Optimization of Generalized 2x2 Mesh Topologies using an Evolutionary Algorithm

L. H. Bonani, F. R. Barbosa, R. R. F. Attux, R. Arthur, E. Moschim

***Abstract*— In this work, we propose and analyze a generalized optimization process for 2x2 mesh topologies based on an algorithm built with the aid of ideas drawn from the field of evolutionary computation. In the proposed framework, this algorithm is employed to minimize a parameter - the average number of hops - which is particularly relevant from the standpoints of increasing the effective network capacity and seeking a better traffic distribution insofar as mesh topologies are concerned. The methodology is simple and can be used for mesh topologies with a wide range of number of nodes.**

***Index Terms*— Optical Networks, Mesh Topologies, Evolutionary Computation.**

***Resumo*— Neste trabalho propomos e analisamos um processo genérico para otimização de topologias em malha 2x2 baseado em um algoritmo construído a partir de idéias da área de computação evolutiva. Neste caso, este algoritmo é empregado para minimizar um parâmetro - o número médio de *hops* -, o qual é particularmente relevante do ponto de vista do aumento da capacidade efetiva e da rede e da melhor distribuição de tráfego. A metodologia é bastante simples e pode ser usada para topologias em malha com uma faixa razoável de número de nós.**

***Palavras chave*— Redes Ópticas, Topologias em Malha, Computação Evolutiva.**

I. Introduction

The performance of minimum path networks is strongly related to the adopted mesh topology and to the control algorithms responsible for establishing connections. It is important to remark that the redundancy of paths in this kind of network - something related to the mesh topology - is a desirable characteristic, since it increases link utilization. Furthermore, the investigation of aspects related to network capacity and link load has shown that these networks should be able to provide fast and efficient packet delivery (connection) with reduced technological complexity. Thus, for a given installed capacity, topology optimization in terms of minimum average number of hops (H) leads to an overall optimization of network capacity and throughput, as well as to the effective usage of node resources, even though in multihop environments a significant amount of the capacity turns out to be used up by the circulating traffic.

The choice of a single parameter, H, is due to the fact that this parameter is directly related to the total network capacity.

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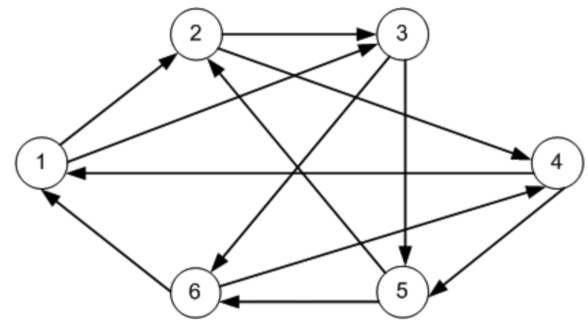

Fig. 1. Topology with 6 nodes.

In fact, the total aggregated capacity is inversely proportional to the average number of hops that a given packet takes from origin to destination [1]. Under these circumstances, a network topology having N nodes with 2 output ports leading to links of bandwidth S bits/s will have total capacity C equal to

$$C = \frac{2.N.S}{H} \tag{1}$$

From Equation 1, we can see that, for a given network with N nodes and links of S bits/s, the total capacity C decreases with the increase of the average number of hops (H). To determine the value of this parameter, we must find the average length considering all possible paths, node by node in a given topology, taking into account the shortest existing path. In the process of capacity calculation, we consider a static routing scenario in which all packets from every connection will travel the same distance and visit the same nodes, as in the case of store-and-forwarding (SF) routing [1].

In order to represent a topology, we use the connectivity matrix (Cm) notation. This matrix, which is square and has a number of rows and columns equal to the number of topology nodes (N), is composed of zeros and ones, the ones representing the existence of a connection and the zeros representing its absence. All the ones in Cm characterize a link between the nodes given by their indices: the rows represent the origin, whereas the columns indicate the destination. In order to illustrate these ideas, let us consider the example of the 6-node topology shown in Fig. 1, which is represented by the Cm below.

$$Cm = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \tag{2}$$

We can calculate, for each one of the $N.(N-1)$ origin-destination connection pairs (applications), the number of hops using the shortest path method, which is presented by the Shortest Path Matrix (Sp), given by Equation 3, and calculated with Floyd-Warshall algorithm [2].

$$Sp = \begin{pmatrix} 0 & 1 & 1 & 2 & 2 & 2 \\ 2 & 0 & 1 & 1 & 2 & 2 \\ 2 & 2 & 0 & 2 & 1 & 1 \\ 1 & 2 & 2 & 0 & 1 & 2 \\ 2 & 1 & 2 & 2 & 0 & 1 \\ 1 & 2 & 2 & 1 & 2 & 0 \end{pmatrix} \quad (3)$$

The average number of hops is the average value of the number of hops in all possible paths. Therefore, H can be defined as the sum of all elements in the Sp matrix divided by the total number of paths, as seen in Equation 4.

$$H = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} Sp(i,j)}{N.(N-1)} \quad (4)$$

For the given example, H is equal to 1.60, since the Sp sum returns 48 and these are 30 origin-destination pairs for a 6-nodes topology. The H parameter is function of the topology, and, for a given number of nodes, we can have several values, depending on how the topology nodes are linked. As the total capacity C is inversely proportional to H, the topologies with a smaller H will lead to better values of capacity and, hopefully, to significant gains in performance. Topologies with the smallest possible values for the average number of hops are named Optimized Mesh Networks (OMT).

II. Optimization Process of Mesh Topologies

The optimized mesh topologies are associated with the smallest H parameter for a given number of nodes. In order to find these topologies, we have considered only matrices with a 2x2 configuration (2 input and 2 output ports), since the OMT must be a topology thus configured. The basis for comparison insofar as OMT are concerned will be the well-established ShuffleNet (SF) and Manhattan Street (MS) [3].

A. Random Search

The first experiment to investigate the topologies with minimum H was based on a random search for topologies with a value of H smaller than that associated with the SN and MS topologies. Therefore, we have selected some topologies with 16 nodes, resultants of MS-16, which have presented better results concerning the H parameter. One of these topologies, achieved with random search, is shown in Fig. 2. In this case, the H parameter for this topology is 2.8125, presenting a smaller average number of hops than in the MS-16 case, whose H is 2.9333. We can also note that the topology presented in the figure exhibits a certain regularity, which is desirable for a geographical distribution of a network.

It is sure that not every obtained topology presents a smaller H parameter than in the MS case: there are topologies whose link distributions lead to an average path much longer than that obtained with the original MS topology. The degree of correspondence between the topology and the H parameter is so strong that smooth modifications impact directly in the H parameter.

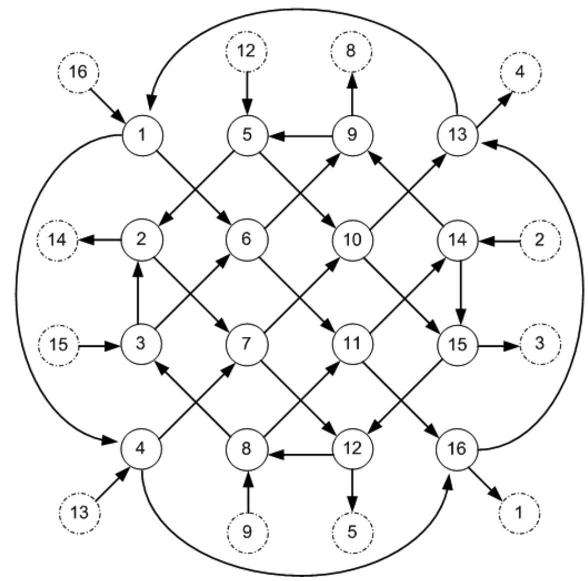

Fig. 2. Topology with 16 nodes acquired with random search, whose H is 2.8120.

Our conclusion is that the random search criterion was not significantly efficient and, moreover, was not so reliable in the task of providing information about the minimum value that the H parameter, for a given number of nodes, could achieve under a 2x2 configuration. In fact, such information could only be assuredly obtained with the aid of an approach founded on the idea of analyzing all the possible topologies with a given number of nodes and, in this set, selecting those whose H parameter is the smallest one.

B. Exhaustive Search

The method in which we consider all possible topologies and select the best one (or ones) is named exhaustive search. We applied this method to the problem at hand by generating all combinations of link distribution that preserve the 2x2 configuration, and, afterwards, analyzing each one with respect to the H parameter.

TABELA I

NUMBER OF TOPOLOGIES AND H PARAMETER

Number of Nodes	Number of Possible Topologies	Number of OMT	Smaller H Parameter
4	9	9	1.3333
5	216	216	1.5000
6	7560	540	1.6000
7	357435	12600	1.7619

This technique is always optimal, but, when the number of nodes is greater than seven, it can be considered to be impractical due to the explosive growth of the repertoire of possible combinations that give rise to topologies in 2x2 configuration with the number of nodes, as we can see in

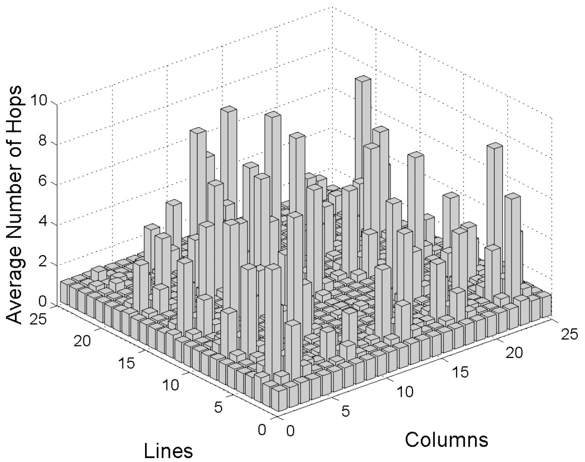

Fig. 3. 3-D Discrete surface of topologies.

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A → population of matrixes in 2x2 configuration
For each element in A: a,
b → matrix of zeros with the same dimension of a;
  While b is different of a,
      Copy a into b,
    For all the columns of b,
            Change the b-th column of b for other
            Verify the 2x2 configuration,
           Calculate the H parameter,
            If H < Ha,
            a = b and Ha = H;
            End
      End
      For all the rows of b,
            Change the b-th row of b for other
            Verify the 2x2 configuration,
           Calculate the H parameter,
            If H < Ha,
            a = b and Ha = H;
            End
      End
  End
End
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Fig. 4. Pseudocode for the mutation of matrices of an initial population.

Table I. The computational effort demanded for the exhaustive search of these topologies with a higher number of nodes is extremely great and the processing time is a factor that should be taken in consideration, since we can remain years searching of topologies without exhausting the possibilities for a number of nodes not so great. As the objective in searching optimized topologies is to use them in real networks, the number of nodes should be considerable, which justifies the main result of this paper: the development of a new bio-inspired search method for finding optimal mesh topologies.

C. Evolutionary Search

An alternative to solve this kind of problem is the utilization of evolutionary computation [4]-[6], a paradigm that is particularly suitable to solving large-scale combinatorial problems. Evolutionary optimization techniques are characterized by the use of concepts drawn from the modern evolution theory in a search process guided by a fitness function built in accordance with the aims underlying the problem to be solved. Our use of techniques belonging to this class was motivated by the fact that there is an enormous combinatorial diversity of the possible topologies as a function of the number of nodes. If each one of these topologies, which, from the standpoint of the evolutionary algorithm, are individuals belonging to a population of feasible solutions, have a single value for the H parameter, it will be possible for us to represent the universe of the H parameter distribution of all possible topologies as a 3-D discrete surface, as shown in Fig. 3. This distribution is of paramount importance, since, in the context of the evolutionary metaphor, it determines the fitness associated with each individual.

Having thus defined the population of solutions and the fitness measure, it is important to choose the genetic operators and a methodology for implementing a selection mechanism. In this work, we decided to employ an algorithm based exclusively on a mutation operator. This was motivated by the idea of reaching an adequate balance between efficiency and parsimony, and, furthermore, by analogies with approaches like evolution strategies [6] and some artificial immune systems [7]. The method starts from a reasonable set of possible topologies, here named initial population (connective matrices). The initial population, which is randomly generated, is formed by 10000 matrices having a degree of diversity as significant as possible. For each one of the individuals of the initial population we apply a mutation that consists of exchanging columns of the original matrix, always respecting the 2x2 configuration. After such changes, the H parameter is evaluated and we always implement the modification that leads to the smallest value of H. Modifications that lead to values of H greater than those associated with the original matrices are not implemented. After it becomes impossible to obtain a different topology with a better value of H by exchanging columns, we start an analogous procedure involving the rows.

When it is not possible to reach a better topology with the row changes, we restart the procedure with the columns and, this goes on until both column and row changes no longer originate improvements. This modus operandi will generate a mutated solution for each element of the population, which means that, from the initial population of 10000 individuals, we will be able to engender 10000 ”improved solutions”. The next step is to select the best individual of the universe of improved solutions and to use it as a basis for a new mutation process. This new process is characterized by the introduction of spurious row and column exchanges: each modification of this kind gives rise to a new individuals and the process is repeated up to 1000 times (we say ”up to”because the number of feasible solutions may be smaller than 1000). These new individuals, which are direct descendants of the fittest one, are then subjected to the same procedure applied to the original 10000-solution population, and, afterwards, the topology (or the topologies) with the minimum value of H is (are) selected. In Fig. 4 we show a simplified pseudo-code of the proposed algorithm.

Using the method of exhaustive search of optimized topologies, we obtained results up to a maximum of seven nodes due to the processing time demanded to obtain all the possible combinations. However, when we use the proposed

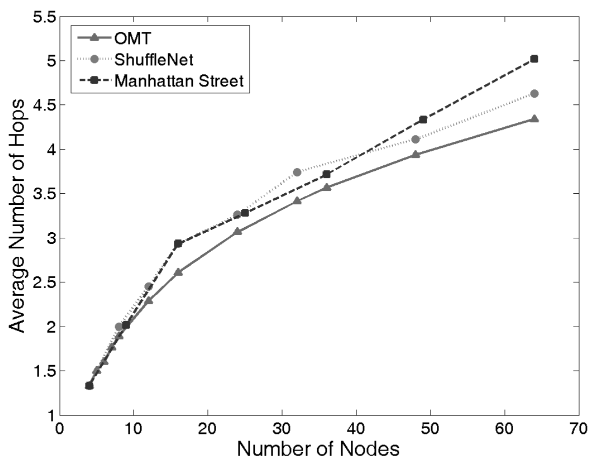


Fig. 5. Comparison of H parameter among topologies.

evolutionary approach, it was possible to determine optimized topologies up to 64 nodes, with the concrete possibility of expanding these results to superior node counts.

The efficiency of the evolutionary methodology becomes even more evident when we compare the results obtained via an exhaustive search of all combinations up to 7 nodes with the results produced by the evolutionary algorithm in the scenario. Both methods have converged to the same results, but the proposed algorithm required a remarkably smaller processing time (the simulation time in the same machine are reduced from the order of some days to the order of a few hours). The reader should notice that it is not possible to ensure that the evolutionary technique will always find the global optima. However, our performance analyses, together with the well-known search potential of evolutionary algorithms and the careful design (having in mind relevant notions like diversity maintenance) of the triad "initialization / operator / selection", give us confidence to assume that the methodology, even in the case of suboptimal convergence, will reach solutions of a very good quality.

III. Optimized Mesh Topologies

The average number of hops H for the OMT obtained via evolutionary search is presented for some node counts and compared with the topologies SN and MS in Fig. 5.

From Fig. 5, we can see that, for 16 nodes, the H parameter obtained by the proposed approach corresponds to an improvement of 12.4% with respect to the SN and MS topologies, which represents, for 2.5 Gb/s links, according to Equation 1, an effective gain of 3.4 Gb/s in the total capacity for OMT-16. Therefore, with small change in the network connections, a significant gain in capacity is achieved. One of the OMT topologies with 16 nodes (OMT-16) built with the aid of the evolutionary algorithm is presented in Fig. 6, and its H parameter is 2.6083. This gain for the OMT-16 is also very interesting since, although larger topologies as the OMT-64 present better gains in comparison with MS and SN, it is known that topologies with a great number of nodes can

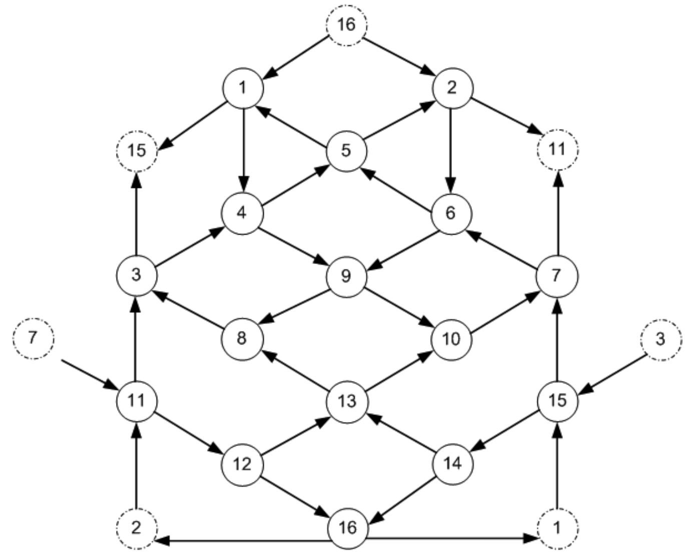


Fig. 6. An Optimized Mesh Topology for 16 nodes, OMT-16. Dashed nodes are virtual only to show connections.

present a bad performance when compared to small topologies, due to the elevated increase of applications (traffic flows) on each link, considering $N.(N-1)$ applications in the network.

The fact that the OMT-16 has the smallest value of H parameter in comparison with other topologies with 16 nodes is reflected by the maximum bitrate per application available for the network users. Therefore, since we have a gain in capacity when using OMT-16 in comparison with MS-16 and MS-16mod, it can be distributed among the network users, increasing the maximum bitrate available to each one. These calculations are based on statistical parameters and the actual maximum available bitrate on each link can be slightly different depending on the link load distribution.

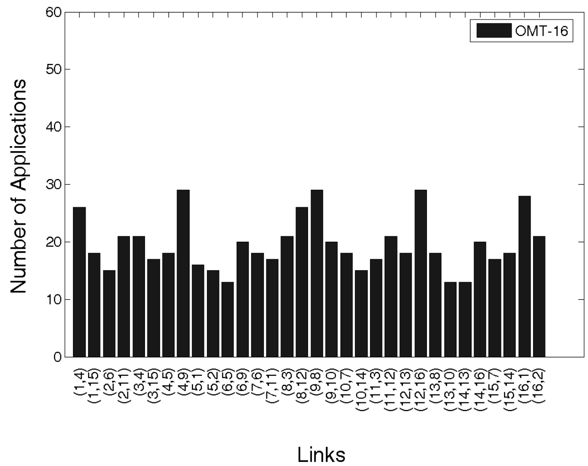
IV. Link Load Distribution

Another observed advantage, besides the smallest H, for the OMT is the better link load distribution. The occupation of each network link, in number of applications per link, is given in Fig. 7, for the MS-16 and the OMT-16 topologies, respectively. These applications are defined as the total amount of traffic flows sharing a given link.

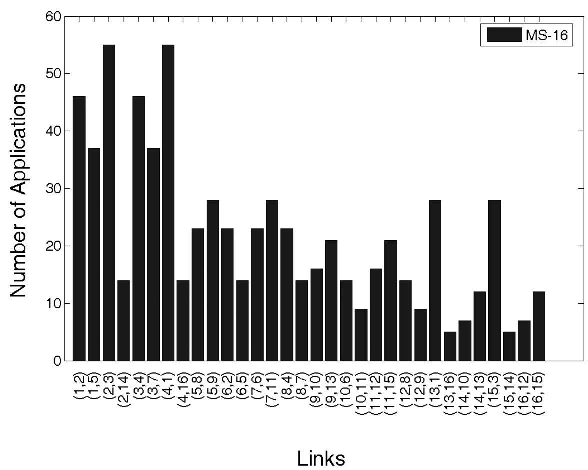
In this context we assume that all applications are in the same wavelength with some granularity, to show that application per link distribution will be a constraint for network performance. For instance, using links of 2.5 Gb/s for MS-16, we can have the more loaded links (2,3; 4,1) which have both 55 applications, with the maximum bitrate of 45.4 Mb/s, while in OMT-16 case, the limit will be 83.3 Mb/s.

The average number of applications per link is always the smallest for the case of OMT topologies than for other kinds of topologies, as we could expect by the smallest H. Some OMT, as OMT-16 and OMT-64 have also a Standard Deviation smallest than that presented by their analogue MS, as we can see by the statistical traffic distribution presented in Table II.

This can reflect, in a positive way, in the performance of these topologies under certain conditions of dynamic routing,



(a) OMT-16



(b) MS-16

Fig. 7. Topology Links usage.

since a Standard Deviation with low value indicates that the absolute number of applications per link has not a great variation relatively to the average.

Although some OMT present a Standard Deviation faintly superior to that of their analogue MS, as in 36-node case shown in Table II, the fact that the OMT present an average number of applications per link better than the MS is a preponderant factor to guarantee the better performance of the OMT.

V. Mesh versus Ring

An interesting characteristic of OMT-16, presented in Fig. 6 (whose link distribution is shown in Fig. 7 is the fact that this topology can be seen as functional a ring topology with 16 nodes. Thus, there is a path that a given application or packet can travel, visiting all the topology nodes just once, regressing the first visited node, closing the ring. This path is shown in Fig. 8, and this OMT-16 characteristic opens a possibility for the use of this kind of topology with synchronous and asynchronous network protocols.

TABELA II

Statistics of Traffic Distribution

Topology	Average H	Traffic Statistics (Applications per Link)	
		Average	Standard Deviation
MS-16	2.9333	22.00	13.85
OMT-16	2.6083	19.75	4.81
MS-36	3.7142	65.00	8.83
OMT-36	3.5650	62.38	9.86
MS-64	5.0158	158.00	66.94
OMT-64	4.3410	136.74	20.97

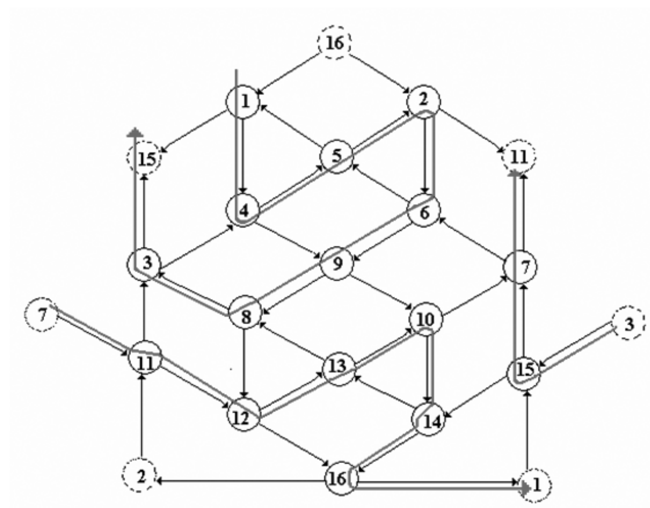


Fig. 8. OMT-16 seen as a functional ring topology.

We can see that the OMT present the smallest number of hops, but they are a little irregular when we analyze the geographical link distribution. However, this cannot be considered a problem, since we expect to use these topologies in access networks, where the link distance is short and we can avoid problems with signal degradation. These new topologies can be combined with a wide number of protocols and technologies and although the studies were carried out considering a single wavelength, we can also extend the results for systems with WDM, increasing the installed capacity. Our propose is to study the impact of these topologies also with systems using Optical Packet Switching and Optical Burst Switching, which will be the focus for further works.

VI. Conclusions

We have presented an evolutionary algorithm from which we successfully obtained Optimized Mesh Topologies for the H parameter. The OMT present a more homogeneous distribution of traffic and a better use of installed capacity. In our analysis, the OMT-16 presents a gain of about 12% in the total network capacity, due to the lower value of the H parameter, indicating that the installed capacity is better used. The increase of capacity can be shared among the network users, offering an extra amount of bitrate without extra technological costs. These solutions represent a more homogeneous distribution of traffic and a better use of installed capacity, presenting considerable gain in the total network capacity. We have focused our analysis in OMT-16, compared to MS-16, since it has a satisfactory number of nodes in the standpoint of Optical Networks. In further works we intend to widely study these topologies, with supplementary node counts and specific technologies, as WDM.

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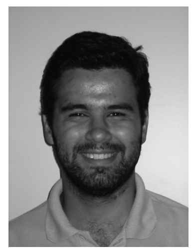

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