Evaluation of the Effects of Co-Channel Interference on the Bit Error Rate of Cellular Networks

Daniel Altamirano & Celso de Almeida

Abstract— This article presents a performance analysis of networks in the presence of co-channel interference with error correcting codes. The performance is evaluated in terms of the bit error rate (BER) for cellular networks using convolutional and turbo codes with BPSK modulation, for AWGN and fading channels. The impact of the co-channel interference is assessed theoretically and also simulated in scenarios where there is one prevailing interferer identical and synchronous to the target user. In order to show the error control coding capability in mitigating the co-channel interference effects, the performance of each coding scheme is presented as a function of the signal-to-interference ratio (S/I).

Index Terms— BER, Co-Channel Interference, Convolutional Codes, Turbo Codes.

Fig. 1.   System Model.

I. INTRODUCTION

With the crescent widespread of cellular networks, the performance evaluation of these systems in the presence of co-channel interference is an important theme that deserves consideration.

Co-channel interference refers to a contamination of an information signal by another undesired signal. This occurs when the radio communication antenna receiver picks up two or more signals that use the same network resources.

In the literature several papers evaluate the BER for different modulation schemes in the presence of co-channel interference and noise [1], [2], [3], [4]. However, in those papers error control coding was not included. Some other papers assess the BER using error control coding as an interference estimate or as a cancellation mechanism [5], [6].

In order to accomplish our goal, we extend the results of [1] by studying the effects of co-channel interference on networks that take advantage of error correcting codes. We assess the BER as a function of E_b/N_0 for networks using convolutional and turbo codes with BPSK modulation in AWGN channels and in fading channels. For convolutional codes, theoretical expressions are also derived.

This paper is organized as follows: section II shows the system description, section III provides BER expressions to evaluate the network performance, section IV shows the results and finally, the conclusions are presented in section V.

II. SYSTEM DESCRIPTION

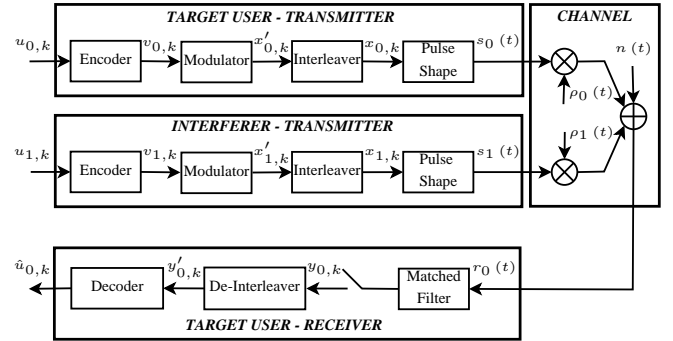Consider the system of Fig. 1. The user transmits u_k random information bits, which assume ± 1 with equal probability.

The information bits are encoded by a rate $R = 1/2$ convolutional or turbo encoder and generate the encoded sequence, $\boldsymbol{v}_k$. The encoder output is multiplexed, and each one of the encoded bits are modulated by a BPSK modulator, generating the $\mathbf{x'}_k$ sequence. The sequence $\mathbf{x'}_k$ undergoes an ideal interleaving with infinity depth, generating at its output the sequence $\mathbf{x}_k$. The pair interleaving/deinterleaving aims to ensure the fading samples are uncorrelated. Finally, the sequence of bits is pulse shaped and then transmitted.

The low-pass equivalent of the target user transmitted signal is given by:

$$s_0(t) = \sum_{k=-\infty}^{\infty} A x_{0,k} p(t - kT_b) \qquad (1)$$

where $p(t - kT_b)$ is the pulse format that satisfies the Nyquist criterion and has unitary energy, $\int_0^{T_b} p^2(t - kT_b) = 1$, where T_b is the bit interval, $R_b = 1/T_b$ is the bit rate and A is the amplitude.

There is a prevailing co-channel interferer whose transmitter performs the same procedure as the target user. Consequently, the low-pass equivalent of the interferer transmitted signal is given by:

$$s_1(t) = \sum_{k=-\infty}^{\infty} \alpha A x_{1,k} p(t - kT_s) \qquad (2)$$

where α is an amplitude factor of the interferer, that is used to vary the signal-to-interference ratio.

Both signals $s_0(t)$ and $s_1(t)$ are transmitted through a slow fading channel. Thus, at the target user receiver, the low-pass equivalent of the received signal is given by:

$$r(t) = \rho_0(t) s_0(t) + \rho_1(t) s_1(t) + n(t) \qquad (3)$$

where $\rho_0(t)$ and $\rho_1(t)$ are random processes that represent the fading of target user and interferer, respectively. Both present Rayleigh PDF with unitary mean power, *i.e.* $E\left[\rho_0^2(t)\right] = E\left[\rho_1^2(t)\right] = 1$, $n(t)$ is the low-pass equivalent of the additive white gaussian noise with variance $\sigma^2 = N_0 R_b/2$, where N_0 is the unilateral noise power spectral density. In case of no fading channel $\rho_0(t) = \rho_1(t) = 1$.

The average received energy per bit of target user is given by:

$$
\begin{aligned}
\overline{E_{b,0}} &= E\left[\rho_0^2(t)s_0^2(t)\right] = E\left[\rho_0^2(t)\right]E\left[s_0^2(t)\right] \\
&= \overline{\rho_0^2}\int_0^{T_b}s_0(t)\,dt = \overline{\rho_0^2}A^2 T_b
\end{aligned}
\tag{4}
$$

In the same fashion, the average received energy per bit of interferer is given by:

$$
\overline{E_{b,1}} = E\left[\rho_1^2(t)s_1^2(t)\right] = \overline{\rho_1^2}\alpha^2 A^2 T_b
\tag{5}
$$

Using coherent detection, the received signal passes through a matched filter with impulse response $p^*(-t)$. Supposing synchronism between the interferer and target user and considering no intersymbol interference at the bit time interval $(k-1)T_b \le t \le kT_b$, then the matched filter output sampled at $t = kT_b$ is given by:

$$
y_k = A\rho_{0,k}x_{0,k} + \alpha A\rho_{1,k}x_{1,k} + n_k
\tag{6}
$$

The receiver also considers a perfect channel estimator that obtains the fading amplitude at each bit time interval in order to be used at the decoding process. The received signal are decoded by the Viterbi algorithm for convolutinal codes and by BCJR algorithm for turbo codes. Finally, an estimate of the transmitted bit $\hat{u}_k$ is presented at the decoder output.

Without considering the noise in (6), the received signal instantaneous power is given by:

$$
P = \left(A\rho_{0,k}x_{0,k} + \alpha A\rho_{1,k}x_{1,k}\right)^2
\tag{7}
$$

Thus, the received mean power is given by:

$$
\begin{aligned}
\overline{P} &= A^2 E\left[\rho_{0,k}^2\right]E\left[x_{0,k}^2\right] + \alpha^2 A^2 E\left[\rho_{1,k}^2\right]E\left[x_{1,k}^2\right] \\
&+ 2A\alpha E\left[x_{0,k}x_{1,k}\right]E\left[\rho_{0,k}\rho_{1,k}\right]
\end{aligned}
$$

where $E\left[x_{0,k}x_{1,k}\right] = 0$, $E\left[\rho_{0,k}^2\right] = E\left[\rho_{1,k}^2\right] = 1$ and $E\left[x_{0,k}^2\right] = E\left[x_{1,k}^2\right] = P_x$ is the sequence mean power.

Therefore, the mean power is given by:

$$
\overline{P} = A^2 P_x + \alpha^2 A^2 P_x
\tag{8}
$$

where the target user mean power is $S = A^2 P_x$ and the interferer mean power is $I = \alpha^2 A^2 P_x$. As a consequence, the signal-to-interference ratio is given by:

$$
\frac{S}{I} = \frac{A^2 P_x}{\alpha^2 A^2 P_x} = \frac{1}{\alpha^2}
\tag{9}
$$

where $\alpha = 1/\sqrt{S/I}$ is the interferer amplitude factor.

III. BER ANALYSIS

A. AWGN Channels

The BER for a BPSK modulation without interference is given by [7]:

$$
P_b = Q\left(\sqrt{2\frac{E_b}{N_0}}\right)
\tag{10}
$$

where $E_b = A^2 T_b$ and $\sigma^2 = N_0/2T_b$, for A and T_b given in (1). BER expressions for different interference scenarios were derived in [1]. It was also shown that the case with only one co-channel interferer is the most significant one to consider. In that case, the BER is given by:

$$
P_b = \frac{1}{2}Q\left((1+\alpha)\sqrt{2\frac{E_b}{N_0}}\right) + \frac{1}{2}Q\left((1-\alpha)\sqrt{2\frac{E_b}{N_0}}\right)
\tag{11}
$$

For convolutional codes, the BER upper bound in terms of the bit WEF (Weight Enumerating Function) was derived in [8]. The BER expression is given by:

$$
P_b < \sum_{d=d_{free}}^{\infty} B_d Q\left(\sqrt{\frac{2dRE_b}{N_0}}\right)
\tag{12}
$$

where d_{free} is the code free distance, B_d are coefficients that represents the total number of nonzero information bits on all weight d paths divided by the number of information bits and R is the code rate.

For a channel with noise and one co-channel interferer, using the same methodology to obtain (11) and (12), we derive a BER upper bound for binary convolutional codes with BPSK modulation in the presence of co-channel interference. Thus, the BER upper bound is given by:

$$
\begin{aligned}
P_b &< \sum_{d=d_{free}}^{\infty} B_d \left[\frac{1}{2}Q\left((1+\alpha)\sqrt{\frac{2dRE_b}{N_0}}\right)\right. \\
&\left. + \frac{1}{2}Q\left((1-\alpha)\sqrt{\frac{2dRE_b}{N_0}}\right)\right]
\end{aligned}
\tag{13}
$$

For turbo codes, theoretical expressions for the BER can be obtained by using the distance properties of the constituents convolutional codes and the interleaver characteristics. However, these expressions are tight just for high signal-to-noise ratios (E_b/N_0) [8] but not for the "*waterfall*" region, where the fast performance increase of turbo codes is achieved. Therefore, only simulation results are shown to evaluate the performance of turbo codes.

B. Fading Channels

The average BER was also analysed in [7] and is given by:

$$
P_b = \int_0^{\infty} Q\left(\rho_0\sqrt{2\frac{E_b}{N_0}}\right)p(\rho_0)\,\partial\rho_0
\tag{14}
$$

where ρ_0 represents the fading. The average BER expression in (14) has a closed-form given by:

TABLE I

CONVOLUTIONAL CODE PARAMETERS

Generator Matrix	G=[5,7]
Code Rate	$R_c = \frac{1}{2}$
Overall Constraint Length	3
d_{free}	5
Bit-Weigth Enumerating Function	$B(x) = x^5 + 4x^6 + 12x^7 + 32x^8$

TABLE II

TURBO CODE PARAMETERS

Constituent Encoders	2 Identical RSC encoders
RSC Parameters	$R_c = \frac{1}{2}$ G=[5,7]
Interleaver	Random $N = 1024$
Puncture	Half of Parity Bits
Iterations	8

$$P_b = \frac{1}{2}Q\left(1 - \sqrt{\frac{E_b/N_0}{E_b/N_0 + 1}}\right) \qquad (15)$$

Performing the same procedure as [1] to obtain (10), we derive an expression for the average bit error rate in a fading channel by considering one co-channel interferer, that is given by:

$$
\begin{aligned}
P_b = & \frac{1}{2}\int_0^\infty \int_0^\infty \left[Q\left((\rho_0 + \alpha\rho_1)\sqrt{2\frac{E_b}{N_0}}\right) \right. \\
& + \left. Q\left((\rho_0 - \alpha\rho_1)\sqrt{2\frac{E_b}{N_0}}\right) \right] p(\rho_1)\,p(\rho_0)\,\partial\rho_1\partial\rho_0
\end{aligned}
\qquad (16)
$$

where ρ_0 and ρ_1 represent the fading of the target user and interferer, respectively. This expression has not a closed-form and needs to be assessed numerically.

For fading channels an average BER upper bound for convolutional codes in terms of the bit WEF was derived in [9] and is given by:

$$P_b < \sum_{d=d_{free}}^{\infty} \int_0^\infty B_d Q\left(\rho_0\sqrt{\frac{2dRE_b}{N_0}}\right) p(\rho_0)\,\partial\rho_0 \qquad (17)$$

For fading channels and one co-channel interferer, an expression using the same method to obtain (13) is not tight and a deeper analysis needs to be performed. Consequently, for this case just simulation results are presented.

The performance of turbo codes in fading channels and one co-channel interferer is also shown in the next section in simulation way.

IV. PERFORMANCE EVALUATION

The parameters of convolutional and turbo codes used in this paper are given in Tab. I and II, respectively.

Fig. 2 presents simulated results of BER as a function of E_b/N_0 for networks using convolutional codes in the presence of one co-channel interferer, for $S/I = 0, 3, 9, 24$ dB. For $S/I = 0$ dB there is a BER floor as for uncoded networks and the system performance can not be improved even increasing
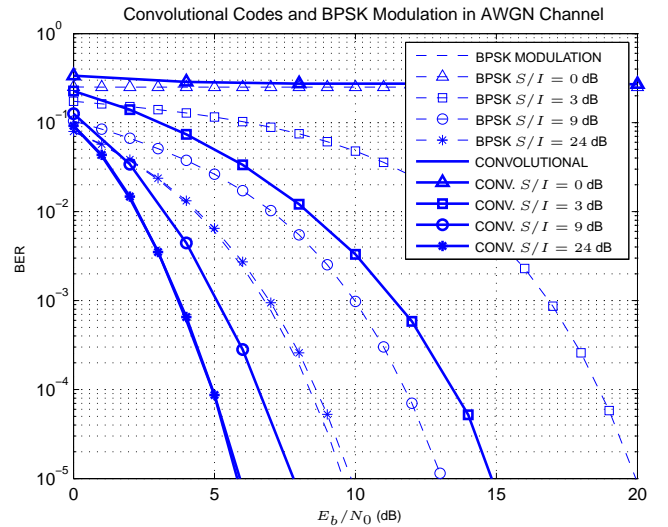

Fig. 2. BER as a function of E_b/N_0 for BPSK modulation and convolutional codes in AWGN channel with co-channel interference.
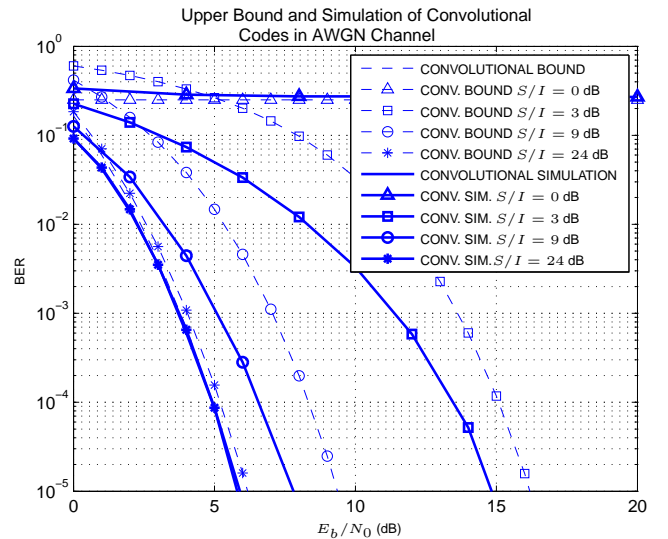

Fig. 3. BER upper bound and simulation as a function of E_b/N_0 for convolutional codes in AWGN channel with co-channel interference.

E_b/N_0. For $S/I = 3, 9$ dB we observe that BER decreases with E_b/N_0 with a cost of some dBs in relation to the free interference case. When $S/I = 24$ dB the case of no interference is achieved and so this happens for any $S/I > 24$. We also plot the results of an uncoded network with BPSK modulation in the presence of one interferer for comparison purposes.

Fig. 3 shows a comparison between the simulation results and the upper bound obtained in (13). In the case of no interference, there is a degradation of less than 1 dB for a BER of 1×10^{-5}, when comparing the upper bound with simulation results. In the presence of interference, the upper bound degradation is about 2 dB. The upper bound is not very tight for low E_b/N_0, but for high E_b/N_0 and S/I it asymptotically merges with the simulation results.

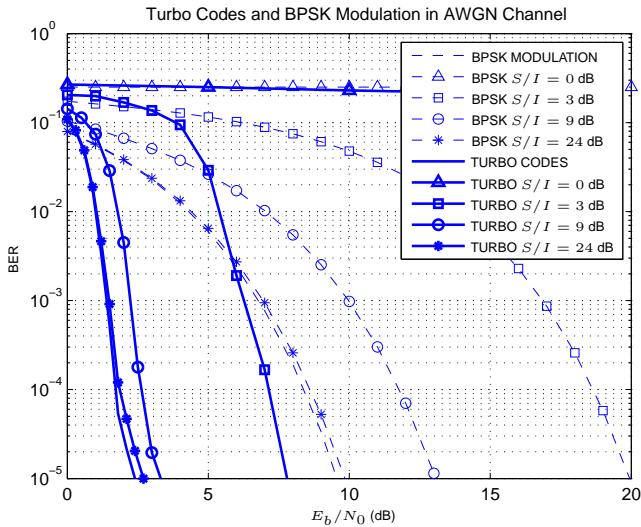Fig. 4 presents the simulated BER as a function of E_b/N_0

Fig. 4. BER as a function of E_b/N_0 for BPSK modulation and turbo codes in AWGN channel with co-channel interference.
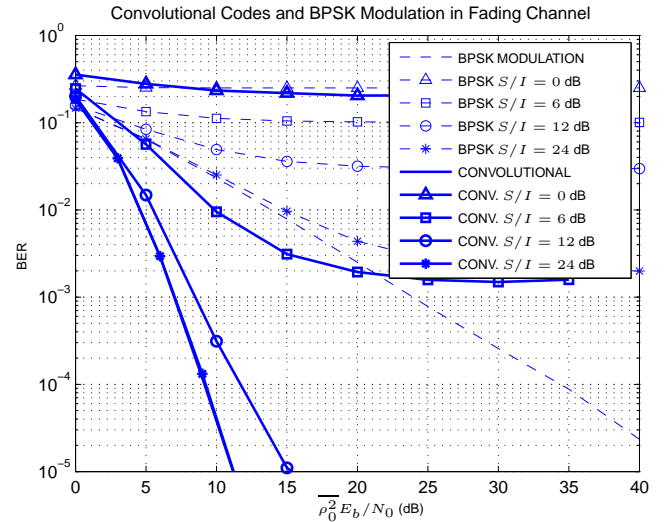

Fig. 5. BER as a function of $\overline{\rho_0^2}E_b/N_0$ for BPSK modulation and convolutional codes in Rayleigh fading channel with co-channel interference.

for networks using turbo codes, for $S/I = 0, 3, 9, 24$ dB. For $S/I = 0$ dB there is a BER floor, like in the uncoded networks. For $S/I = 3, 9$ dB we observe that the BER decreases rapidly at the cost of some dBs in relation to the free interference case. For $S/I = 24$ dB the case of no interference is achieved and so this happens for any $S/I > 24$ dB. For comparison purposes, we also plot the results of an uncoded network.

Using convolutional and turbo codes, for $S/I = 24$ and $S/I = 0$, we have observed the same behavior in relation to uncoded systems. For $S/I = 24$ the behavior is equivalent to a system without interference and for $S/I = 0$ there is a BER floor equal to $1/4$, that is shown in [1] for BPSK modulation.

Using simulation, Fig. 5 and 6 show the BER of convolutional and turbo codes, respectively as a function of mean E_b/N_0 by considering Rayleigh fading channel and co-channel interference. These figures also shown the BPSK modulation performance, which presents a BER floor for any S/I.

Fig. 5 presents simulated results for a network using convolutional codes for $S/I = 0, 6, 12, 24$ dB. As for uncoded networks in AWGN channels, for $S/I = 0$ dB there is a BER floor. For $S/I = 6$, dB we observe another BER floor that is lower in comparison with the uncoded case. For $S/I = 12$, dB the BER is improved and a BER floor is not observed until 1×10^{-5}. For $S/I \geq 24$ dB the case of no interference is approximately achieved for BER$\geq 1 \times 10^{-5}$, but a floor is expected for lower BER.

Fig. 6 presents the results for networks using the turbo codes for $S/I = 0, 6, 12, 24$ dB. Again, for $S/I = 0$ dB there is a BER floor and for $S/I \geq 24$ the no interference behavior is approximately achieved. For $S/I = 6, 12$, dB have good performance, although there is a floor for lower BER.

V. CONCLUSIONS

Due to the channel coding, the presented curves show that good coding gains and diversity are obtained even for low S/I values. Thus, we have proved the effectiveness of
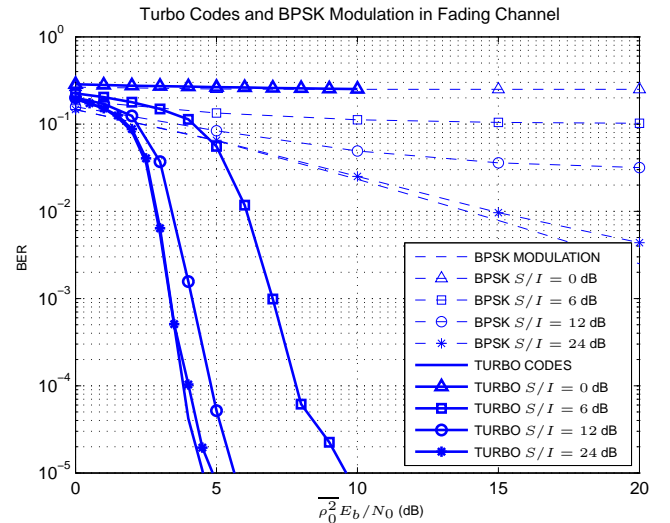

Fig. 6. BER as a function of $\overline{\rho_0^2}E_b/N_0$ for BPSK modulation and turbo codes in Rayleigh fading channel with co-channel interference.

error correcting codes in order to mitigate the co-channel interference.

At a cost of higher complexity, turbo codes are more robust to co-channel interference, and present better performance when compared with convolutional codes. Turbo codes present "*waterfall*" performance even in channels with co-channel interference.

It is important to emphasize that networks in fading channels with co-channel interference presents BER floors, that can be lowered but not eliminated by convolutional or turbo codes.

Future work could be interesting in obtaining expressions for fading channels and in assessing high spectral efficiency networks.

REFERENCES

[1] D. Altamirano and C. de Almeida, "Evaluation of the Effects of the Co-Channel Interference on the Bit Error Rate of Cellular Systems for BPSK Modulation," Proceedings of International Telecommunications Symposium, 2010.

[2] D. Altamirano, G. Quelal, D. Moya, E. Benitez, and C. de Almeida, "Evaluation of the Effects of Co-Channel Interference on the Bit Error Rate for QPSK and M-QAM Modulations," Proceedings of International Telecommunications Symposium, 2010.

[3] V. Tralli and R. Verdone, "Performance Characterization of Digital Transmission Systems with Cochannel Interference," IEEE Trans. Veh. Technol., vol. 48, pp. 733–745, May 1999.

[4] H. Roelofs, R. Srinivasan and W. van Etten, "Performance Estimation of M-ary PSK in Co-Channel Interference Using Fast Simulation," IEEE Proc.-Commun., Vol. 150, no. 5, pp. 335-340, Oct. 2003.

[5] Y. Sanada, W. Qiang, "A co-channel interference cancellation technique using orthogonal convolutional codes," Communications, IEEE Transactions on , vol.44, no.5, pp.549-556, May 1996.

[6] H. Hisata, J. Horikoshi, "Turbo codes with RLS-MLSE channel estimation in the co-channel interference environment," Personal, Indoor and Mobile Radio Communications, 2000. PIMRC 2000. The 11th IEEE International Symposium on , vol.2, no., pp.903-907 vol.2, 2000.

[7] J. G Proakis, Digital Communications, McGraw-Hill, 2008.

[8] S. Lin e D. S. Costello Jr., "Error Control Coding: Fundamentals and Applications." Prentice Hall, 2004.

[9] J. Modestino and S. Mui, "Convolutional code performance in the Rician fading channel," IEEE Trans. on Comm., vol. 24, no. 6, pp. 592-606, Jun., 1976.

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