# Performance of Network of Queues with Traffic Modeled by Heavy-tailed Distributions

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***Abstract*— Markovian models are not suitable for traffic modeling in some modern telecommunications networks. Among the new proposed models, those based on heavy-tailed distributions offer lower complexity. There are a lot of investigations of this kind of model considering a stand alone queue, but there is a lack of analysis for networks of queues. In this paper we analyze the performance of networks of queues under traffic modeled by heavy-tailed distributions, considering G/M/1 and G/G/1 models, with G modeled by Pareto, Lognormal and Weibull distributions. We analyze open networks with and without add/drop traffic.**

## I. INTRODUCTION

Traffic in telecommunications networks has evolved from voice traffic to multimedia traffic, including voice, data and video. In this new scenario, the traditional Markov models are not suitable to characterize the traffic in telecommunications networks.

In 1994, Leland et al [1] demonstrate that Ethernet Local Area Network traffic is statistically self-similar and that none of the traditional traffic models is able to capture this behavior. Since then, several studies were conducted to propose new traffic models to telecommunications networks. These works can be classified in three categories:

a) Based on measurements.
b) Based on fractal models.
c) Based on generic models.

The approach based on generic models is less complex than fractal models [2][3] and is the subject of this work. In this kind of model, the arrival processes is modeled by a heavy-tailed distribution, like Pareto, Lognornal or Weibull distributions, and the service time can be modeled by an exponential distribution (G/M/1 queue), by a heavy-tailed distribution (G/G/1 queue) or can be considered constant (G/D/1 queue).

Several works have analyzed the performance of isolated single server queues with the traffic modeled by a heavy-tailed distribution, but there is a lack of analysis for networks of queues in this scenario.

The goal of this paper is to evaluate, based on simulations, the performance of networks of queues with the traffic modeled by Pareto, Lognormal and Weibull distributions. Two scenarios have been considered:

a) Scenario I: an open network of queues without add/drop traffic.

b) Scenario II: an open network of queues with add/drop traffic after each queue.

The parameters used to evaluate the performance of the networks are the mean waiting time of each queue, as a function of the position of the queue, and the total network delay. For both parameters, we present the results as a function of the utilization factor in each queue.

There are three approaches to vary the utilization factor of the queue in simulations involving traffic modeling with heavy-tailed distributions [4][5]. In this work, we opted to vary the utilization factor by varying the service time of the server. Thus, we can use fixed shape parameters of the heavy-tailed distributions, thus maintaining the control over the auto-similarity of the traffic. Due this, it is necessary to normalize the time/delays by the service time. Thus, in all results presented in this paper, the mean waiting time and the total network delay are normalized by the service time.

The remaining of this paper is organized as follow: Section II presents some characteristics of the heavy-tailed distributions used in this paper; Section III describes the scenarios used in our simulations; Section IV presents the results for the scenario without add/drop traffic; Section V presents the results considering add/drop traffic; and, finally, Section VI presents the conclusions.

## II. HEAVY-TAILED DISTRIBUTIONS

Let X a random variable (R.V) with Probability Density Function (PDF) *f(x)* and Cumulative Distribution Function (CDF) F(x). The R.V. X has a heavy-tailed distribution if: [6]

$$P(X > x) \approx L(x)x^{-\alpha}, \quad \alpha > 0, \quad x \to \infty \qquad (1)$$

where $L(x)$ is a function which decays slowly, tending to infinity when:

$$\lim_{x \to \infty} \frac{L(cx)}{L(x)} = 1, \quad \forall c > 0 \qquad (2)$$

Some important heavy-tailed distributions used to traffic modeling in telecommunications networks are Pareto, Lognomal and Weibull distributions. The main characteristics of these distributions are resumed below.

### A. Pareto Distribution

Pareto distribution is widely used for traffic modeling in telecommunications networks. This distribution can be represented using one, two or three parameters. Results presented in [7] show that the use of Pareto with two parameters results in a lower mean queuing time, compared with the one parameter distribution. In our work, we opted to use de Pareto Distribution with one parameter.

The Probability Density Function of Pareto distribution with one parameter is given by: [7]

$$f(x) = \frac{\alpha}{(1+x)^{\alpha+1}}, \quad 1 < \alpha < 2, \quad x \geq 0 \tag{3}$$

The parameter α is the shape parameter of the distribution. If this parameter takes values between one and two, the expected value of the R.V. is finite, its variance is infinity and the process is self-similar. The expected value can be computed by

$$E(x) = \frac{1}{\alpha - 1} \tag{4}$$

### B. Lognormal Distribution

Although Lognormal distribution is mentioned in several works as a heavy-tailed distribution, it does not have infinite variance, which is the main characteristic of a heavy tailed distribution [8][9]. However, as their moments increase very rapidly, it has also been used for traffic modeling.

The Probability Density Function for Lognormal distribution is given by:

$$f(x) = \frac{1}{x\sqrt{2\pi\beta^2}} e^{\left[-\frac{(\ln x - \mu)^2}{2\beta^2}\right]}, \quad \beta^2 > 0, \quad \mu \in R, \quad x \in (0, +\infty) \tag{5}$$

where μ and β are the shape parameters of the distribution. The expected value for this distribution is given by:

$$E(x) = e^{\mu + \beta^2/2} \tag{6}$$

### C. Weibull Distribution

Weibull distribution has also been used to traffic modeling in telecommunications networks [5][10]. The PDF of this distribution is given by:

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^{\alpha}}, \quad 0 < \alpha \leq 1, \quad \beta > 0, \quad x \in (0, +\infty) \tag{7}$$

where α and β are the shape parameters of the distribution. To characterize a heavy-tailed distribution, the parameter α must take values between zero and one [11].

The expected value of the Weibull distribution is given by equation:

$$E(x) = \beta \cdot \Gamma\left(1 + \frac{1}{\alpha}\right) \tag{8}$$

## III. Scenarios for the Simulations

In our simulations, we have used the software Arena 11.0 Professional. This tool does not provide the possibility to generate Pareto distributions directly. Thus, for this distribution we have used the Percentile Transformation Method [12].

Gross et al [13] show that there are some difficulties in simulating queues with Pareto service. To overcome these problems, it is necessary to consider a truncated expected value, obtained from a truncated CDF, for the distribution. This truncated expected value is given by [12]:

$$E_T(x) = \frac{\alpha}{F(T)}\left[\frac{1}{\alpha(1+T)^{\alpha}} - \frac{1}{(\alpha-1)(1+T)^{\alpha-1}} + \frac{1}{\alpha(\alpha-1)}\right]$$

(9)

where *T* is the truncation parameter and *F(T)* is given by:

$$F(T) = 1 - \frac{1}{(1+T)^{\alpha}} \tag{10}$$

We have considered two scenarios in our analysis. In the first one, called scenario I, we consider an open network of queues without add/drop traffic; in the second one, called scenario II, we consider an open network of queues with add/drop traffic after each queue. Figures 1 and 2 illustrate the scenarios I and II, respectively.

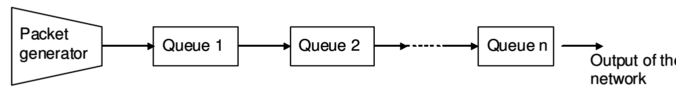



Fig. 1: *Scenario 1 – Open network of queue without add-drop.*

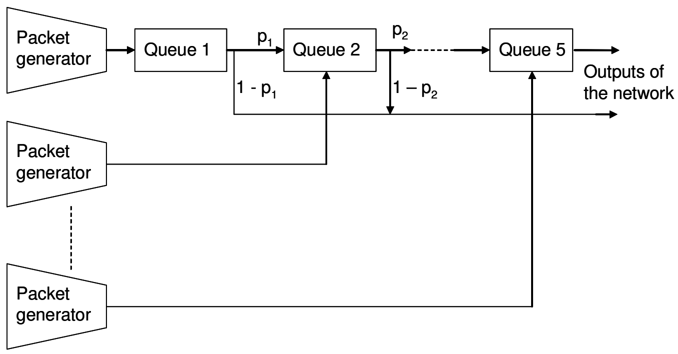



Fig. 2: *Scenario I1 – Open network of queues with add-drop.*

In both scenarios we analyze the networks considering G/M/1 queues or G/G/1 queues. In G/G/1 model, the packet generator and service time in each queue is modeled by a heavy-tailed distribution: Pareto, Lognormal or Weibull. In G/M/1 model, the packet generator is modeled by a heavy-tailed distribution and the service time is considered with Exponential distribution. All queues are single server queue.

In both scenarios, the shape parameters of the heavy-tailed distributions used in packet generators are: Pareto, $\alpha = 1.3$; Lognormal, $\alpha = 1.015$ and $\beta = 2$; Weibull, $\alpha = 0.257$ and $\beta = 1$.

In both scenarios, the service time is varied in such a way to vary the utilization factor of the queues.

## IV. Results for Scenario I

In this scenario, the focus of our investigation is the behavior of the queue as a function of its position in the network. The parameter used to define the performance of the queue is the normalized mean waiting time in each queue.

Figure 3 shows the waiting time as a function of the position of the queue in the network, considering G/M/1 queues, with G modeled by Pareto distribution. For comparison, we plot the performance of an M/M/1 queue in the same figure. We can see that as we walk away from the traffic generator, the queue tends do behave like an M/M/1 queue.

Figure 4 shows the waiting time as a function of the position of the queue in the network, considering now G/G/1 queues, with G modeled by Pareto distributions. Comparing with Figure 3, we can see that, in this case, the performance tends to M/M/1 system in a very slow way.

Similar conclusions have been obtained for Lognormal and Weibull distributions, considering G/M/1 and G/G/1 queues. Figures 5 and 6 show the results for Lognormal distributions and Figures 7 and 8 for Weibull distributions.

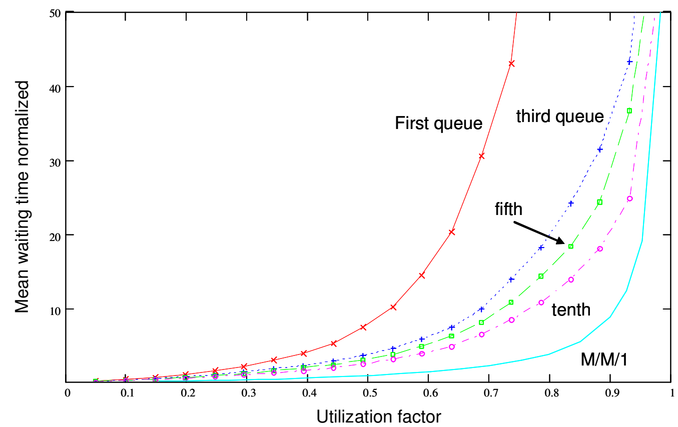



Fig. 3: *Normalized mean waiting times for first, third, fifth and tenth queues of the network considering Pareto/M/1 and M/M/1 models.*

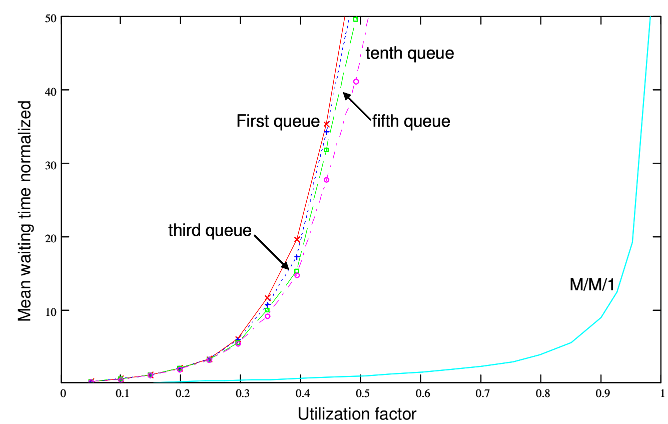



Fig. 4: *Normalized mean waiting times for first, third, fifth and tenth queues of the network considering Pareto/Pareto/1 and M/M/1 models.*

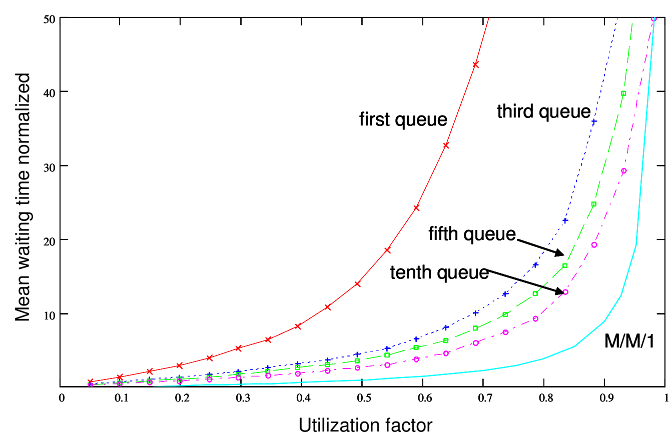



Fig. 5: *Normalized mean waiting times for first, third, fifth and tenth queues of the network considering Lognormal/M/1 and M/M/1 models.*

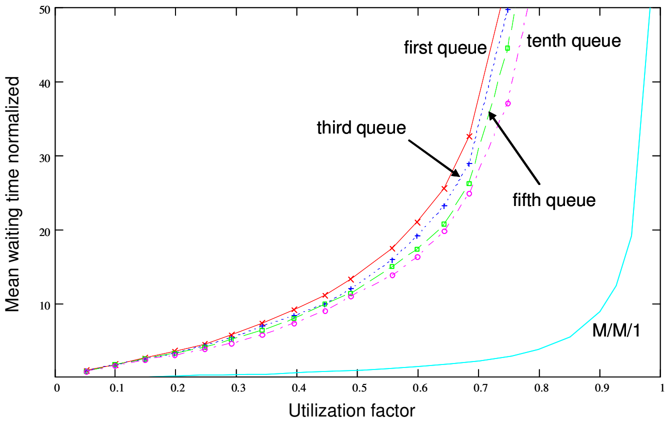



Fig. 6: *Normalized mean waiting times for first, third, fifth and tenth queues of the network considering Lognormal/Lognormal/1 and M/M/1 models.*

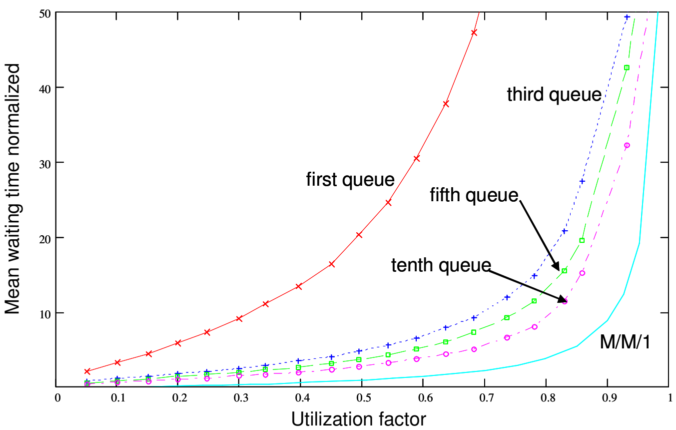



Fig. 7: *Normalized mean waiting times for first, third, fifth and tenth queues of the network considering Weibull/M/1 and M/M/1 models.*

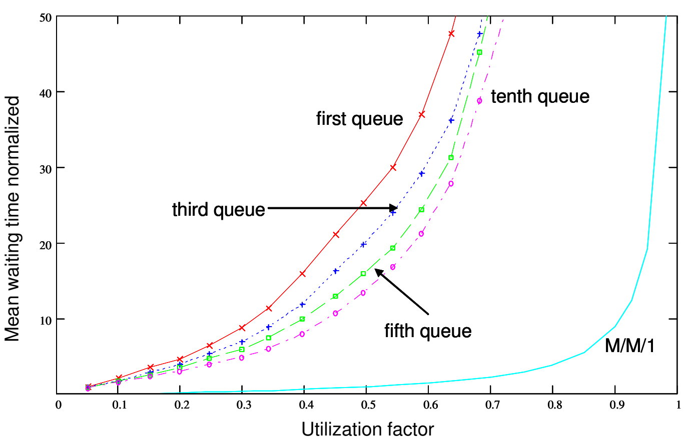



Fig.8: *Normalized mean waiting times for first, third, fifth and tenth queues of the network considering Weibull/Weibull/1 and M/M/1 models.*

To finalize this section, Figure 9 compares the total network delay for G/M/1, G/G/1 and M/M/1 for a network with ten queues, with G modeled by Pareto, Lognormal and Weibull distributions. Based on this figure, we can see that, for G/G/1 model, the Lognormal distribution results in a closer to the M/M/1 model than the other distributions. Considering G/M/1 model, the performances for all distributions are similar.

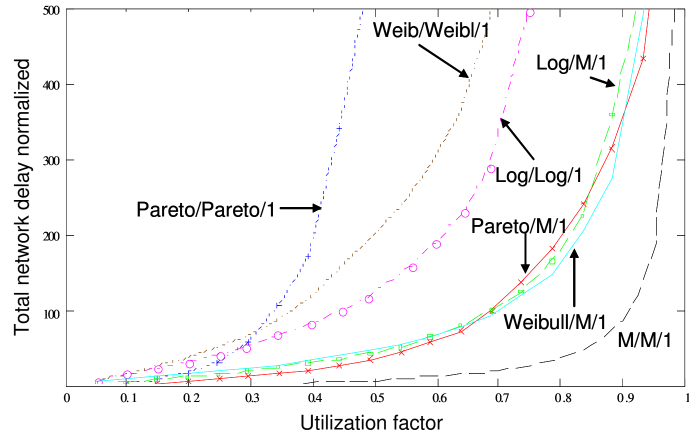



Fig. 9: *Total Network delay for G/M/1, G/G/1 and M/M/1 models.*

## V. Results for Scenario I I

In this section we analyze the normalized mean waiting time in each queue as a function of the position in the network, but considering add-drop traffic in each queue. We consider that the new traffic added to the network is equal to the traffic dropped in the same point.

Figure 10 shows the results considering a 50% add/drop traffic after each queue, while Figure 11 shows the results considering 5% add/drop traffic. The queue models in both figures are G/M/1, with traffic generators modeled by a Pareto distribution and service time modeled by an exponential distribution. For comparison, the result for M/M/1 model is plotted too.

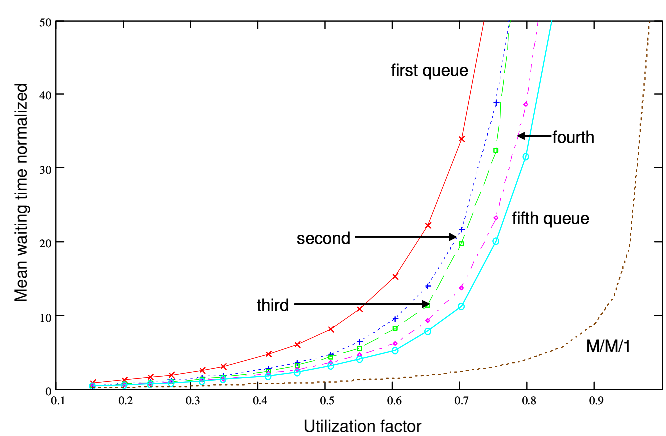



Fig. 10: *Normalized mean waiting times in each queue considering Pareto/M/1 and M/M/1 queues with 50% add/drop traffic.*

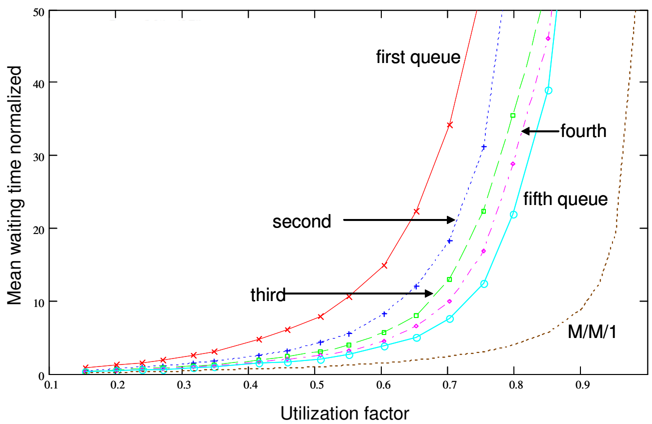



Fig. 11: *Normalized mean waiting times in each queue considering Pareto/M/1 and M/M/1 queues with 5% add/drop traffic.*

Comparing figures 10 and 11, we can see that the model Pareto/M/1 has performance closer to the M/M/1 model when the percentage of add/drop is smaller.

Figures 12 and 13 show the normalized mean waiting time in each queue, considering now a G/G/1 model, with the traffic generator and service time modeled by a Pareto distribution. In Figure 12 the percentage of add/drop is 50%, while in Figure 13 this percentage is 5%.

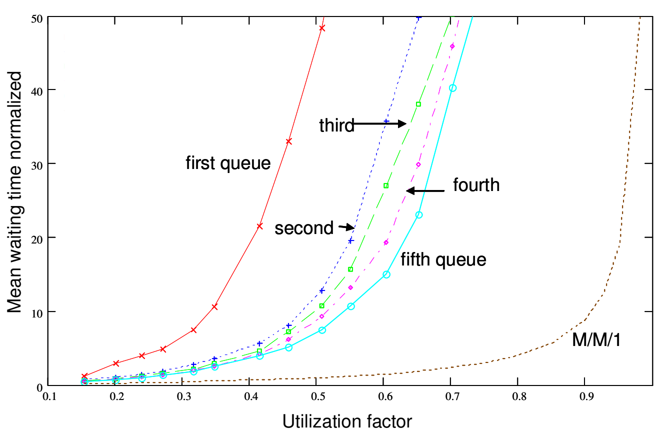



Fig. 12: *Normalized mean waiting times in each queue considering Pareto/Pareto/1 and M/M/1 queues with 50% add/drop traffic.*

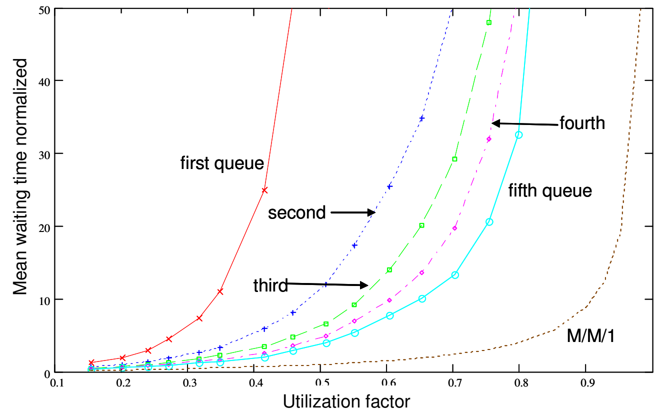



Fig. 13: *Normalized mean waiting times in each queue considering Pareto/Pareto/1 and M/M/1 queues with 5% add/drop traffic.*

Comparing figures 12 and 13, we can see that the model Pareto/Pareto/1 has performance closer to the M/M/1 model when the percentage of add/drop is smaller.

Thus, in both models, G/M/1 and G/G/1, with G modeled by Pareto distribution, the performance is closer to the M/M/1 model when the percentage of add/drop is smaller. Similar conclusions are obtained using Lognormal and Weibull distributions.

Figures 14 and 15 show the results for Lognormal/M/1 model, figures 16 and 17 for Lognormal/Lognormal/1 model, figures 18 and 19 for Weibull/M/1 model and figures 20 e 21 for Weibull/Weibull/1 model.

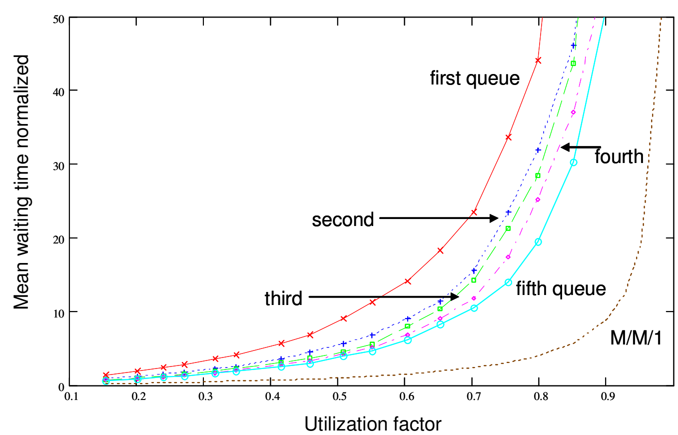



Fig. 14: *Normalized mean waiting times in each queue considering Lognormal/M/1 and M/M/1 queues with 50% add/drop traffic.*

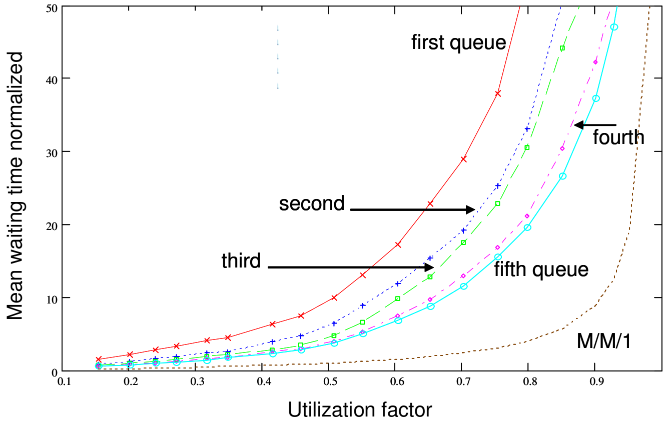



Fig. 15: *Normalized mean waiting times in each queue considering Lognormal/M/1 and M/M/1 queues with 5% add/drop traffic.*

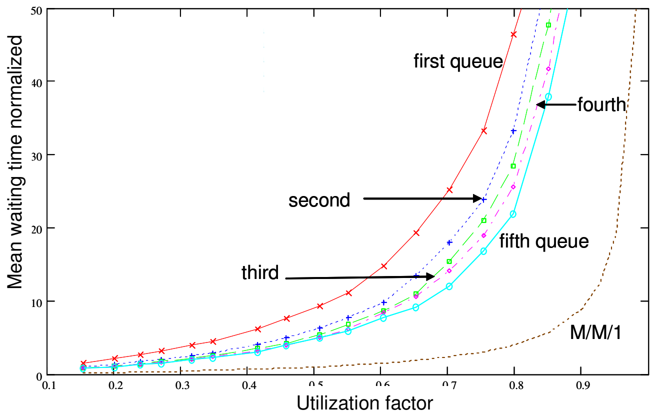



Fig. 16: *Normalized mean waiting times in each queue considering Lognormal/Lognormal/1 and M/M/1 queues with 50% add/drop traffic.*

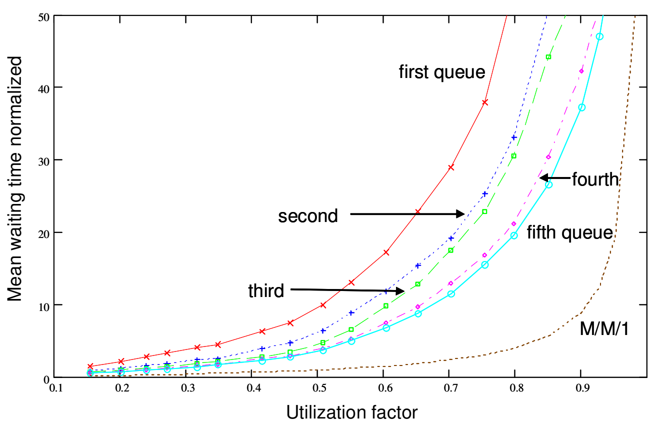
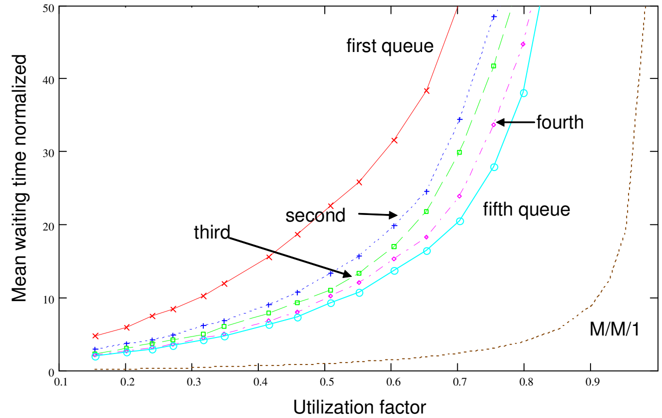



Fig. 17: *Normalized mean waiting times in each queue considering Lognormal/Lognormal/1 and M/M/1 queues with 5% add/drop traffic.*




.Fig. 18: *Normalized mean waiting times in each queue considering Weibull/M/1 and M/M/1 queues with 50% add/drop traffic.*

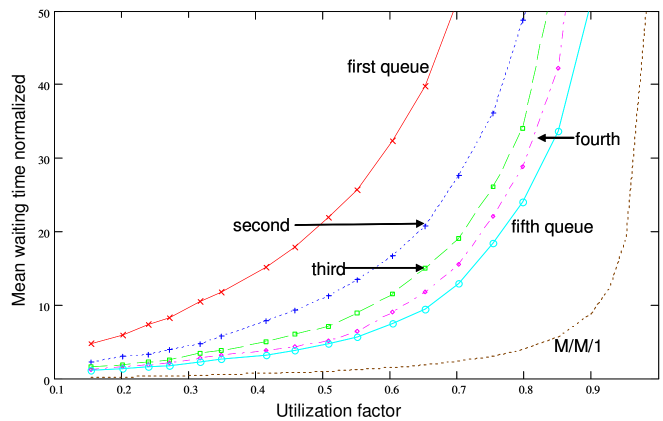



Fig. 19: *Normalized mean waiting times in each queue considering Weibull/M/1 and M/M/1 queues with 5% add/drop traffic.*

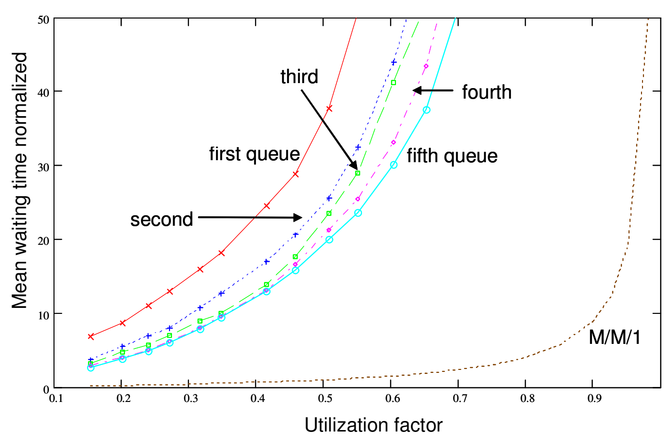



Fig. 20: *Normalized mean waiting times in each queue considering Weibull/Weibull/1 and M/M/1 queues with 50% add/drop traffic.*

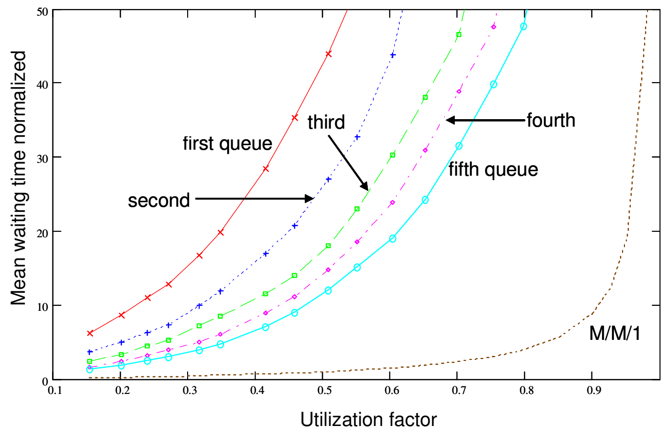



Fig. 21: *Normalized mean waiting times in each queue considering Weibull/Weibull/1 and M/M/1 queues with 5% add/drop traffic.*

## VIII. Conclusions

In this paper we analyzed the performance of networks of queues under traffic modeled by heavy-tailed distributions.

We consider open networks with and without add/drop traffic after each queue.

The models used in simulations are G/M/1 and G/G/1, with G modeled by Pareto, Lognormal or Weibull distributions.

We conclude that the mean waiting time in each queue tends to the performance of a M/M/1 system as we move away from the first traffic source, with the velocity of the trend depending of the type of the queue (G/M/1 or G/G/1), of the type of the distribution and of the percentage of the add/drop traffic.

Also, analyzing the results, we can observe the influence of the heavy-tailed distributions (Pareto, Lognormal or Weibull) in the performance of the system.

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