- Satellite reception, microwave point-to-point links, etc.;

Front-Fed

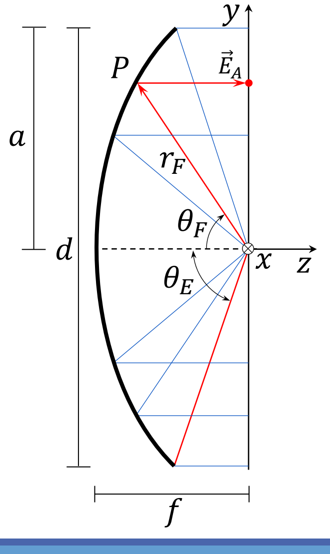




The reflector antena's aperture can be defined by the projection of the main reflector on a constant phase plane .

The approximate analysis can be made using Aperture Method (ApM) with aperture field given para by Geometrical Optics;

Front-Fed



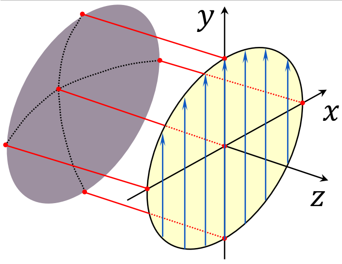
$$r_F = \frac{2f}{\cos\theta_F + 1} \qquad \tan\left(\frac{\theta_E}{2}\right) = \frac{d}{4f}$$

$$r_F = f\sec^2(\theta_F/2) \qquad r_F + \overline{PA} = 2f$$

Maximum directivity:

- **Uniform Phase:**
  - Constant optical path length ($r_F + \overline{PA} = \text{const.}$);
- **Uniform amplitude:**
  - Feeder with $D(\theta_F) \propto \sec^4(\theta_F/2)$
- **Uniform Polarization:**
  - Feeder with circularly symmetric radiation pattern;

Front-Fed



Uniform Aperture in Free Space

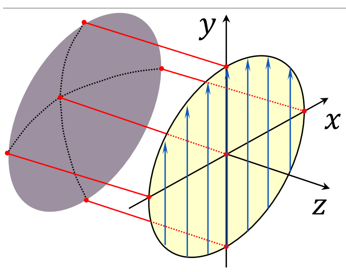
$$\vec{E}_A = E_o\hat{y} \qquad \vec{M}_S = +E_o\hat{x}$$

$$\vec{H}_A = -\frac{E_o}{\eta}\hat{x} \qquad \vec{J}_S = +E_o\hat{y}$$

$$E_\theta \simeq 2\pi a^2 E_o \left(\frac{jk}{2\pi}\right)\frac{e^{-jkr}}{r}\left\{\cos\phi\,(1+\cos\theta)\left[\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right]\right\}$$

$$E_\phi \simeq -2\pi a^2 E_o \left(\frac{jk}{2\pi}\right)\frac{e^{-jkr}}{r}\left\{\sin\phi\,(1+\cos\theta)\left[\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right]\right\}$$

Front-Fed



Uniform Aperture in Free Space

$$D(\theta,\phi) = \left(\frac{\pi}{\lambda}d\right)^2 (1+\cos\theta)\left[\frac{J_1(\Omega)}{\Omega}\right]$$

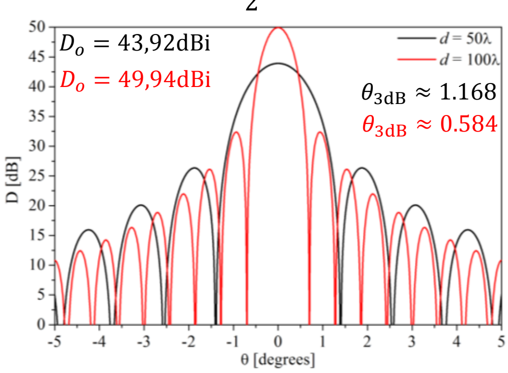
$$\Omega = k\frac{d}{2}\sin\theta$$

Maximum directivity:

$$D_o(\theta = 0) = \left(\frac{\pi}{\lambda}d\right)^2$$

3dB Beamwidth:

$$\theta_{3dB} \approx 58.4\left(\frac{\lambda}{d}\right) \quad [\text{deg.}]$$



Front-Fed: Aperture Efficiency

Aperture efficiency (e_{ap}) is defined as the ratio of the maximum directivity D_o (or effective area A_{em}) to an uniform aperture with the same geometry and radiated power (or its physical area A_p):

$$\epsilon_{ap} = \frac{A_{em}}{A_p} = \frac{D_o}{\left(\frac{\pi}{\lambda} d\right)^2} = D_o \left(\frac{\lambda}{\pi d}\right)^2$$

Front-Fed: Aperture Efficiency

$$\epsilon_{ap} = \cot^2\left(\frac{\theta_E}{2}\right) \left| \int_0^{\theta_E} \sqrt{G_f(\theta')} \tan\left(\frac{\theta'}{2}\right) d\theta' \right|^2$$

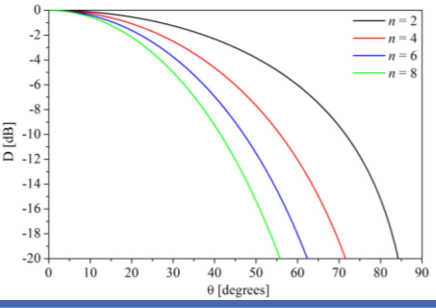
The aperture efficiency is a function of the subtended angle θ_E and the feed pattern $G_f(\theta')$ of the reflector. Thus for a given feed pattern, all paraboloids with the same ratio f/d have identical aperture efficiency.

$$\frac{f}{d} = \frac{1}{4} \cot\left(\frac{\theta_E}{2}\right)$$

Front-Fed: Aperture Efficiency

To illustrate the variation of the aperture efficiency as a function of the feed pattern and the angular extent of the reflector function $G_f(\theta')$ is defined as:

$$G_f(\theta') = \begin{cases} G_o \cos^n \theta' & 0 \le \theta' \le \dfrac{\pi}{2} \\ 0 & \dfrac{\pi}{2} \le \theta' \le \pi \end{cases}$$



Front-Fed: Aperture Efficiency

$$\epsilon_{ap} = \epsilon_s \epsilon_t \epsilon_p \epsilon_x \epsilon_b \epsilon_r$$

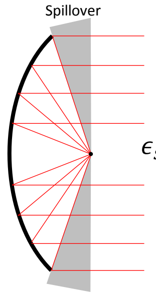
Aperture Efficiency:

- **Spillover Efficiency (ϵ_s)**
- **Taper Efficiency (ϵ_t)**
- Phase Efficiency (ϵ_p)
- Polarization Efficiency (ϵ_x)
- Blockage Efficiency (ϵ_b)
- Random error Efficiency (ϵ_r)

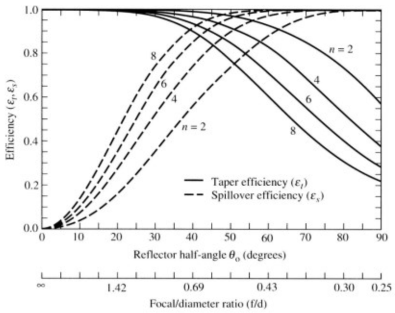
Front-Fed: Aperture Efficiency

Spillover Efficiency (ϵ_s): fraction of the total power that is radiated by the feed, intercepted, and collimated by the reflecting surface;

Taper efficiency (ϵ_t): uniformity of the amplitude distribution of the feed pattern over the surface of the reflector.

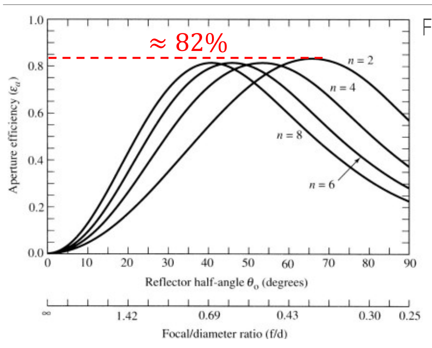


$$\epsilon_s = \frac{\int_0^{\theta_E} G_f(\theta') \sin\theta' d\theta'}{\int_0^{\pi} G_f(\theta') \sin\theta' \, d\theta'}$$



C. A. Balanis, Antenna Theory: Analysis and Design

Front-Fed: Aperture Efficiency



C. A. Balanis, Antenna Theory: Analysis and Design

For a given feed pattern (*n* = constant):

- There is only one reflector with a given angular θ_E aperture or f/d ratio which leads to a maximum aperture efficiency.
- Each maximum aperture efficiency is in the neighborhood of 82–83%.

- As the feed pattern becomes more directive (*n* increases), the angular aperture of the reflector that leads to the maximum efficiency is smaller.

n	θ_E	f/d	$A_B(dB)$
2	67°	0.4	−11.4
4	54°	0.5	−11.2
6	46°	0.6	−10.9
8	42°	0.7	−11.5

Front-Fed: Design Example

1. Design a Front-Fed reflector antenna with maximum gain $G_o = 50\text{dBi}$;

$$G_o = \epsilon_s \epsilon_t \left(\frac{\pi}{\lambda} d\right)^2 \qquad 10^{50/10} = 0.8\left(\frac{\pi}{\lambda} d\right)^2 \qquad d \approx 113\lambda$$

2. Determine θ_E for a feeder with $n = 9$;

$$A_B(\text{dB}) \approx 10\log\left[\cos^4\left(\frac{\theta_E}{2}\right)\cos^n \theta_E\right] \qquad \epsilon_{ap} = 0.8 \rightarrow A_B \approx -11\text{dB}$$

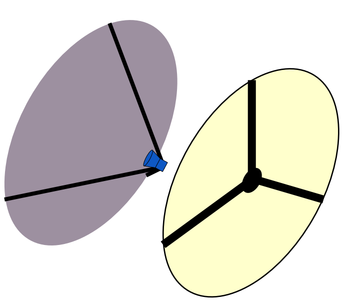
$$10\log\left[\cos^4\left(\frac{\theta_E}{2}\right)\cos^9 \theta_E\right] \approx -11 \qquad \theta_E \approx 39°$$

3. Determine f/d for a feeder with $n = 9$;

$$\frac{f}{d} = \frac{1}{4}\cot\left(\frac{39°}{2}\right) = 0.706 \qquad f \approx 80\lambda$$

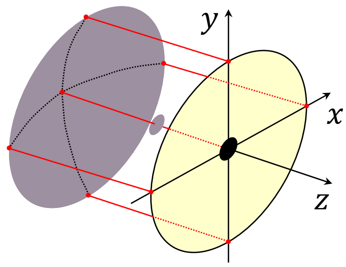
Front-Fed: Aperture Method

The Aperture Method is useful to provide physical understanding and initial reflector antenna design. However, the method has several inaccuracies, for example:

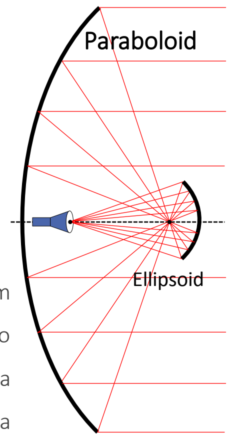


- It does not take into account feed and supports blockage.
- Diffractions at border reflector
- Coupling effects between reflector surface, supports and feeder;
- Direct fields from feeder;

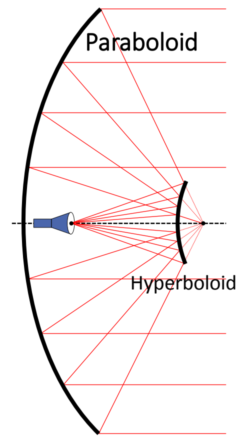
Axis-symmetric Dual-Reflector



Another way to obtain uniform phase in the aperture is using two reflectors. The main reflector is a paraboloid and the subrefletor is a ellipsoid (Gregorian) or hyperboloid (Cassegrain).

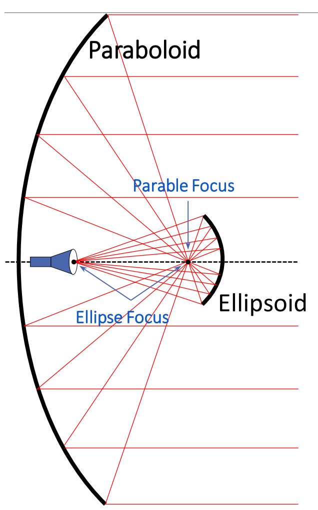


Gregorian



Cassegrain

Dual-Reflector: Gregorian



Paraboloid

Rx Frequency	Rx Gain	Tx Frequency	Tx Gain
3.625 GHz	46.5	6.175 GHz	50.9
4.000 GHz	47.4	6.425 GHz	51.1
4.200 GHz	47.8	6.725 GHz	51.4

Beamwidth, Mid-band, Degrees	*C-Band*
3 dB Receive (Transmit)	.74 (.45)
15 dB Receive (Transmit)	1.46 (.87)

Dual Reflector: Gregorian

Andrew Antenna 7.6m, Ku-band

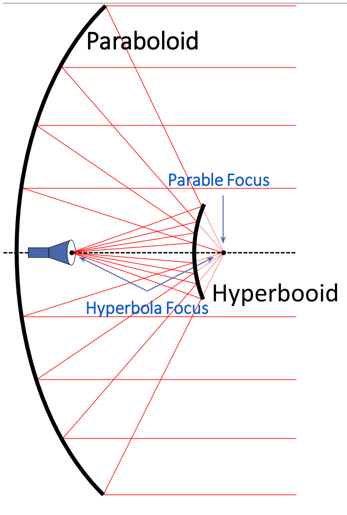
Optics Configuration: Gregorian

Frequency
- Transmit: 14 to 14.8 GHz
- Receive: 10.7 to 13.25 GHz

Antenna Gain
- Transmit: 59.4dBi (14.5 GHz)
- Receive: 58.3 dBi (12.75 GHz)

3 dB Beamwidth
- Transmit: 0.18°
- Receive: 0.22°

Dual-Reflector: Cassegrain



Vertex Antenna 6.1m C-band

Frequency:
- Rx 3.6 to 4.2 GHz / Tx. 5.8 to 6.4GHz

Gain:
- Rx 47.1 dBi (4.2GHz) / Tx.: 50.0 dBi (6.4 GHz)

3dB Beamwidth
- Rx.: 0.82° / Tx.: 0.56°

Dual Reflector: Cassegrain

Source: http://www.sky-brokers.com

GDSatcom 13.2M Ka-Band Antenna

Optics Configuration: Cassegrain

Frequency

- Transmit: 28 to 30 GHz
- Receive: 18 to 20 GHz

Antenna Gain

- Transmit: 69dBi (30 GHz)
- Receive: 66dBi (20 GHz)

3 dB Beamwidth

- Transmit: 0.06°
- Receive: 0.09°

Single Reflector x Dual Reflector

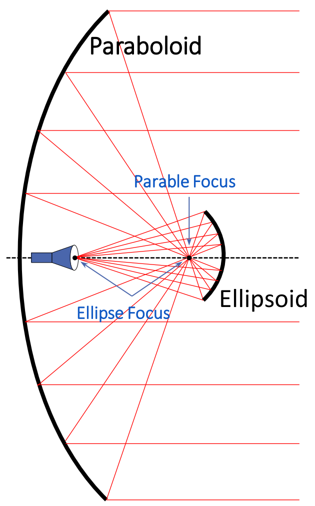
Advantages

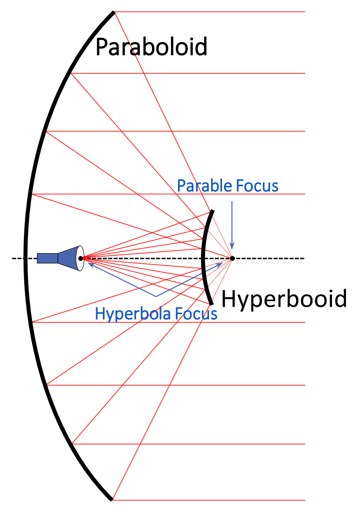
- Easy access to the feeder;
- Less thermal noise. The feeder is pointed at the sky.
- Greater control over the aperture fields (2 reflectors);
- **More efficient**

Disadvantages

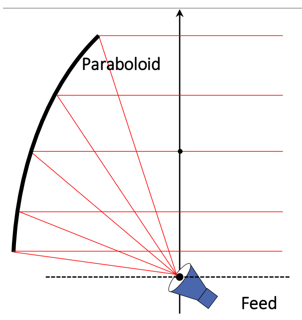
- Problems to access the feeder
- Higher manufacturing cost;
- More complex, bigger and directive feeder;
- Support structures for subreflector surface;
- Aperture blockage;

Dual-Reflector Symmetric





Offset Reflector

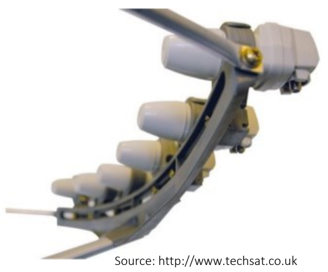


- Geometry more complex then Front-Fed.
- No aperture blockage by feeder and its supports;
- Satellite communications, radar, etc. ;
- This geometries generates cross-polarization;

Offset Reflector

Source: http://www.techsat.co.uk

Offset reflector antennas can use over a feeder in order to produce a multibeam oppering in many satellites at the same time.



Source: http://www.techsat.co.uk

Frequency range: 10,7 - 12,75 GHz

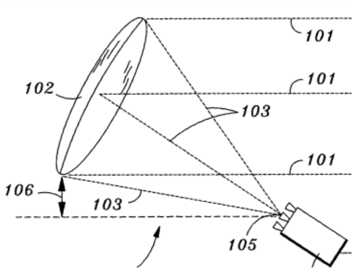
Antenna width / height: 91 cm / 70 cm

Multi beam capability: +/- 20° orbital

Gain (dB): 36 dBi

3 dB beamwidth: 2,3°

Offset Reflector



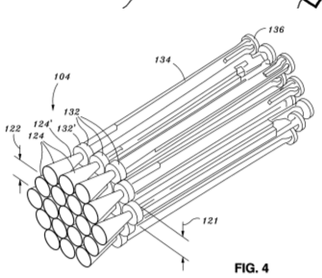
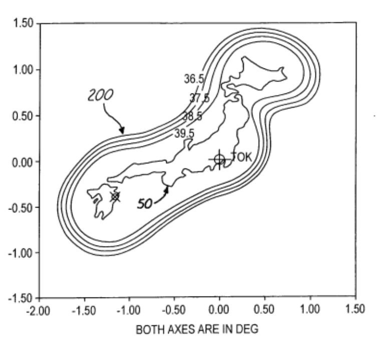
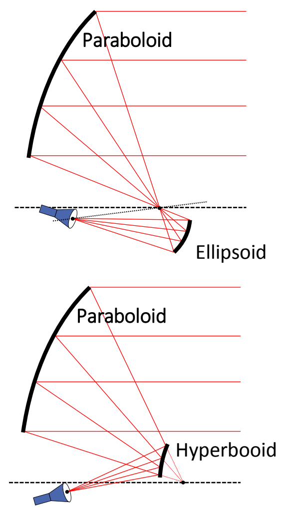


FIG. 4

Offset reflector antennas can use with many feeder in order to produce a shaped coverage area.



Offset Dual-Reflector

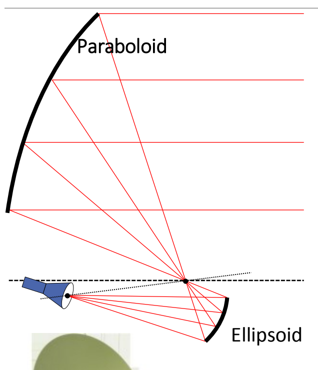


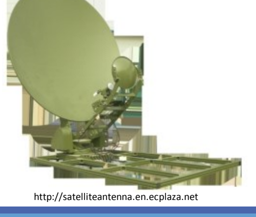
- Geometry very complex.
- No aperture blockage;
- Satellite communications
- This geometries generates cross-polarization;
- Antennas with very high efficiency

http://www.networkinv.com

http://news.cision.com/swe-dish-satellite-systems

Offset Dual-Reflector: Gregorian





http://satelliteantenna.en.ecplaza.net

https://teradium.com

https://teradium.com

https://teradium.com

TD-120 Ku 1.20 m

Frequency :

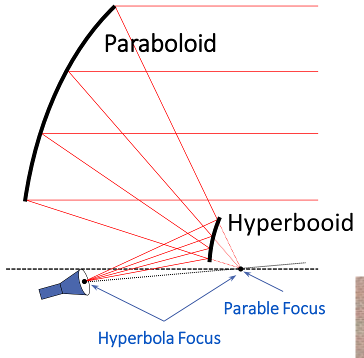
- Transmit: 13.75 to 14.5 GHz
- Receive: 10.7 to 12.75 GHz

Antenna Gain

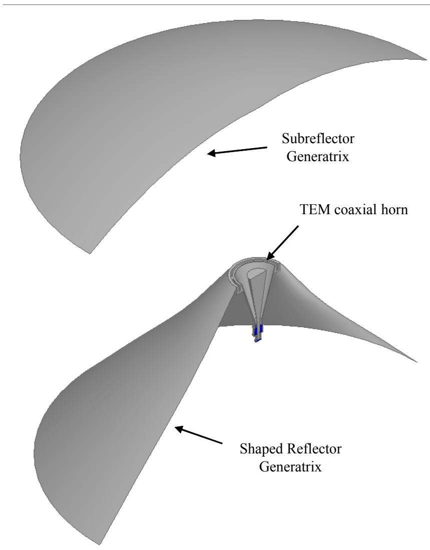
- Transmit: 42.5 dBi @ 14.25 GHz
- Receive: 41.8 dBi @ 11.7 GHz

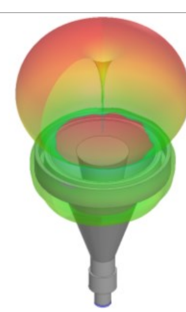
https://teradium.com

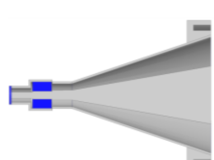
Offset Dual-Reflector: Cassegrain

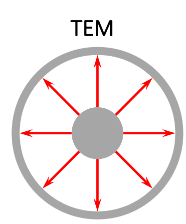


Omnidirectional Dual-Reflector





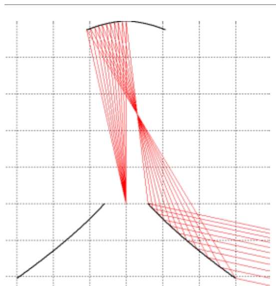




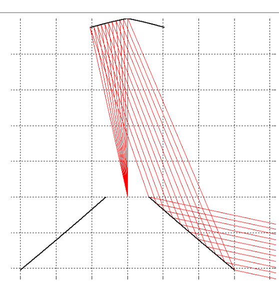
Omnidirectional Dual-Reflector

Application	• WiMax • LMDS (*Local Multipoint Distribution Service*) • 5G (28GHz / 38GHz)
Main benefits	• Broadband behavior • High gain antennas • Compact structures
Shaping procedure	• Evaluation of an ordinary differential equation • Concatenation of short local conic sections

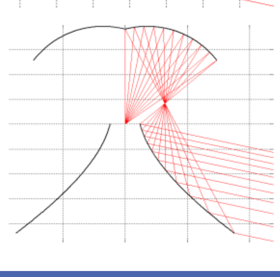
Omnidirectional Dual-Reflector



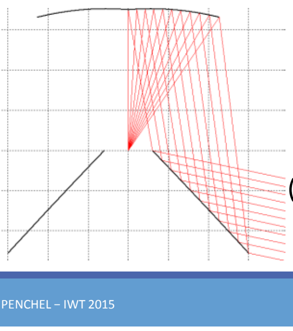
OADG
(Gregorian)



OADH
(Hyperbola)

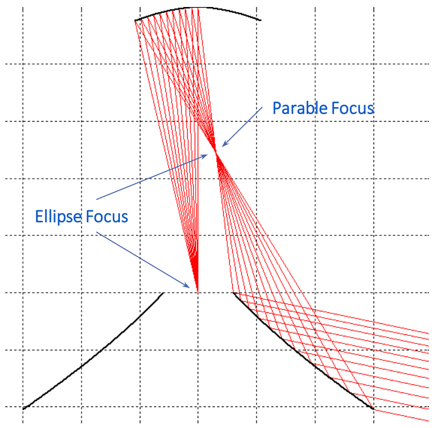


OADE
(Ellipse)



OADC
(Cassegrain)

Omnidirectional Dual-Reflector



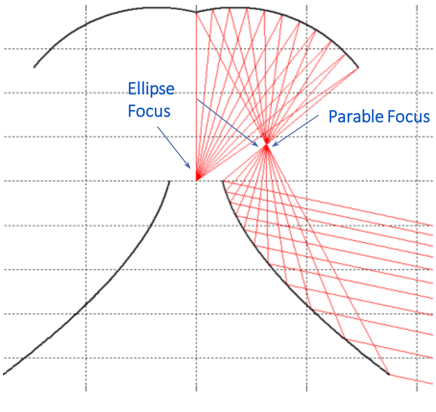
OADG

(Omnidirectional Axis-Displaced Gregorian)

Real Ring Caustic

The ray mapping is made between the negative side of the ellipse and the positive side of the parable.

Omnidirectional Dual-Reflector



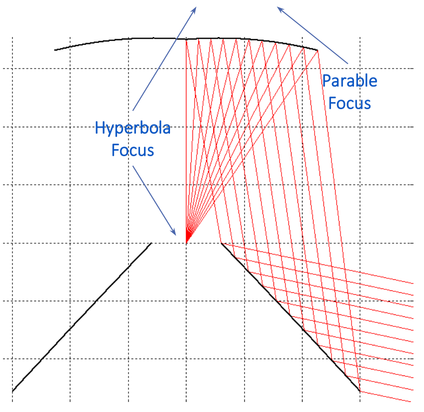
OADE

(Omnidirectional Axis-Displaced Ellipse)

Real Ring Caustic

The ray mapping is made between the positive side of the ellipse and the positive side of the parable.

Omnidirectional Dual-Reflector



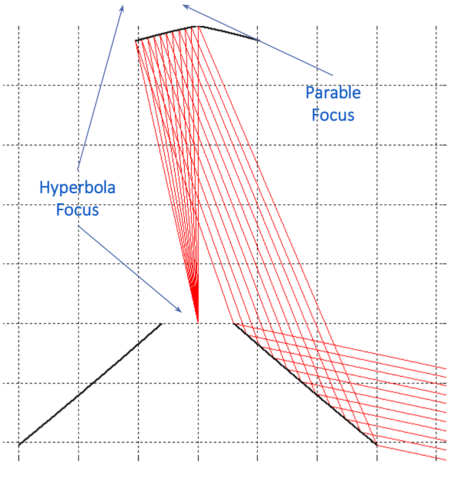
OADC

(Omnidirectional Axis-Displaced Cassegrain)

Virtual Ring Caustic

The ray mapping is made between the positive side of the hyperbola and the positive side of the parable.

Omnidirectional Dual-Reflector



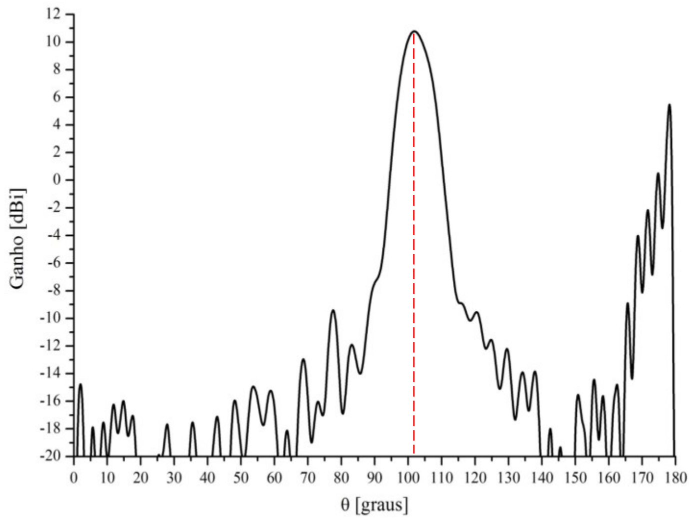
OADH

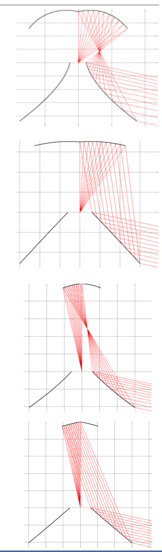
(Omnidirectional Axis-Displaced Hyperbola)

Virtual Ring Caustic

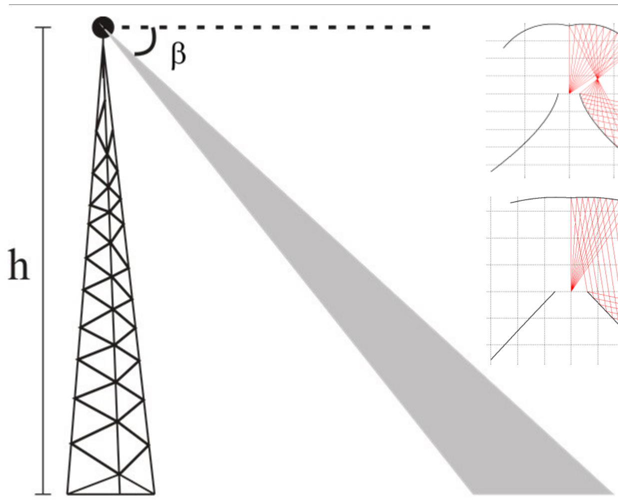
The ray mapping is made between the negative side of the hyperbola and the positive side of the parable.

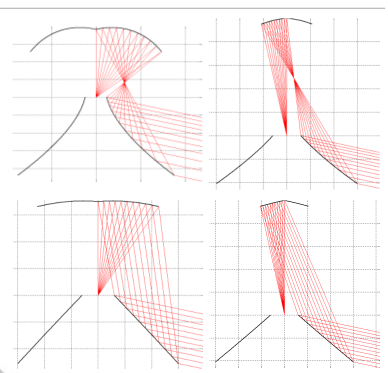
Directive Radiation Pattern



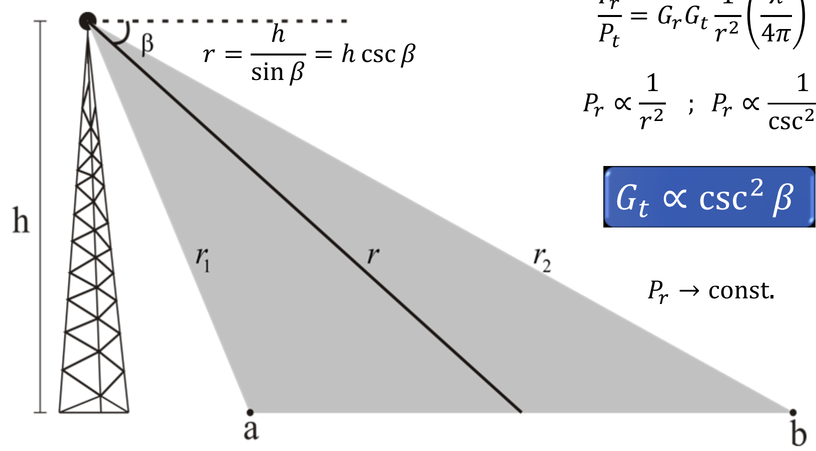


Directive Radiation Pattern





Cosecant Square Radiation Pattern



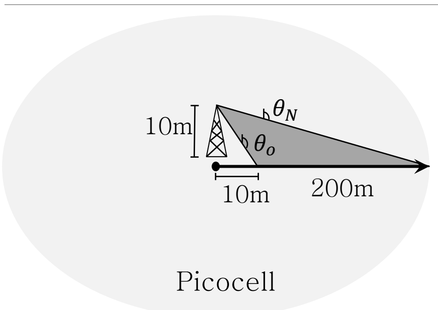
$$\frac{P_r}{P_t} = G_r G_t \frac{1}{r^2}\left(\frac{\lambda}{4\pi}\right)^2$$

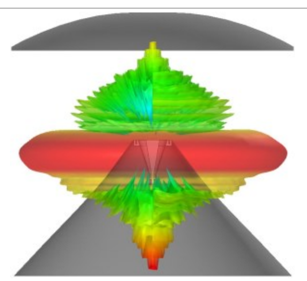
$$P_r \propto \frac{1}{r^2} \quad ; \quad P_r \propto \frac{1}{\csc^2\beta}$$

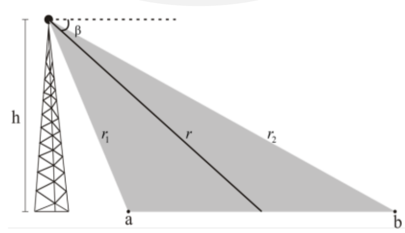
$$G_t \propto \csc^2\beta$$

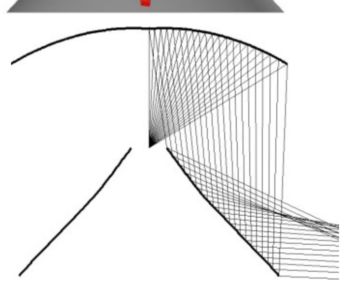
$$P_r \rightarrow \text{const.}$$

Cosecant Square Radiation Pattern



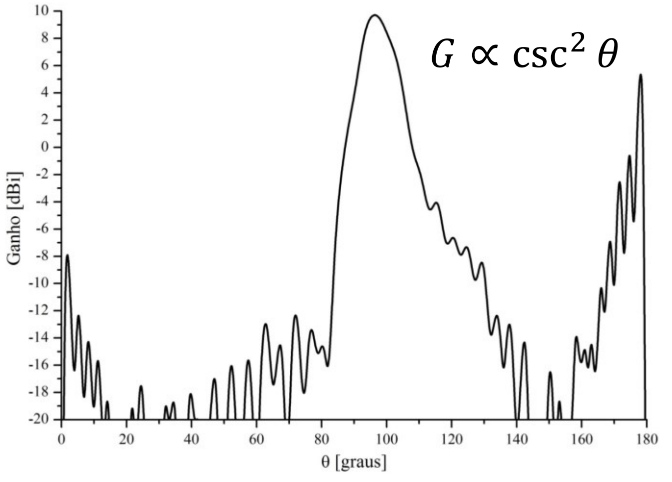


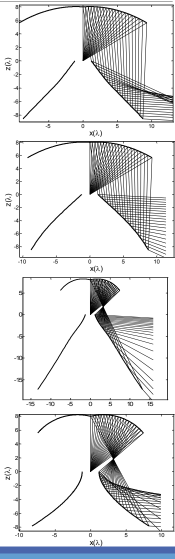




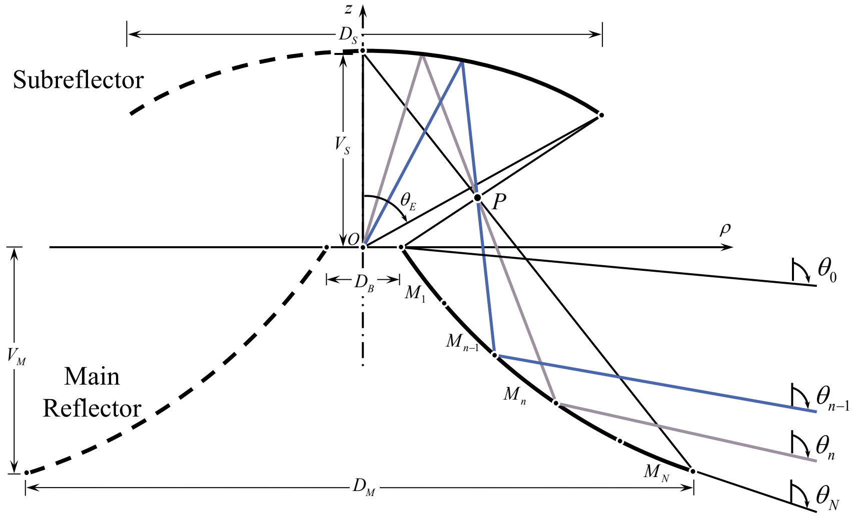
Cosecant Square Radiation Pattern

$$G \propto \csc^2 \theta$$

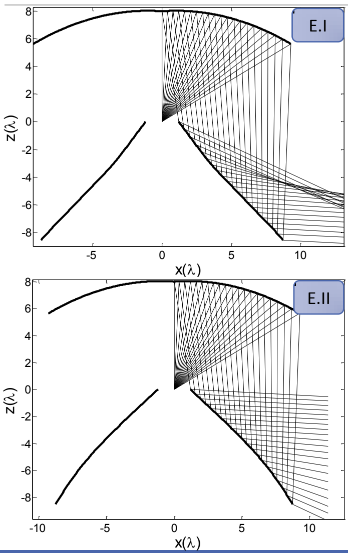


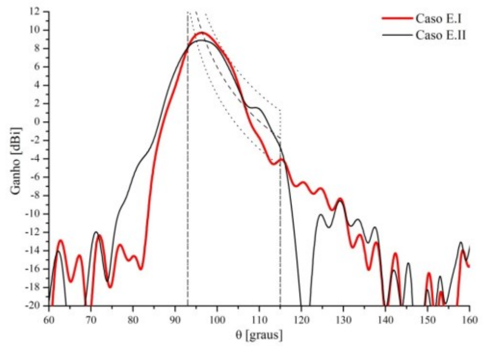


Shaping Procedure



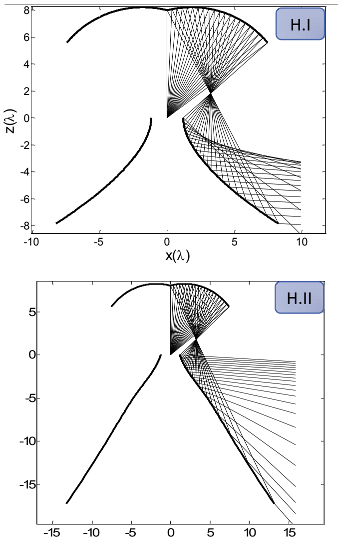
Cosecant Square ($93° < \theta < 115°$)

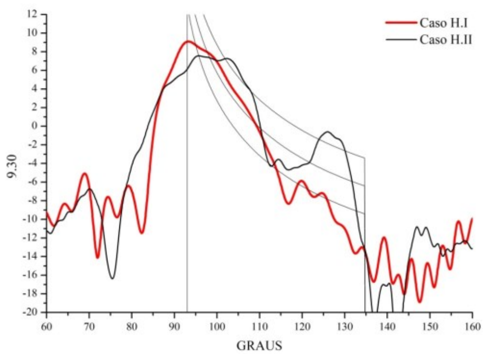




Parameters	OADC Real Caustic	OADC Virtual Caustic
D_M (λ)	17,515	17,521
V_M (λ)	8,566	8,482
VOL.($\lambda^3 10^3$)	4,295	4,275

Cosecant Square ($93° < \theta < 115°$)





Parameters	OADC Real Caustic	OADC Virtual Caustic
D_M (λ)	**16,532**	**26.424**
V_M (λ)	7,834	17,308
VOL.($\lambda^3 10^3$)	3,057	8,299

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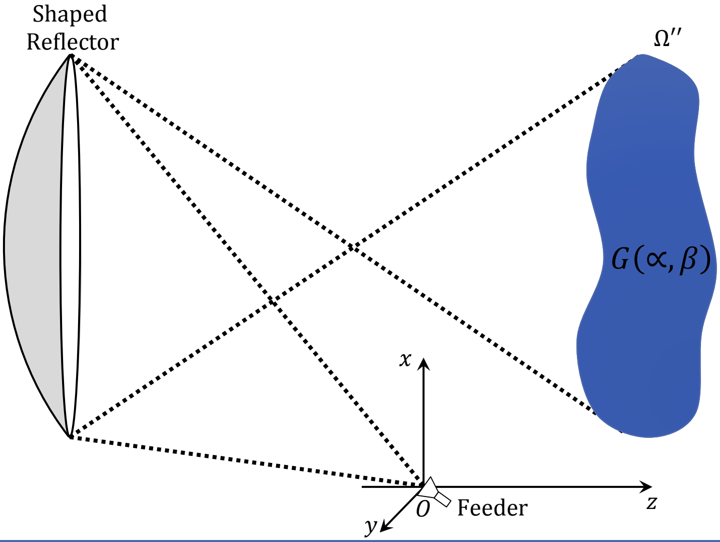
Shaped Offset

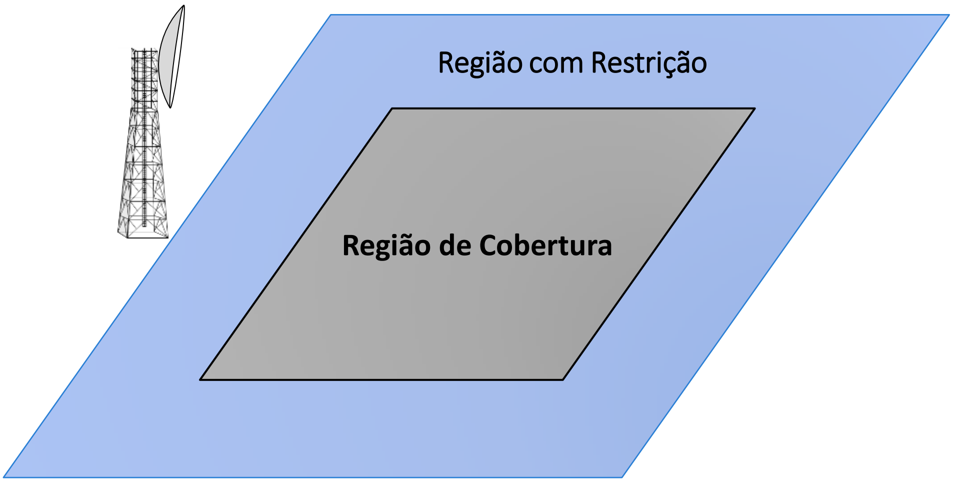


Shaped Offset Reflector

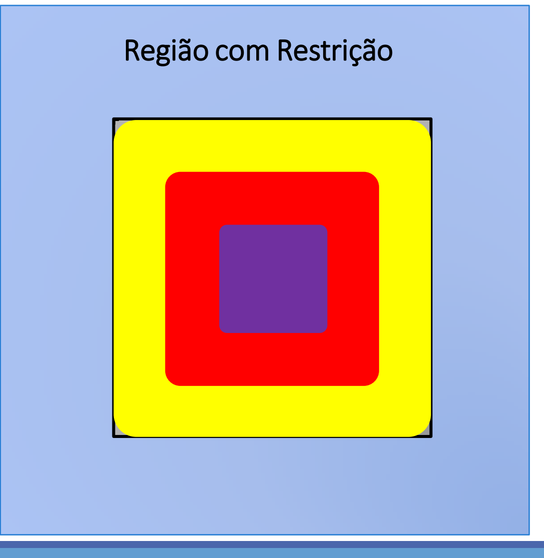
Aplication	• Satellite Communications • Restrict cellular coverage area
Main benefits	• Optimized coverage • Low co-channel interference

Numerical Technique	• Monge-Ampère Differential Equation
	  • Finite Differences
	  • Local Confocal Quadric Surfaces

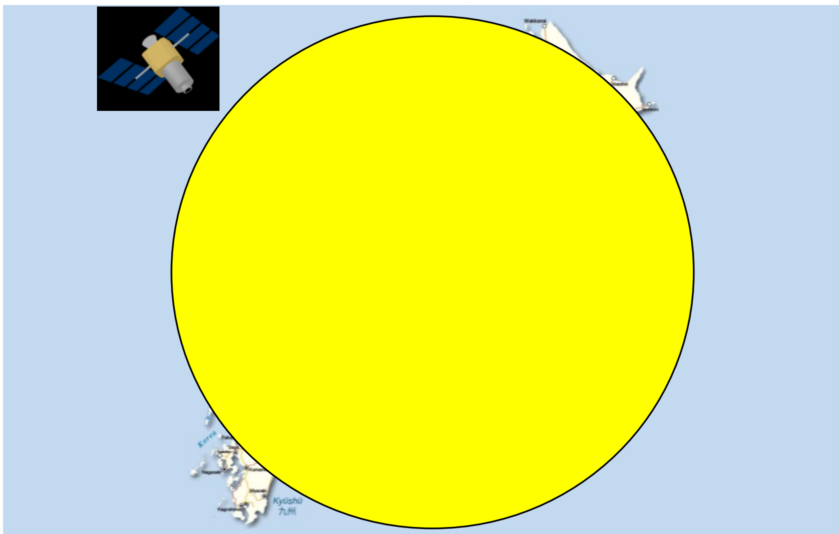
Shaped Offset: Super-Elliptical Coverage



Shaped Offset: Super-Elliptical Coverage



Shaped Offset: Generic Coverage



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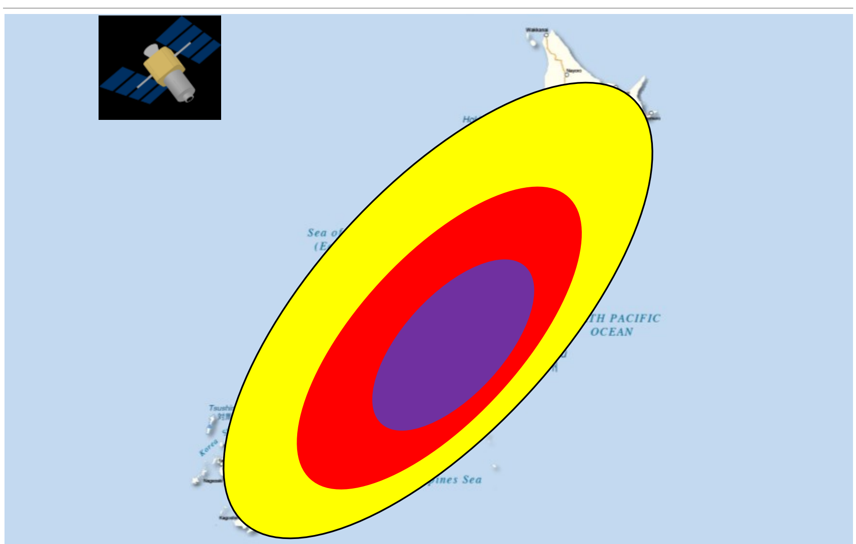
Shaped Offset: Generic Coverage



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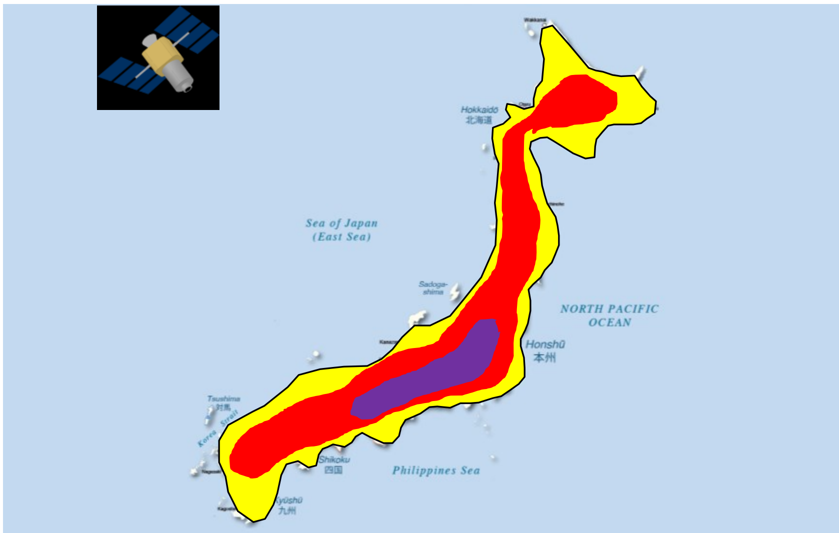
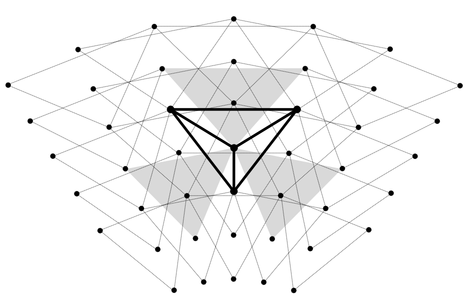
Shaped Offset: Generic Coverage

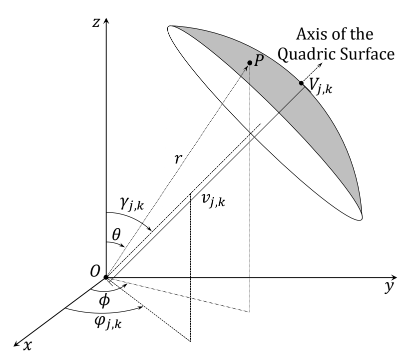


Shaping Procedure

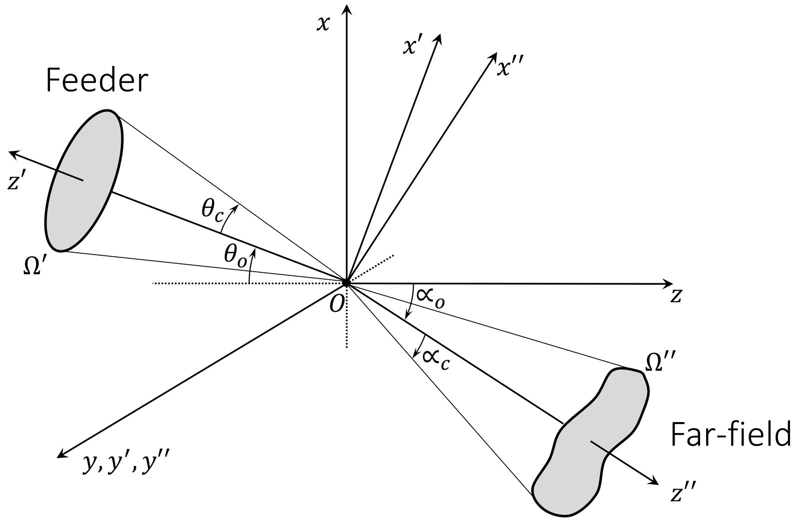


Polar Grid

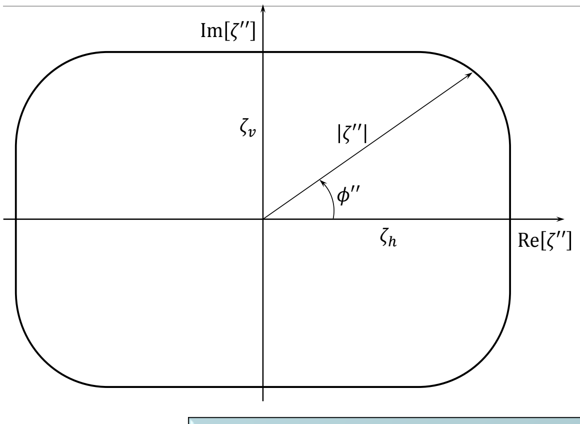
Confocal Quadrics



Shaped Offset: Contour



Shaped Offset: Contour

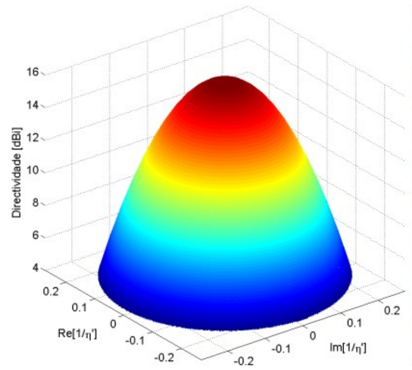


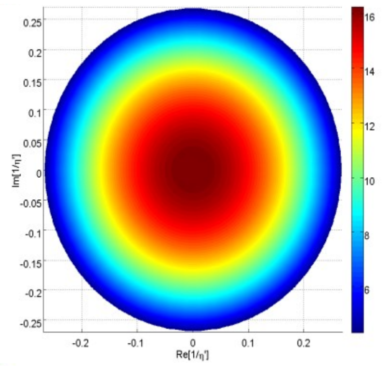
$$\zeta_v = \cot\left(\frac{\alpha_v}{2}\right)$$

$$\zeta_u = \cot\left(\frac{\alpha_u}{2}\right)$$

$$\left|\frac{\mathrm{Re}[\zeta'']}{\zeta_u}\right|^{2\sigma} + \left|\frac{\mathrm{Im}[\zeta'']}{\zeta_v}\right|^{2\sigma} = 1$$

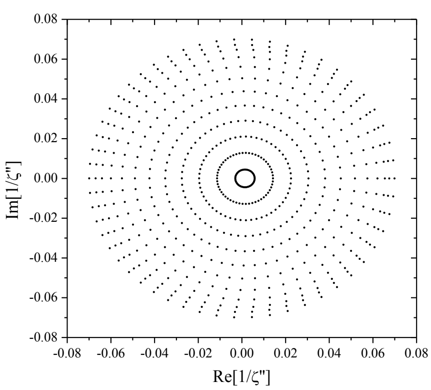
Shaped Offset: Feeder



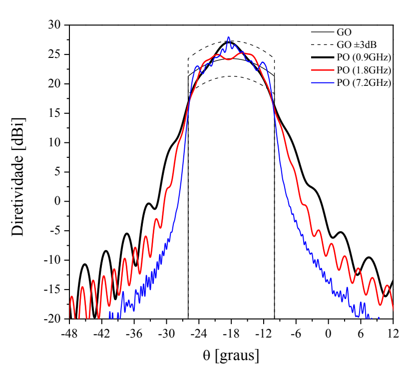


$$I(\theta') = I_0 \cos^{2n} \theta' °, n = 9.6$$

Shaped Offset: Circular Contour

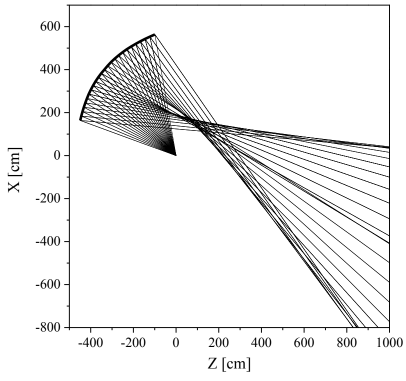


$$\alpha_c = 8°, \alpha_o = 18°$$



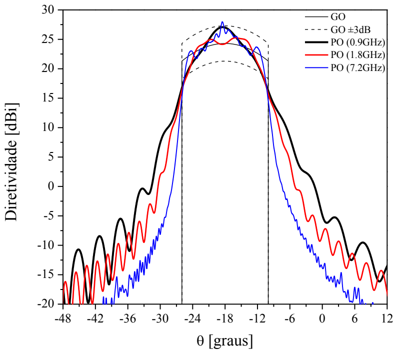
$$\psi = 0{,}69, L_G = -3\text{dB}$$

Shaped Offset: Circular Contour

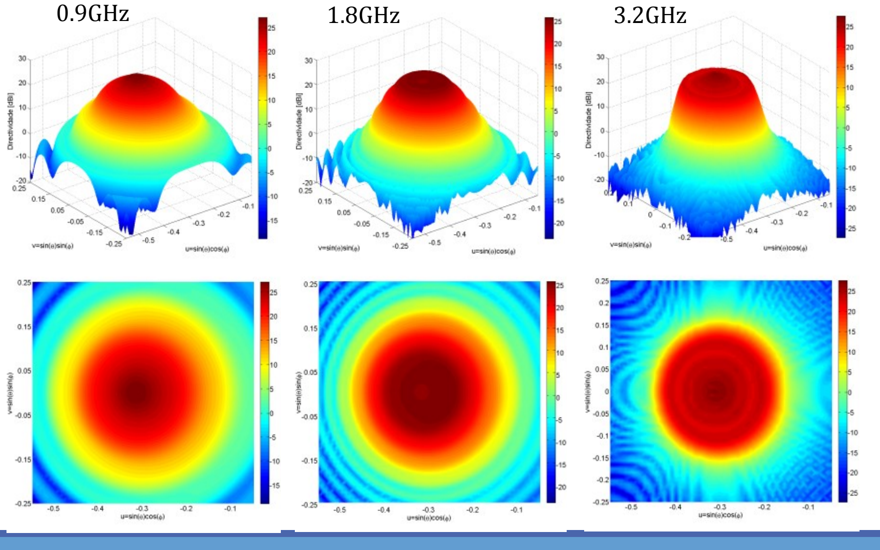


$\alpha_c = 8°, \alpha_o = 18°$

$\psi = 0{,}69, L_G = -3\text{dB}$

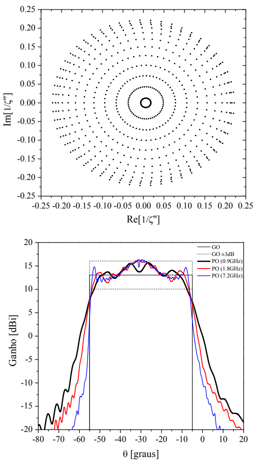


Shaped Offset: Circular Contour

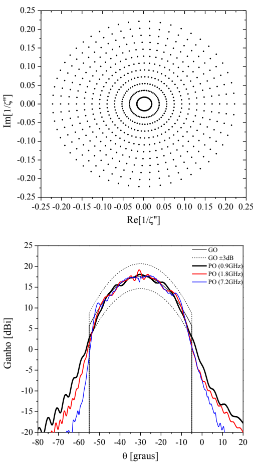


Shaped Offset: Circular Contour

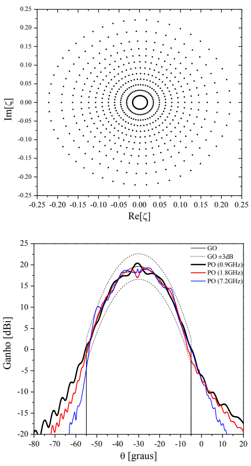
$L_G = 0\text{dB}$



$L_G = -12\text{dB}$

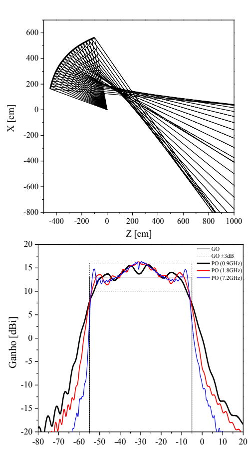


$L_G = -20\text{dB}$

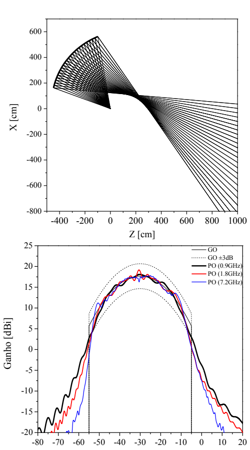


Shaped Offset: Circular Contour

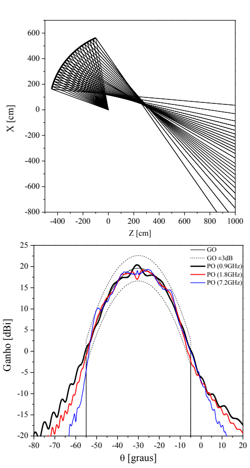
$L_G = 0\text{dB}$



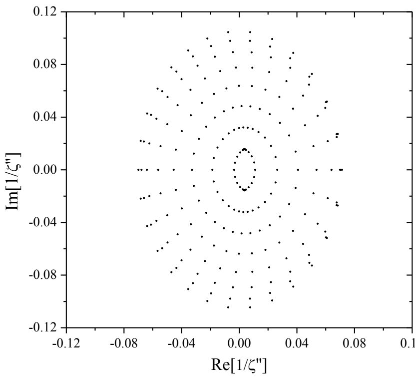
$L_G = -12\text{dB}$



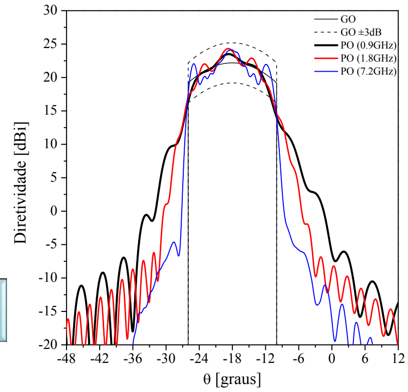
$L_G = -20\text{dB}$



Elliptic Coverage Contour



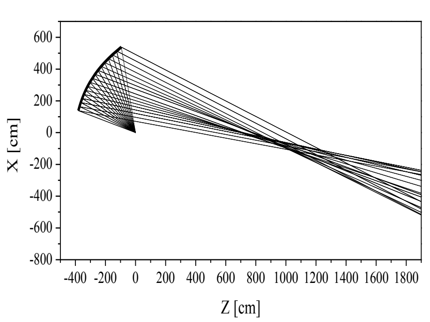
$$\alpha_o = 18°$$
$$\alpha_u = 8°, \alpha_v = 12°$$

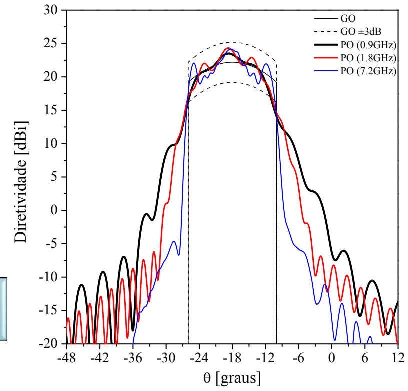


$$\psi = 0{,}69, L_G = -3\text{dB}$$

Shaped Offset: Elliptical Contour

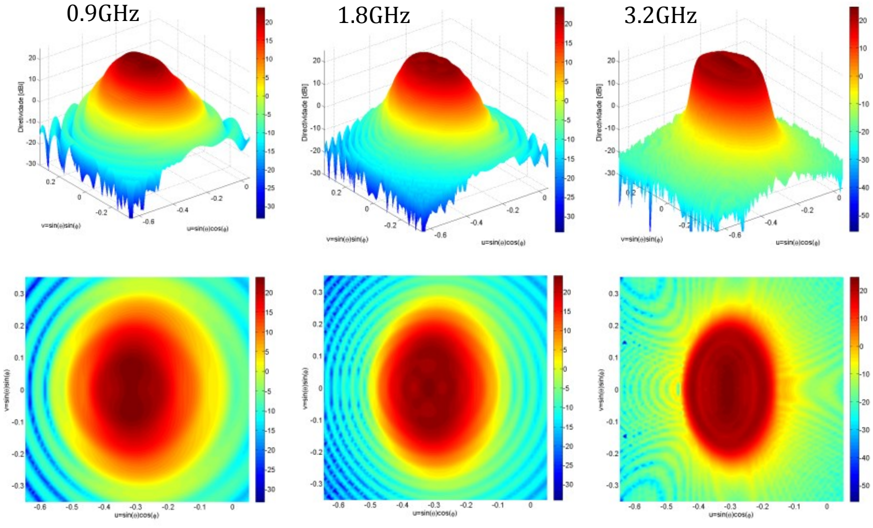
$$\alpha_o = 18°$$
$$\alpha_u = 8°, \alpha_v = 12°$$



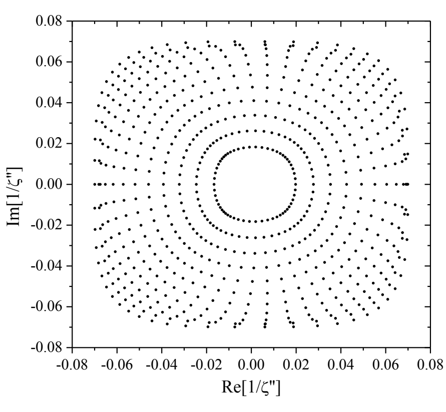


$$\psi = 0{,}69, L_G = -3\text{dB}$$

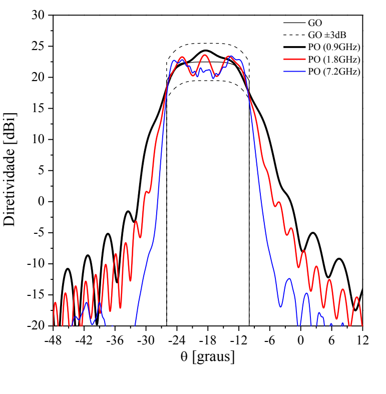
Shaped Offset: Elliptical Contour



Shaped Offset: Super-Elliptical Contour



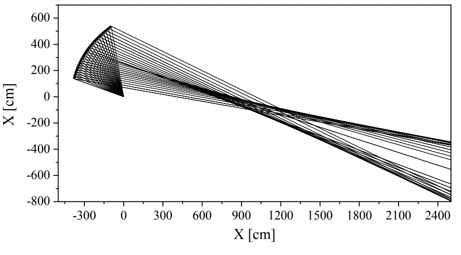
$$\alpha_o = 18°, \sigma = 2$$
$$\alpha_u = 8°, \alpha_v = 8°$$

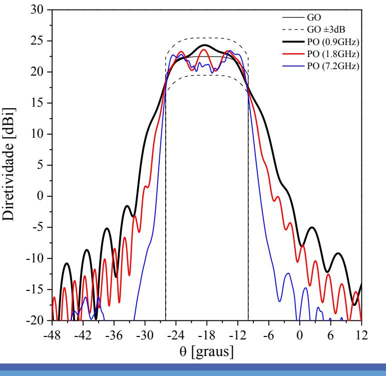


$$\psi = 0{,}69, L_G = -3\text{dB}$$

Shaped Offset: Super-Elliptical Contour

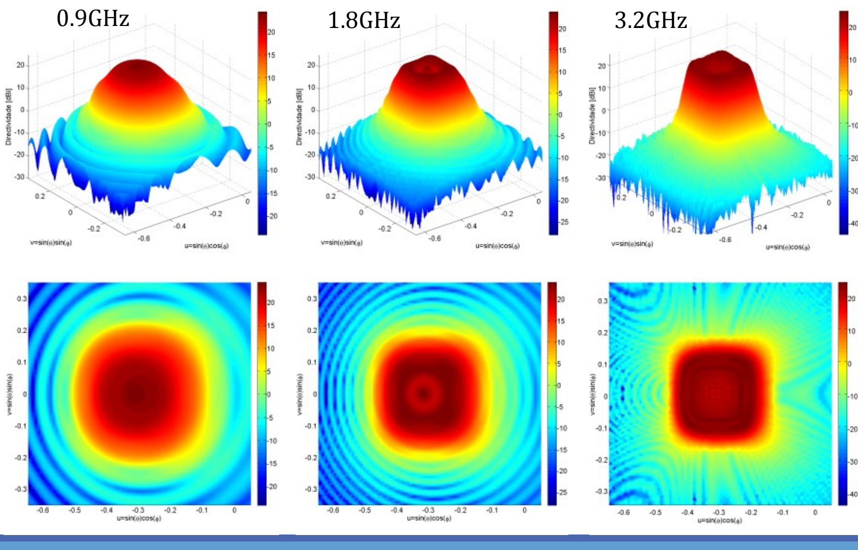
$\alpha_o = 18°, \sigma = 2$
$\alpha_u = 8°, \alpha_v = 8°$





$\psi = 0{,}69, L_G = -3\text{dB}$

Shaped Offset: Super-Elliptical Contour



Numerical and Asymptotic Methods

GO (Geometrical Optics):

- Asymptotic Method;
- Extremely low computational cost;
- It does not take into account diffractions effects;
- It does not take into account electromagnetic coupling effects;
- Phase and polarization of feeder;
- Direct fields from feeder;
- Initial design of reflector and lenses antenna;

Numerical and Asymptotic Methods

ApM (Aperture Method) :

- Asymptotic Method;
- Low computational cost;
- It takes into account some diffractions effects;
- It does not take into account border diffractions;
- There are some techniques to describe more accurately the edge currents and, consequently, border diffraction effects;
- Analysis of electromagnetic scattering of aperture antennas with big electrical dimensions.

Numerical and Asymptotic Methods

PO (Physical Optics) :

- Asymptotic Method;
- Medium computational cost;
- It takes into account some diffractions effects;
- It does not take into account correctly border diffractions;
- There are some techniques to describe more accurately the edge currents and, consequently, border diffraction effects;
- Analysis of electromagnetic scattering of bodies with big electrical dimensions.

Numerical and Asymptotic Methods

MoM (Method of Moments) :

- Numerical Method;
- High computational cost;
- It takes into account accurately all diffractions effects;
- It takes into account electromagnetic coupling effects;
- Analysis of electromagnetic scattering;

Numerical and Asymptotic Methods

FEM (Finite Element Method) :

- Numerical Method;
- Extremely high computational cost;
- It takes into account accurately all diffractions effects;
- It takes into account electromagnetic coupling effects;
- Many electromagnetic analysis;

References

[1] C. A. Balanis, Antenna Theory: Analysis and Design, Third Edition, John Wiley and Sons, Inc., New York, 2005.

[2] S. Silver (ed.), Microwave Antenna Theory and Design, IEE Electromagnetic Wave Series Vol. 19, Peter Peregrinus, London, 1984.

[3] A. W. Love (ed.), Reflector Antennas, IEEE Press, New York, 1978.

[4] A. W. Rudge and N. A. Adatia, "Offset-Parabolic-Reflector Antennas: A Review," *Proceedings of the IEEE,* vol. 66, n. 12, pp. 1592-1618, December 1978.

[5] P. Clarricoats and G. Poulton, "High-efficiency microwave reflector antennas – a review," *Proceedings of the IEEE*, vol. 65, pp. 1470–1504, Oct 1977.

Thank you!!