

Channel Capacity Estimation for a Multi-Carrier Code-Division Multiple Access System[♦]

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Abstract—This paper presents some results concerning the investigation of the Shannon channel capacity for a Multi-Carrier, Direct-Sequence, Code-Division Multiple Access (MC-DS-CDMA) system operating in a multi-path, frequency-selective, Rayleigh-fading channel. The channel capacity estimation uses a Monte Carlo technique. The results reveal that, for the considered system, it is preferable to explore the maximum order of diversity instead of the maximum parallelism of the data stream. The results also indicate that one possible solution for a channel coding scheme to be used along with the system is a concatenation of a powerful low-rate outer code with the inherent inner repetition code of the system.

Index Terms—Channel Capacity, Multi-Carrier CDMA.

Resumo—Neste artigo são apresentados alguns resultados referentes à capacidade de canal de Shannon de um sistema CDMA multiportadora operando em um canal Rayleigh seletivo em frequência. A estimativa da capacidade de canal é realizada por meio do método de Monte Carlo. Os resultados apresentados revelam que, para o sistema sob análise, é preferível explorar a máxima ordem de diversidade em vez de explorar o máximo paralelismo do feixe de dados. Tais resultados também indicam que uma possível solução para codificação de canal para o sistema em questão corresponde à concatenação de um código externo potente e de taxa baixa com o código de repetição inerente à estrutura do sistema MC-DS-CDMA.

Palavras chave—Capacidade de canal, sistemas CDMA multiportadora.

I. INTRODUCTION

It is a fairly collective opinion that multi-carrier modulation, especially when combined with the code-division multiple access (CDMA) technique, is a potential candidate for the next generation (4G) of wireless communication systems. But the scarce of spectrum and the multitude of foreseen services will demand new supporting technologies for these systems that will make their real utilization feasible. Among these technologies, efficient channel coding schemes can be highlighted. Nevertheless, after the conception of the system, it is advantageous if one knows its limits of

Manuscript received on November 9, 2006.

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[♦]Part of this work was presented during the 4th International Workshop on Multi-Carrier Spread-Spectrum, MC-SS 2003, Oberpfaffenhofen, Germany, September 17-19, 2003, and published in its proceedings: *Multi-Carrier Spread-Spectrum*, FAZEL, Khaled and S. KAISER (editors), pp. 115-122, ISBN: 1-4020-1837-1, Kluwer Academic Publishers. The Netherlands, 2004, under the title: *Turbo Product Codes for an Orthogonal Multicarrier DS-CDMA System*.

transmission of information in order to support the code construction and to evaluate its performance.

After the remarkable contribution from Shannon [1], several authors have extended his studies in order to estimate the channel capacity for fading channels. Among them we can mention [2] [3] [4] [5] [6] [7], for unlimited input alphabet, and [8] [9] for BPSK signaling. It also can be found some studies related to channel capacity calculations for multi-carrier fading channels [10].

In this paper, the channel capacity for the Multi-Carrier Direct-Sequence Code Division Multiple Access (MC-DS-CDMA) system of [11], is estimated using a Monte Carlo technique, and the results for some choice of the system parameters are presented. The Monte Carlo technique is applied here to make possible an alternative calculation of capacity formulae.

The results lead to important conclusions about the choice of the parameters of the considered system, unveiling that it is preferable to explore the maximum order of diversity instead of exploring the maximum parallelism of the data stream, and giving an insight about some characteristics of an error correcting code scheme to be designed for this system.

II. SYSTEM DESCRIPTION

Fig. 1 and Fig. 2 [11] show, respectively, the block diagrams for the transmitter and receiver of the MC-DS-CDMA system under consideration.

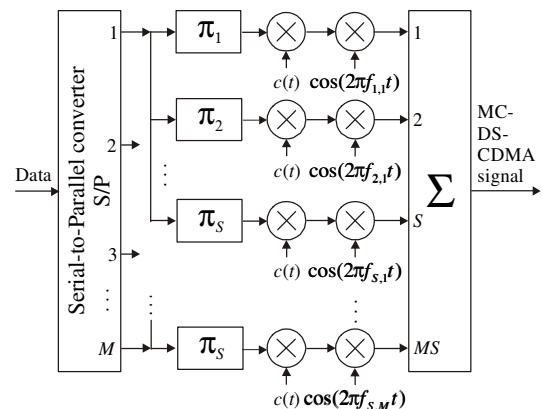


Fig. 1. MC-DS-CDMA Transmitter.

In the system suggested in [11], transmitted data bits are serial to parallel converted to M parallel branches. On each branch, each bit is repeated S times and the replicas feed

different block interleavers (π). Then, these identical bits are direct-sequence spread spectrum BPSK modulated and transmitted using orthogonal carries. Hence, there are a total of MS carriers and time-frequency diversity is obtained. At the receiver, the matched filters outputs of the S identical-bit carriers are combined prior to decoding. The system of [11] can be viewed as a combination or generalization of the *copy-type* and *S/P-type* configurations described in [12]. The main attributes of this system are: 1) the possibility of overcoming the performance of the conventional single-carrier CDMA system, and 2) the reduction of complexity through the use of one matched filter per carrier, instead of a RAKE receiver, situation that is achieved if the number of carriers satisfies [11]

$$MS \geq 2L_1 - 2 \quad (1)$$

where L_1 is the number of resolvable propagation paths for a single-carrier CDMA system with the same total bandwidth as for the MC-DS-CDMA.

In [11] it is allowed a 50% overlap of adjacent modulated carriers, still maintaining orthogonality. Fig. 3 illustrates the main lobe spectrums of the transmitted signal for $M = 4$ and $S = 3$. In this figure, the modulated carriers identified with the same pattern transport the replicas of bits and can be viewed as diversity carriers.

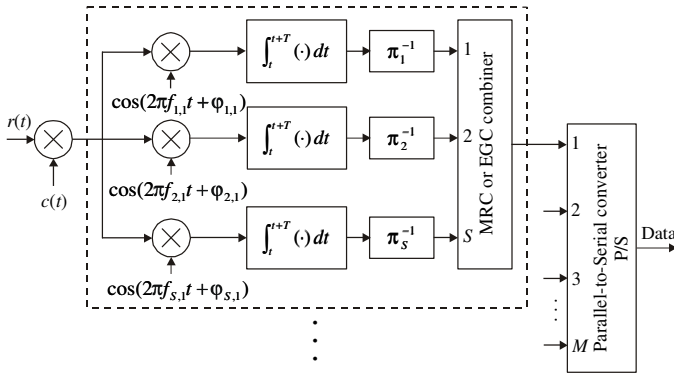


Fig. 2. MC-DS-CDMA receiver.

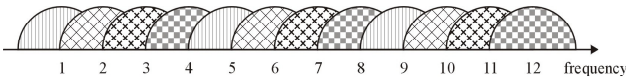


Fig. 3. MC-DS-CDMA spectrum for $M = 4$, $S = 3$.

III. CHANNEL CAPACITY ESTIMATION

In this section, the method for estimating the channel capacity for the MC-DS-CDMA system is described. Initially, a short review of some previous published results is presented, followed by a description of the Monte Carlo technique used for solving the capacity formulas.

A. Background

According to [5], whose credit was given to Wolfowitz [13], let $g[i]$ be a stationary and ergodic stochastic process associated to the channel state information, assuming values on a finite set U of discrete memoryless channels. Let $C(u)$ denote the capacity of a channel $u \in U$, and $p(u)$ denote the

probability, or fraction of time, that the channel is in state u . The capacity for this time-varying channel is given by [5]

$$C^* = \sum_{u \in U} C(u)p(u) \quad (2)$$

Now, consider an AWGN-fading channel and assume that the channel gains $g[i]$ can be considered as a stationary and ergodic stochastic process, known by the transmitter and receiver. A time-invariant AWGN channel with average received signal-to-noise ratio (SNR) γ and unconstrained input alphabet has capacity, in bits per second, given by

$$C_\gamma = B \log_2(1 + \gamma) \quad (3)$$

Let $p(\gamma) = p(\gamma[i] = \gamma)$ be the probability density function (p.d.f.) of γ . Then, based on (2) and (3), the capacity of the time-varying channel is [5]

$$C = \int_\gamma B \log_2(1 + \gamma) p(\gamma) d\gamma \quad (4)$$

Still following [5], if the *compatibility constraint* is satisfied, then the capacity of the channel with *side information at the receiver only* is also given by the average capacity formula (4). The compatibility constraint is satisfied if the channel sequence $g[i]$ is independent and identically distributed (i.i.d.), and if the input distribution that maximizes the mutual information is the same, regardless the channel state. The result (4) will be considered later on in this paper.

Now, consider a BPSK modulation for which the symbols, with energy E_s , are sent through an AWGN channel with equal probability. In this case, it can be shown that the channel capacity, in bits per channel use, can be calculated by [14]

$$C_{BPSK} = \int_{-\infty}^{\infty} p(y | \sqrt{E_s}) \log_2 \frac{p(y | \sqrt{E_s})}{p(y)} dy \quad (5)$$

where

$$p(y | \sqrt{E_s}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(y - \sqrt{E_s})^2}{2\sigma^2}\right] \quad (6)$$

$$p(y) = \frac{1}{\sqrt{8\pi\sigma^2}} \left\{ \exp\left[-\frac{(y - \sqrt{E_s})^2}{2\sigma^2}\right] + \exp\left[-\frac{(y + \sqrt{E_s})^2}{2\sigma^2}\right] \right\} \quad (7)$$

and σ^2 is the noise power of the samples at the output of the optimum receiver (matched filter or correlator).

If the receiver knows the channel state information, the capacity of a memoryless Rayleigh fading channel with BPSK signaling can be calculated as [8]

$$C_{BPSK} = - \int_g \int_y p(g)p(y | \sqrt{E_s}, g) \log_2[f(g, y)] dy dg \quad (8)$$

where $p(g)$ is the p.d.f. of the fading amplitude,

$$f(g, y) = \frac{1}{2} \left\{ 1 + \exp[-(4/N_0)yg\sqrt{E_s}] \right\} \quad (9)$$

and $N_0/2 = \sigma^2$ is the bilateral noise power spectral density at the receiver input.

The results (5) and (8) will also be considered latter on in this paper.

Finally, it is known [15, p. 344] that the capacity of a set of U parallel and independent AWGN channels is equal to the sum of the individual capacities. So, if the multi-carrier, frequency-selective, fading channel is approximately considered as a set of independent parallel and conditioned AWGN channels (conditioned on the fading amplitudes), the total capacity can be estimated as the sum of U capacities calculated via (4), if the input alphabet is unconstrained, and via (8) for BPSK signaling.

The inherent problem with (4) and (8), even if numerical integration is used, is that the probability density functions operated into each of these expressions are not always easily obtained, specially in the case of (8). For example, if it is of interest to consider the fading statistics at the output of an EGC combiner with S inputs, the task of obtaining the p.d.f. of the sum of Rayleigh envelopes becomes more and more complex as the value of S increases. In this context, we suggest a Monte Carlo method for channel capacity estimation, without the need for operating such density functions. The method is described in the next subsection.

B. Applying the Monte Carlo Method

From this point on, it is assumed that the receiver has perfect knowledge of the channel state information and that the transmit power is constant (no power adaptation scheme). It is further assumed that the compatibility constraint described in Section II is satisfied.

Let $g[i]$ represent the channel state information at the discrete-time instant i , and assume that it is possible to generate by computer, directly or indirectly, a sufficient large number X of values for g , based on a specific and known probability distribution. Then, it is claimed here that the capacity expressed by (4) can be estimated using the discrete-time average

$$C = \frac{1}{X} \sum_{i=1}^X B \log_2 (1 + \gamma g^2[i]) \quad (10)$$

By a sufficient large value of X it is meant a value enough for convergence in (10).

Then, for BPSK signaling in a fading channel, the capacity expressed by (5) can be adapted to this Monte Carlo technique, leading to

$$C_{BPSK} = \frac{1}{X} \sum_{i=1}^X \int_{\Lambda[i]}^{\Lambda[i]} p(y | \psi[i]) \log_2 \frac{p(y | \psi[i])}{p(y)} dy \quad (11)$$

where $\psi[i] = g[i]\sqrt{E_s}$, $\Lambda[i] = \psi[i] + 6\sigma$,

$$p(y | \psi[i]) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(y - \psi[i])^2}{2\sigma^2}\right] \text{ and } \quad (12)$$

$$p(y) = \frac{\exp\left[-\frac{(y - \psi[i])^2}{2\sigma^2}\right] + \exp\left[-\frac{(y + \psi[i])^2}{2\sigma^2}\right]}{\sqrt{8\pi\sigma^2}} \quad (13)$$

Obviously, for an AWGN channel $g[i] = 1$. The results obtained through (10) and (11) demonstrate perfect agreement [14] with those obtained through their counterparts (4) and (8), respectively, showing the applicability of the method for both unconstrained and constrained input alphabets. Fig. 4 shows some results concerning this comparison.

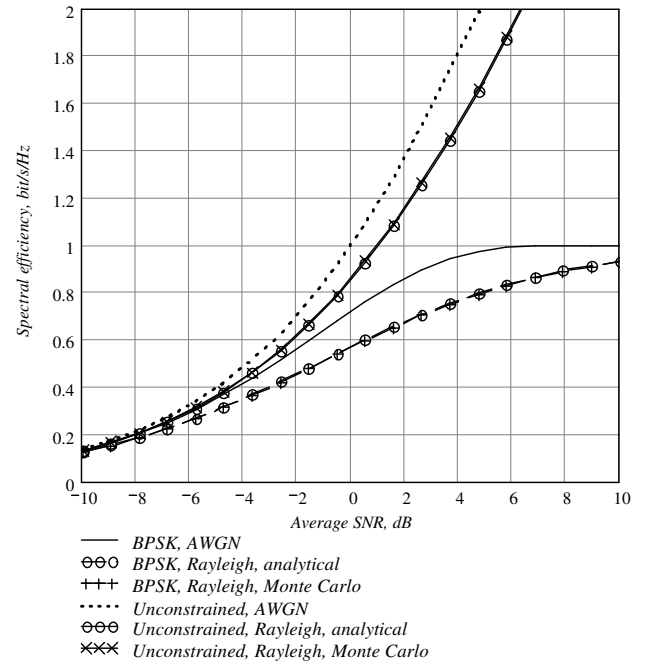


Fig. 4. Comparison between *analytical* and *Monte Carlo* methods used for channel capacity estimation.

The reverse link of the MC-DS-CDMA system for a user of reference can be interpreted, in one hand, as a set of M parallel channels with BPSK signaling. These channels are defined from each of the M S/P converter outputs at the transmitter to each of the M combiner's outputs at the receiver. On the other hand, this link can be interpreted as a set of MS parallel channels defined from each of the MS modulator inputs at the transmitter to each of the MS matched filters outputs at the receiver. Then, according to previous statements (see Section II), the channel capacity for the MC-DS-CDMA system can be estimated as the sum of M or MS individual capacities, depending on the case under consideration. This sum is possible if it is presumed independence among the M or MS channels, a reasonable assumption when the bandwidth occupied by each modulated carrier is smaller than the *coherence bandwidth* of the channel and a frequency interleaver is applied.

Then, if the sum of the interference terms at the receiver input is modeled as Gaussian, the capacity for *each* of the M or MS channels of the MC-DS-CDMA system can be approximately estimated using (11). Furthermore, the value of $\sqrt{E_s}$ in this expression should be replaced by [11] $T\sqrt{P/2}$, where P is the average transmitted power per carrier and T is the BPSK symbol duration.

Table I shows the values for $g[i]$ and for the variances of the interference terms plus noise, σ^2 , to be operated in (11), according to each case taken into consideration here. The values of the average signal-to-noise ratio γ , used to calculate the average transmitted power per carrier, P , in each situation are also given in Table I. The value of $\beta[i]_\nu$, $i = 1, 2, \dots, X$, corresponds to the i -th value of the computer generated Rayleigh random variable, i.i.d. for all i and ν , and J_ν accounts for the interference variances at the output of each matched filter, at the receiver. In fact, the values of J_ν are different for different combiner's outputs [11], but if this difference is not taken into consideration, the channel capacity results are not significantly affected [14].

Table I – Values operated directly or indirectly in (11), (12) and (13).

	γ	$g[i]$	σ^2
M ch. EGC	$\frac{PT}{N_0 S} E \left[\left(\sum_{\nu=1}^S \beta_\nu \right)^2 \right]$	$\sum_{\nu=1}^S \beta[i]_\nu$	$\sum_{\nu=1}^S J_\nu + \frac{N_0 T S}{4}$
M ch. MRC	$\frac{PT}{N_0} E \left[\sum_{\nu=1}^S \beta_\nu^2 \right]$	$\sum_{\nu=1}^S \beta[i]_\nu^2$	$g[i] \left[\frac{1}{S} \sum_{\nu=1}^S J_\nu + \frac{N_0 T}{4} \right]$
MS ch.	$\frac{PT}{N_0} E(\beta^2)$	$\beta[i]$	$\frac{1}{S} \sum_{\nu=1}^S J_\nu + \frac{N_0 T}{4}$

IV. RESULTS AND DISCUSSION

In this paper we consider three situations for the MC-DS-CDMA system analyzed: Equal Gain Combining (EGC), Maximal Ratio Combining (MRC) and no combining. The first two situations are interpreted as M parallel channels. The last one is interpreted as MS parallel channels. In all cases, the total capacity was calculated as the sum of M or MS capacities, according to the case under consideration, versus the average SNR at the output of each channel. The results were converted to spectral efficiency versus the Shannon minimum average SNR per information bit for error-free transmission.

Fig. 5 shows capacity results for $MS = 6$ and variable M and S . For M channels with diversity, the results shown are for MRC combining. The systems with EGC combining have capacities identical to those of systems with MRC combining, but, for a given SNR, the necessary transmitted power per carrier with EGC is greater than that necessary for MRC.

It is possible to note from Fig. 5 that it is more advantageous to explore the maximum order of diversity instead of exploring the maximum parallelism of the data stream. This indicates that one possible coding scheme is a concatenation of a powerful outer code with the inherent repetition inner code of the system. At the receiver, the outputs

of the combiners can be viewed as soft-inputs for the outer decoder.

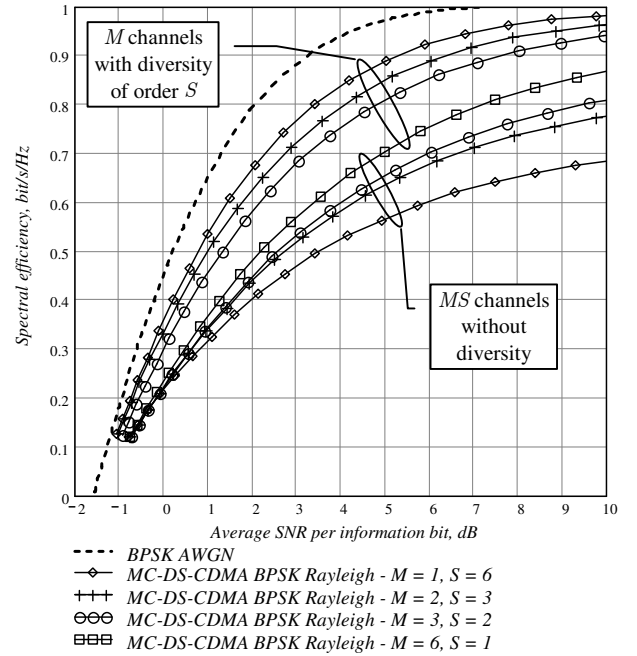


Fig. 5. Spectral efficiency for the MC-DS-CDMA system on the Rayleigh channel; $MS = 6$, variable M and S .

The length N_1 of the spreading code for a single-carrier CDMA system taken for reference was made equal to 60, and the number of resolvable paths L_1 for this single-carrier case was made equal to 4. In this case (1) is satisfied and the number L of resolvable paths per carrier reduces to 1.

The multi-path intensity profile was considered uniformly distributed and the number of active users, K , was made equal to 10. The expressions that permit the estimation of the variances J_ν in Table I can be obtained from the Appendix.

It can also be seen from Fig. 5 that the use of diversity is critical to system performance improvement and that, as the diversity order S increases, the channel capacity approximates the AWGN one. The observation of MS channels by the receiver, without diversity, significantly reduces the capacity, especially for high information rates.

Still referring to the results presented in Fig. 5, it can be seen that the best choice for the system parameters is $M = 1$ and $S = 6$. However, in real channels it is almost impossible to guarantee low correlation between diversity carriers if they are adjacent and, furthermore, overlapping. This fact leads us to the conclusion that the best choice for M is the minimum value such that the diversity carriers experience low correlation. The value of S is then readily obtained.

The use of the concatenated coding scheme suggested as an option for this MC-DS-CDMA system not necessarily increases the transmission bandwidth or reduces the information data rate. The length of the spreading code per carrier, N , can be adjusted [14] to compensate for the reduced coded symbols duration $R_c T$ due to coding of rate R_c , keeping unchanged both the bandwidth and the information rate, relative to the uncoded system. In this case, however, the

channel capacity is reduced, since the total variance of the interference terms in the decision variable is increased.

Fig. 6 shows some results for the spectral efficiency of the MC-DS-CDMA system for $M = 1$, $S = 6$ and variable number of active users, K . As expected, the capacity reduces as the number of users is increased, all other parameters unchanged. The results shown in Fig. 6, and also those presented in Fig. 5, indicate that, referring to the concatenated coding scheme already mentioned, it is preferable to use a low-rate outer code in order to aim the best performance, since the capacity is changed less than 1 dB for code rates below 0.2. In [14] it is suggested a low-rate product code with turbo-decoding as an outer coding/decoding scheme for the MC-DS-CDMA system and it is shown that good performance results can be obtained, even if the length of the spreading code per carrier is changed to maintain the information rate and the occupied bandwidth.

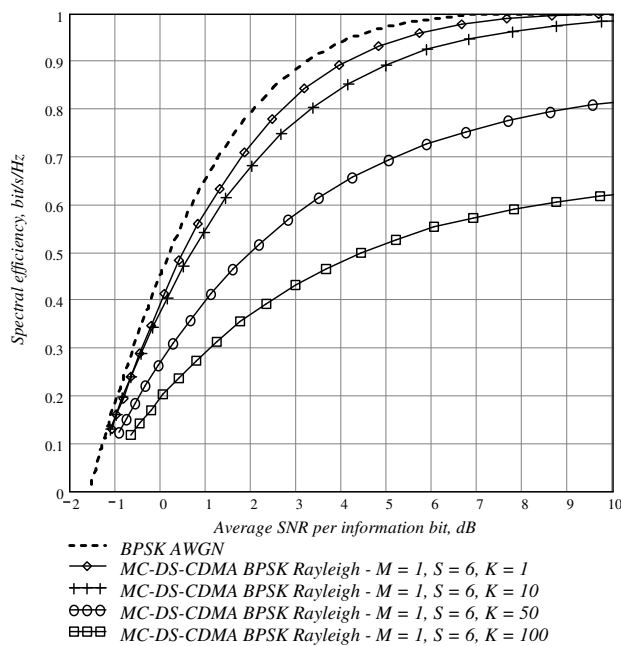


Fig. 6. Spectral efficiency for the MC-DS-CDMA system on the Rayleigh channel; $M = 1$, $S = 6$, variable K .

V. CONCLUSIONS

This paper presented some results of the investigation about the Shannon channel capacity for the MC-DS-CDMA system proposed in [11], considering that the fading amplitudes are independent and identically distributed in time and frequency, and that the channel state information is available at the receiver. The channel capacities were estimated through a Monte Carlo technique, which can avoid the characteristic manipulation of probability density functions typically found in channel capacity expressions. One drawback of this method is that it can take a long time of computer processing to demonstrate convergence, obviously depending on the complexity of the capacity formula.

It was concluded that, for the MC-DS-CDMA system under consideration, it is preferable to explore the maximum order of diversity instead of exploring the maximum parallelism of the data stream. It was also concluded that one possible solution

for a channel coding scheme to be used along with this system is a concatenation of a powerful outer code with the inherent inner repetition code of the system. At the receiver, the combiner's outputs serve as soft-inputs for the outer channel decoder. It was also pointed out that it would be preferable to choose an outer low-rate code in order to aim better performance, from the point of view of capacity achieving results.

REFERENCES

- [1] C. E. Shannon, A Mathematical Theory of Communication, *Bell System Technical Journal*, 27, 1948, 379-423, 623-656.
- [2] T. Huschka, M. Reinhardt & J. Lindner, Channel Capacities of Fading Radio Channels. in *7th IEEE Int. Symp. Personal, Indoor, and Mobile Radio Communication, PIMRC'96*, Taipei, Taiwan, 1996, 467-471.
- [3] M. S. Alouini & A. Goldsmith, Capacity of Nakagami Multipath Fading Channels, in *Proc. IEEE Veh. Technol. Conf. VTC'97*, Phoenix, AZ, 1997, 358-362.
- [4] C. G. Günther, Comment on "Estimate of Channel Capacity in Rayleigh Fading Environment", *IEEE Transactions on Vehicular Technology*, 45(2), 1996, 401-403.
- [5] A. J. Goldsmith & P. P. Varaiya, Capacity of Fading Channels with Channel Side Information, *IEEE Transactions on Information Theory*, 43(6), 1997, 1986-1992.
- [6] L. H. Ozarow, S. Shamai & A. D. Wyner, Information Theoretic Considerations for Cellular Mobile Radio, *IEEE Transactions on Vehicular Technology*, 43(2), 1994, 359-378.
- [7] E. Biglieri, J. Proakis & S. Shamai, Fading Channels: Information-Theoretic and Communications Aspects, *IEEE Transactions on Information Theory*, 44(6), 1998, 2619-2692.
- [8] E. K. Hall & S. G. Wilson, Design and Analysis of Turbo Codes on Rayleigh Fading Channels, *IEEE JSAC*, 16(2), 1998, 160-174.
- [9] T. F. Wong, Numerical Calculation of Symmetric Capacity of Rayleigh Fading Channel with BPSK/QPSK, *IEEE Communications Letters*, 5(8), 2001, 328-330.
- [10] L. Goldfeld & V. Lyandres, Capacity of the Multicarrier Channel with Frequency-Selective Nakagami Fading, *IEEE Transactions on Communication*, E83-B(3), 2000, 697-702.
- [11] E. Sourour & M. Nakagawa, Performance of Orthogonal Multicarrier CDMA in a Multipath Fading Channel, *IEEE Trans. on Comm.*, 44(3), 1996, 356-367.
- [12] S. Kaiser, *Multi-carrier CDMA mobile radio systems - analysis and optimisation of detection, decoding and channel estimation* (Ph.D. Thesis, VDI Verlag GmbH. Düsseldorf, 1998).
- [13] J. Wolfowitz, *Coding Theorems of Information Theory* (New York: Springer-Verlag, 2nd ed., 1964).
- [14] D. A. Guimarães, *A Class of Product Codes and Its Turbo Decoding Applied to a Multicarrier CDMA System*, Ph.D. thesis, Unicamp, SP, Brazil, 2003 (in Portuguese).
- [15] R. G. Gallager, *Information Theory and Reliable Communication* (John Wiley & Sons, 1968).

VI. APPENDIX

The following expressions permit the calculation of the variances J_v shown in Table I and used for the channel capacity estimations considered in this paper. A more complete mathematical analysis, as well as generic expressions for interference calculation for the MC-DS-CDMA system can be obtained from [11].

$$J_\nu = \frac{PT^2}{6N^3}r + \frac{1}{M} \sum_{p=1}^M \frac{PT^2}{2\pi^2 N^3} \mu Q_{p,\nu} \quad (14)$$

where

$$N = \frac{2M}{MS + 1} N_1 \quad (15)$$

$$r = 2(K - 1)N^2 \quad (16)$$

$$\mu = (K - 1)N^2 \quad (17)$$

$$Q_{p,\nu} = \sum_{\substack{m=1 \\ m \neq p + (\nu-1)M}}^{MS} \frac{1}{[m - p - (\nu - 1)M]^2} \quad (18)$$



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