Wavelength Assignment and Upgrading Strategies for WDM Rings

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***Resumo* – Investigamos as melhorias no desempenho em anéis WDM pela aplicação de algoritmos de alocação de comprimento de onda. Apresentamos um algoritmo computacionalmente barato que minimiza: a) a probabilidade de bloqueio imediatamente após cada alocação; e b) o incremento da probabilidade de bloqueio de canal no comprimento de onda alocado. Propomos métricas simples para a escolha da melhor alocação de comprimento de onda que são baseadas totalmente em informações locais. Discutimos também estratégias de ampliação da capacidade para anéis WDM, mostrando que um aumento modesto no tamanho da grade de comprimentos de onda com respeito à carga na fibra pode produzir quase a mesma melhoria de desempenho conseguida pela convertibilidade plena de comprimentos de onda.**

***Palavras-chave* – Multiplexação por divisão de comprimento de onda, alocação de comprimento de onda, anéis ópticos.**

***Abstract* – We investigate the performance improvements imparted by some wavelength assignment algorithms to optical path WDM rings. A computationally inexpensive algorithm is presented that minimizes: a) the blocking probability immediately after each assignment; and b) the increment of the channel blocking probability at the assigned wavelength. Simple metrics are proposed for the choice of the best wavelength assignment that are based totally on local information. We also discuss upgrading strategies for the WDM ring, showing that a modest wavelength pool size excess with respect to fiber load can produce almost the same improvement as full wavelength conversion.**

Keywords – Wavelength division multiplexing, wavelength assignment, optical rings.

I. INTRODUCTION

We consider optical path networks over ring topologies. Multiple link paths are set up and may be taken down under demand from upper layers in the network hierarchy.

Routing in the ring is not complex, since there are only two routes between any two points, one of them being often much shorter than the other. Wavelength assignment, however, may imply a choice among many alternatives, which may or may not favor the capacity of the ring to meet future traffic demands. At first, one might think that balancing the load among all W wavelengths would be a good policy. This can be easily achieved by just randomly choosing among all available wavelengths (or all available wavelength sequences, when conversion is allowed somewhere) when assigning a new path under request: this is called the *random algorithm* [5],[6]. Early simulations, however, have shown that other algorithms provide better performance than the random algorithm, even though they put more load on some wavelengths than on others (or rather for this reason) [3].

This paper discusses some criteria to derive good wavelength assignment algorithms. The discussions focus on comparisons between the performance improvements that can be obtained with increasing algorithmic complexity, growing wavelength pool size, and providing wavelength conversion capabilities at the nodes. Once good algorithms are identified in Section 3, they are used as reference in Section 4 to investigate strategies for upgrading the capacity of optical rings. Such strategies may include increasing the wavelength load on the fiber and using multiple fiber rings. In the latter case, one might consider requesting paths to each ring separately, sequentially or not; jointly managing wavelength assignment with a single algorithm on both rings; or interconnecting the ring at the hardware level with cross-connecting nodes in lieu of OADMs.

II. BASIC MODEL

Let W and $L \leq W$ be the maximum number of wavelengths allowed in a ring and in each of its links, respectively, and let the wavelengths be numbered $1,2,3,...,W$ [2]. If there is no wavelength conversion, the network may be thought of as W separate, but jointly load-constrained, single-wavelength sub-rings. If there is full wavelength convertibility in all nodes, any optical path is free to switch from any wavelength to any other available wavelength in the next link. In this case, the ring is equivalent to a ring of trunks with L wires in each trunk, hence there is no point in making $W > L$. Under sparse and/or partial wavelength conversion capabilities, wavelength switching is constrained to the allowable alternatives at each node.

One or more upper layers request paths. A request for a path is called illegal if one of the requested links is already busy on L wavelengths. Illegal requests are always blocked. If the ring has full conversion capability on all nodes, all legal requests will be served. Otherwise some legal requests will have to be blocked, but good wavelength assignment algorithms may reduce the frequency of this occurrence.

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III. Wavelength Assignment Algorithms

In this section, we investigate the performance improvements imparted by first-fit wavelength assignment algorithms on blocking WDM rings without wavelength conversion. We still consider optical path networks over the ring topology. Multiple link, single wavelength optical paths are activated and taken down according to the demand for traffic from an upper layer in the network hierarchy. An available wavelength must be assigned to each path.

The performance of these networks is critically dependent on the wavelength assignment algorithm used to set up new paths for the incoming calls. Although no algorithm can produce a higher performance than full convertibility, it is interesting to investigate the maximum performance provided by conversion because the cost of the intelligence present in the algorithm is much smaller than the cost of the conversion.

Several algorithms have been proposed in the literature, each of them having a different heuristics [1]. The next subsections present some definitions and heuristics used to define the algorithm we propose in this work. The simulation results show the relative importance of the proposed heuristics to improve the blocking performance of the ring.

III.1 Definitions

The number of possible paths that may be requested in a WDM ring is $N(N-1)$, where N is the number of nodes. Each of these paths may be activated in any of the W colors. In addition, no more than L paths may traverse the same link at the same time. Given the different meanings that may be implied by the notion of a path, it is useful to define:

Definition 1 - A *network path* is a sequence of nodes in the network, such that any node is physically connected to the previous and succeeding nodes in the sequence.

Definition 2 - A *channel path* (or “colored” path) is a path in the sub-network of a given wavelength (“channel” or “color”).

When there is no wavelength conversion, a channel path is available if all its links are free. A network path is available if at least one of its corresponding W channel paths is available. When two or more channels are available for the requested path, then a wavelength assignment algorithm will choose between them.

Whenever any one wavelength is not being used anywhere in a network, instantaneous blocking probability is zero. This may be the reason why good wavelength assignment algorithms tend to unbalance the load, using some wavelengths more than “others”. Such “good” algorithms are generally comprised in the class of *first-fit algorithms*, defined below.

Definition 3 - An algorithm is first-fit if it assigns a wavelength that is not being used in the network only when the requested path cannot be accommodated by one of the wavelengths that are already being used somewhere.

Several first-fit algorithms have been studied and compared in the literature [3]. The simplest one uses an *a priori* wavelength list: the algorithm will then look up the list and pick the first wavelength under which the path can be accommodated. This will be called the *fixed priority algorithm* (FP). Other algorithms favor the use of the wavelength that is being most used in the network at assignment time. A good survey of routing and wavelength assignment algorithms for WDM networks is given in [1].

III.2 Paths in Ring and Linear Topologies

Let $l_1, l_2, \ldots, l_m$ be the m wavelengths available to accommodate a given path request on a ring. This means that in each of the m corresponding sub-rings there is a “hole” (i.e. a maximal sequence of free adjacent links) that can accommodate the request. If an available wavelength is currently unused everywhere in the ring, then its corresponding hole is the whole ring. Let $C_1, C_2, \ldots, C_m$ denote the available holes where a given path request may be accommodated.

Theorem 1 - If $C_i \subseteq C_j$ for some $1 \le i, j \le m$ then assigning wavelength l_i for the requested path minimizes the instantaneous path blocking probability immediately after the assignment.

Proof: All paths in C_i are also in C_j.Therefore, assigning l_i for the requested path will not change the set of available network paths, thus keeping the path blocking probability unchanged.

Theorem 1 means that the minimization of path blocking probability will often lead to multiple assignment choices. For example, if one wavelength is currently unused all over the ring, then any of the remaining available wavelengths may be chosen to accommodate the requested path without incrementing the instantaneous path blocking probability.

III.3 Minimization of Path Blocking Probabilities

Nevertheless, there may be situations in which more than one wavelength are available, but no hole is contained in any other available hole. This can only

happen if each available wavelength is being used somewhere in the ring. Therefore, each hole C_i, if and when it is used to accommodate the requested path, will leave two other holes with (possibly null) sizes a_i and b_i to the left and right sides of the path, respectively.

Lemma 2 - Let the available holes be such that no hole is contained in any other, and let them be indexed such that $a_1 > a_2 > a_3 > \ldots > a_m$. Then:

$b_1 < b_2 < b_3 < \ldots < b_m$.

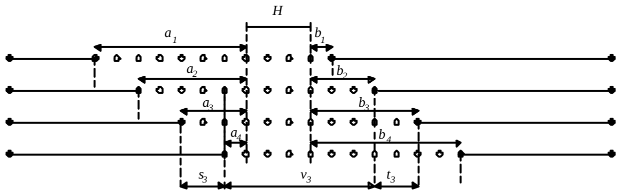

Figure 1 – Typical situation in which lemma 2 holds.

Proof: If $b_i \geq b_j$ for some $i < j$, then $C_j \subset C_i$, which is a contradiction.

Fig. 1 illustrates a typical situation in which Lemma 2 holds. In this situation, any assignment will block some new paths that were not blocked before. The algorithm must then choose that assignment which blocks the least probable set of paths. An inspection of Fig. 1 shows that paths that connect nodes from two disjoint node sets form these sets.

Consider sets of $s+v+t+1$ succeeding nodes in the ring. Let $f(s,t;v)$ be the probability of a path being requested from any of the first s nodes to any of the last t nodes, passing through all of the central $(n+1)$ nodes.

Theorem 3 - Let the available wavelengths be such that no hole is contained in any other, and let them be indexed as in Lemma 2. Then, given a request for an H-hop path, the assignment that minimizes the ensuing path blocking probability minimizes $f(s_i, t_i; v_i)$, where:

$$s_i = a_i - a_{i+1}, \qquad i = 1, 2, \ldots, m-1 \tag{1}$$

$$s_m = a_m + H \tag{2}$$

$$t_i = b_i - b_{i-1}, \qquad i = 2, 3, \ldots, m \tag{3}$$

$$t_1 = b_1 + H \tag{4}$$

$$v_i = |C_i| - s_i - t_i, \quad i = 1, 2, \ldots, m. \tag{5}$$

Proof: Paths that are blocked by the assignment of l_i are those that can only be provided by l_i. Each one connects one of the leftmost s_i nodes of C_i to one of its rightmost t_i nodes.

Theorem 3 shows that the optimal assignment, when a path blocking probability increment must be accepted, results from the minimization of metric $f(s_i, t_i; v_i)$, which is dependent on traffic statistics. We now derive specific assignment rules for the cases of uniform and exponential traffics.

III.3.1 UNIFORM TRAFFIC

In uniform traffic, we assume that all paths on the ring are equally likely to be requested. Therefore:

$$f(s_i, t_i; v_i) = \frac{s_i t_i}{N^2}, \tag{6}$$

where N is the number of nodes on the ring.

Therefore, the best assignment is the one(s) that minimize $s_i t_i$, regardless of v_i.

III.3.2 EXPONENTIAL TRAFFIC

In exponential traffic, the probability of a given path being requested decreases exponentially with its number of hops H. For $0 < r < 1$ and $i = 1, 2, \ldots$:

$$prob(H = i) = p(i) = \left(\frac{1-r}{r}\right) r^i. \tag{7}$$

We assume very large N, so that the truncation of the exponential distribution may be neglected. Without loss of generality, let $s_i \leq t_i$. Then:

$$f(s_i, t_i; v_i) = N^{-1}[h(v_i) - h(v_i + s_i) - \\ - h(v_i + t_i) + h(v_i + s_i + t_i)], \tag{8}$$

where

$$h(x) = \sum_{k=1}^{\infty} k p(x + k + 1) = \frac{r^{x+1}}{1-r}. \tag{9}$$

Therefore:

$$f(s_i, t_i; v_i) = \frac{1}{N}\left(\frac{r}{1-r}\right) r^{v_i} \left(1 - r^{s_i}\right)\left(1 - r^{t_i}\right). \tag{10}$$

It is enough, in this case, to assign l_i such that

$$m_i = r^{v_i}\left(1 - r^{s_i}\right)\left(1 - r^{t_i}\right) \tag{11}$$

is minimized, since m_i is a sufficient decision metric. The assignment should then favor large n_i, but small s_i and t_i. It is easy to show that, when r approaches 1, minimizing m_i is equivalent to minimizing $s_i t_i$, with vanishing influence from n_i, as suggested by (6). However, for small r the prevailing influence comes from n_i, with vanishing influence from s_i and t_i.

III.4 MINIMIZATION OF CHANNEL PATH CAPACITIES

The occurrence of multiple holes contained in other holes will be frequent on a ring operating under a low blocking probability. Since all corresponding wavelengths could be assigned with no path blocking probability increment, some other algorithm must be used to choose between them. Hence the idea of minimizing, among these wavelengths, the increment in channel blocking probability. The motivation is to preserve the ability of remaining channel paths to support future paths requests. In this way, the assignment will not only minimize the current path blocking probability, but also keep the network better prepared to minimize it in the future.

Let $C_1, C_2, \ldots, C_q$ denote all available holes that are contained in some other hole, and let $n_j = |C_j|$ be the size of C_j. Let H be the number of hops in the requested path. Accommodating the requested path in C_j will generate two new holes on its left and right sides with sizes a_j and b_j respectively, with:

$$a_j + b_j + H = n_j. \tag{12}$$

Let $g(n)$ be the probability that a channel path request of any size be accommodated in a hole of size $n < N$ of the requested wavelength.

Theorem 4 - Minimization of the increment in channel path blocking probability is achieved by assigning wavelength l_j that minimizes:

$$\Delta_j = g(n_j) - g(a_j) - g(b_j). \tag{13}$$

Proof: Δ_j is the loss in the probability that a request for a l_j path be accommodated.

The actual metric to be used to guide the assignment choice is derived from Theorem 4 and the traffic first-order statistics. We now derive it for the same two cases considered in the subsection III.3.

III.4.1 UNIFORM TRAFFIC

There are n 1-hop, $(n-1)$ 2-hop, $(n-2)$ 3-hop, ..., one n-hop paths that may be accommodated in an n-hop hole. Since they are all equally likely to be requested in uniform traffic, we have:

$$g(n)=\frac{1}{N^2}\sum_{i=1}^{n} i=\frac{n(n+1)}{2N^2}. \quad (14)$$

So we have

$$\Delta_j=\frac{1}{2N^2}\left[n_j^2+n_j-a_j^2-a_j-b_j^2-b_j\right]= \quad (15)$$

$$=\frac{1}{2N^2}\left[H+n_j^2-a_j^2-b_j^2\right]. \quad (16)$$

Therefore, it is enough to minimize $n_j^2-a_j^2-b_j^2$, which is equal to $2(Hn_j+a_jb_j)-H^2$. A simple and sufficient metric is then:

$$r_j=Hn_j+a_jb_j. \quad (17)$$

The assignment should then favor: a) smaller holes; and b) asymmetric insertion of the path in the hole, which yields small a_jb_j. Since a_jb_j is at least zero and at most $(n_j-H)^2/4$, we have

$$Hn_j\le Hn_j+a_jb_j\le Hn_j+\frac{(n_j-H)^2}{4}. \quad (18)$$

If the maximum possible metric for hole size n_j is less than the minimum possible metric for hole size n_j+1, then the decision may take hole size n_j as a sufficient metric.

This will happen if and only if:

$$Hn_j+(n_j-H)^2/4\le H(n_j+1) \quad (19)$$

or

$$n_j\le H+2\sqrt{H}. \quad (20)$$

As long as (20) is met for some available hole, the smallest hole should be assigned. The decision between two holes with the same size should always favor the most asymmetric insertion.

Since all requests are equally likely in uniform traffic, this assignment will also minimize the loss in the number of channel paths available for future requests, which is the objective of the MaxSum algorithm proposed by [4].

III.4.2 EXPONENTIAL TRAFFIC

The same arguments as in the previous subsection will now provide:

$$g(n)=\frac{1}{N}\sum_{i=1}^{n}(n-i+1)\left(\frac{1-r}{r}\right)r^i. \quad (21)$$

As shown in the Appendix:

$$g(n)=\frac{1}{N}\left[n-\frac{r}{1-r}+\left(\frac{1-r}{r}\right)r^n\right]. \quad (22)$$

Theorem 4 will then yield:

$$\Delta_j=\frac{1}{N}\left[H+\frac{r}{1-r}+\left(\frac{r}{1-r}\right)\left(r^{n_j}-r^{a_j}-r^{b_j}\right)\right], \quad (23)$$

yielding the following equivalent metric to be minimized

$$s_j=r^{n_j}-r^{a_j}-r^{b_j}. \quad (24)$$

Let $d_j=\min(a_j,b_j)$. If $r\le 1/2$, the term r^{d_j} is dominant in this equation, meaning that d_j is a sufficient metric to be minimized, i.e. the best assignment will place the path as close as possible to another path with the same wavelength. In case of a tie, then the smallest hole should be chosen.

Again, insertion asymmetry and hole size are the important decision parameters. However, asymmetry becomes most important for small r, and sufficient for $r\le 1/2$. As r exceeds 1/2, size becomes more and more important, but it never reaches overall sufficiency: for $r=1$, size is sufficient only below $H+2\sqrt{H}$, as seen in subsection III.4.1. Notice that the metric r_j may be obtained from s_j in the limit when r approaches 1.

III.5 FIRST-FIT ALGORITHMIC GAINS

The following algorithm, to be applied whenever two or more wavelengths are available for assignment, results from the full application of all results obtained above:

1. Index the available wavelengths in the order of increasing hole size, forming list *A*;

2. Check if each hole is contained in some succeeding one, and put it in list *B* (initially empty) if it does;

3 If *B* has only one member, assign it; if *B* is empty, go on to step 5; otherwise, continue;

4. Assign a wavelength in *B* according to the following rules:

a) Uniform Traffic $(r=1)$. If the smallest hole(s) is (are) smaller than $H+2\sqrt{H}$, assign the wavelength with the most asymmetrical path insertion among the smallest holes. Otherwise, assign l_i that minimizes r_i from (17);

b) Exponential Traffic. If $r\le 1/2$, assign a wavelength with an existing path closest to the requested path, choosing the smallest hole in case of a tie. If $1/2<r<1$, assign l_i that minimizes s_i from (24);

5. Reorder the available wavelengths in the order of decreasing a_i, forming the list *C*;

6. Calculate s_i, t_i, v_i from (1),(2),(3),(4) and (5) for each wavelength in *C*;

7. For uniform traffic, assign the wavelength that minimizes s_it_i. For exponential traffic, assign the wavelength that minimizes m_i from (11).

This algorithm, called *minimal blocking* (MB), minimizes: a) the instantaneous path blocking probability after each assignment; and b) the increment in channel blocking probability induced by the assignment, subject to the minimization of the path blocking probability. Applying step 4 directly to list *A* is the MaxSum or the *maximal sum of channel capacities* (MC) algorithm. Our simulations compare the performances of the RD, FP, MC and MB algorithms on a 16-node ring. A network with the same topology and full wavelength conversion capability on all nodes gives a lower bound (LB) on the performance of all algorithms.

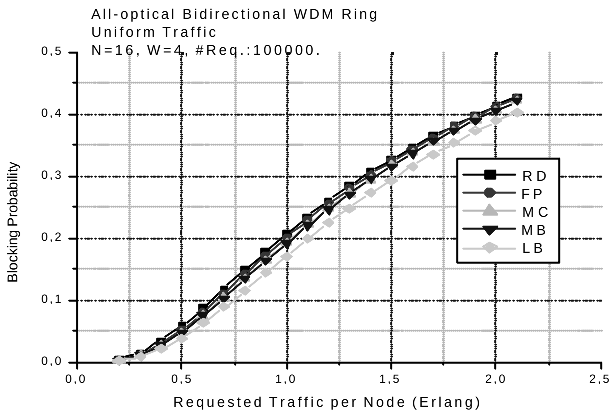

Figure 2 –Blocking probability for uniform traffic.

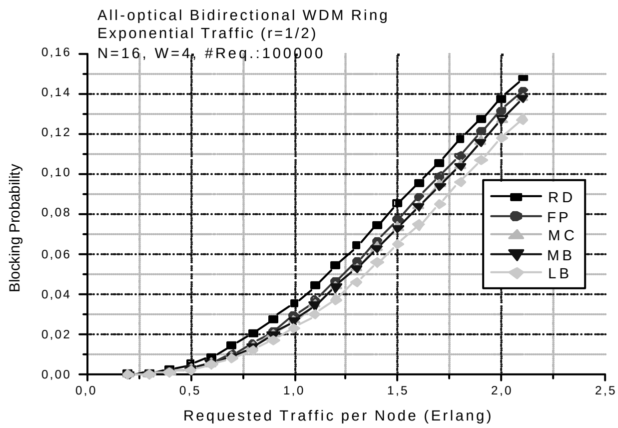

Figure 3 –Blocking probability for exponential traffic.

Figs. 2 and 3 compare the simulated performances of RD, FP, MC, MB and LB for uniform and exponential $(r = 1/2)$ traffics, respectively. The simulations were made on a 16-node ring with shortest-path routing. Each data point in the simulation was obtained from 10^5 random requests. Both the load L and the wavelength pool size W are equal to 4.

Shortest-path routing in the ring effectively truncates the traffic distributions considered above at $H = N/2$ instead of $H = N$, so the criteria derived in the previous sections keep their approximate validity.

IV. UPGRADING STRATEGIES

In this section, we discuss two upgrading strategies for the optical path ring [2]:

1. Enhancing the wavelength pool size W, which may be done up to the fiber load $W = L$ or beyond it; and

2. Duplicating the fiber ring and integrating the resources of both rings, which may be done either by duplicating the OADMs and integrating them or not at the management level, or by replacing the OADMs by OXCs with full routing capability.

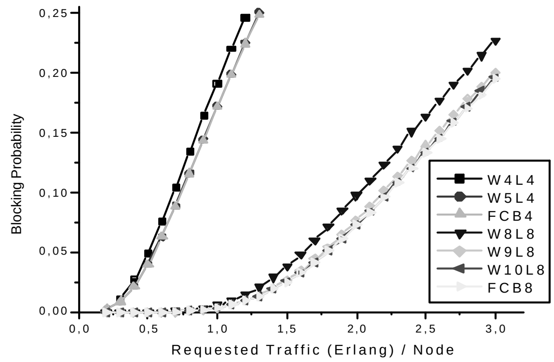

Figure 4 – Influence of upgrading the fiber load for several levels of integration between both rings in the blocking probability on a WDM ring.

The purpose of this discussion is to evaluate the trunking gains obtained by each upgrading strategy. Comparing these gains with the costs of each strategy may then guide the system planning decisions. For the sake of fairness in the comparison, the same algorithm was used in all simulations. Considering the results of previous Section, the MaxSum (MC) algorithm was chosen for this purpose.

IV.1 UPGRADING THE FIBER LOAD AND THE WAVELENGTH POOL

Let us consider a four-wavelength ring with no wavelength conversion capability. If the number of wavelengths is upgraded to eight both in the fiber and in the pool of available wavelengths, how much more traffic can now be requested under the same blocking probability? Simulations shown in Fig. 4 address this and related questions. Curve $WxLy$ shows the results obtained by simulating the ring performance with a wavelength pool with $W = x$ wavelengths and a fiber load of $L = y$ wavelengths. For comparison with the full convertibility bound, curve $FCBz$ shows the ring performance with z wavelengths when all nodes have full conversion capability, in which case there is no gain in making $W > L$.

Comparing curves $W4L4$ and $W8L8$ in Fig. 4 shows that requested traffic for the same blocking probability (and therefore the serviced traffic too) is approximately trebled when the number of wavelengths is doubled. The resulting trunking gain is almost the same as would be obtained with full conversion capability, as can be seen from comparison with $FCB4$ and $FCB8$.

IV.2 UPGRADING THE FIBER PLANT

Let us say that the ring fiber plant is doubled, either with the installation or the appropriation of one additional fiber at each hop. As for the nodes, they may be either duplicated along with the fibers, with or without integration between their managing functionalities, or replaced at each node location by a routing node, i.e. a node with full routing capabilities. In the latter alternative, a path might exchange fibers when passing through a node location, thus enhancing the routing ability to avoid blocking.

IV.2.1 NODE DUPLICATION

We consider now the duplication of the same four-wavelength ring already considered in the last Section. The resulting ring capacity upgrading depends on the way the two rings are managed to support the requested traffic. We have compared three levels of integrated management:

1. No Integration (NI). In this configuration, the traffic is randomly split between both rings. When blocked by its destination ring, a request is blocked forever, and no further attempt is made to accommodate it;
2. Sequential Request (SR). The request is initially addressed to a randomly chosen ring. If blocked by this ring, it is then submitted to the other ring;
3. Joint Management (JM). In this case, one single manager controls both rings. The manager applies the MaxSum algorithm to the set of $2L$ (fiber, wavelengths) pairs. Notice that this option is equivalent to the fiber load upgrading considered in Section IV.1. Under a first-fit algorithm based on a fixed priority wavelength list, joint management would have the same performance as sequential request.

IV.2.2 ROUTING NODES (RN)

When routing nodes are used to connect both rings, more alternatives are opened to the routing of physical paths. While duplicating the fiber plant is equivalent to double the number of wavelengths, routing nodes will effectively associate each of these wavelengths with another one to and from which it can be converted.

Strictly speaking, there are 2^H shortest routes for a request with H hops, which apparently raises a routing assignment problem to be solved prior to the wavelength assignment one. This is only apparent, however, since routing through any fiber is equivalent whenever a wavelength is available in both fibers at a hop. Therefore, MaxSum may keep being applied with the shortest path routing, but a proper meaning must be given to the concept of available routes for the purpose of counting them. We have considered a route to be available in a given wavelength if each of its links is available in at least one fiber.

Fig. 5 compares the single fiber ring performance with the two-fiber one for several levels of integration between both rings: no integration (NI), sequential request (SR), joint management (JM) and routing nodes (RN). With no integration, the serviced traffic is just doubled, thus yielding no trunking gains, as expected. All other cases yield similar trunking gains, suggesting that the added expense of the routing nodes may not be warranted. Joint management yields some gain over sequential request, but only if the assignment algorithm is sophisticated (MaxSum).

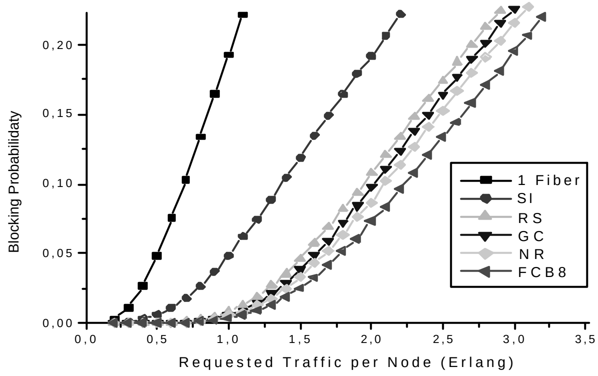

Figure 5 – Influence of upgrading the fiber plant for several levels of integration between both rings in the blocking probability on a WDM ring.

V. CONCLUSIONS

We have demonstrated a wavelength assignment algorithm that minimizes the blocking probability immediately after each assignment in linear topologies. This algorithm, however, has provided no significant improvement in the long-term blocking probability over the best known heuristics (MaxSum), at least for 16-node rings.

Nevertheless, the main contribution has been the derivation of simple metrics to implement these algorithms in distributed management environments, for two important spatial traffic distributions: uniform and exponential. In both cases, the assignment favors smaller hole sizes and asymmetric insertion into them, but the priority of these features depends on traffic spatial distribution. These metrics are based totally on local information.

Upgrading strategies for the ring have also been investigated, showing that increasing the wavelength pool size by about 25\% above the fiber load will yield almost the same performance improvement that may be obtained from full wavelength conversion capability in all nodes.

VI. APPENDIX

This Appendix derives Eq. 22.

Let $q(n)=\sum_{i=1}^{n}(n-i+1)r^i$. Then:

$$q(n)=rn+rq(n-1). \qquad (25)$$

Consider the following related series:

$$a(n)=q(n)-\frac{r}{1-r}n+\left(\frac{r}{1-r}\right)^2. \qquad (26)$$

In (25), let us express $q(n)$ and $q(n-1)$ in terms of $a(n)$ and $a(n-1)$, respectively, getting:

$$a(n)=ra(n-1). \qquad (27)$$

Therefore:

$$a(n)=r^{n-1}a(1)=\left(\frac{r}{1-r}\right)^2 r^n. \qquad (28)$$

Substituting (28) back into (26) and the resulting expression for $q(n)$ into (21) will then yield (22).

REFERENCES

[1] H.Zang, J.P. Jue and B. Mukherjee, "A review of routing and wavelength assignment approaches for wavelength-routed optical WDM networks," *Optical Networks Magazine*, vol. 1, n.1, pp. 47-60, Jan. 2000.

[2] H. Waldman, D.R. Campelo and R. Camelo, "Dynamic priority strategies for wavelength assignment in WDM rings", *Proc. IEEE GLOBECOM'00*, San Francisco, CA, pp. 1288-1292, Nov. 2000.

[3] R. Ramaswami and K.N. Sivarajan, "Optical Networks: A Practical Perspective", Morgan Kauffman Publishers, 1998.

[4] S. Subramaniam and R.A. Barry, "Wavelength assignment in fixed routing WDM networks", *Proc. IEEE ICC'97*, pp. 406-410, Jun. 1997.

[5] R.A. Barry and P.A. Humblet, "Models of blocking probability in all-optical networks with and without wavelength changers", *IEEE J. Select Areas Commun*, vol. 14, pp. 858-867, Jun. 1996.

[6] M. Kovacevic and A. Acampora, "Benefits of wavelength translation in all-optical clear-channel networks", *IEEE. J. Select Areas Commun.,* vol. 14, pp. 868-880, Jun. 1996.

[7] D.R. Campelo, R. Camelo and H. Waldman, "Estratégias de ampliação da capacidade de anéis ópticos com bloqueio", *Anais do XVIII Simpósio Brasileiro de Telecomunicações*, Gramado, RS, Brazil, Sept. 2000 (*in Portuguese*).

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