Double Walsh-Hadamard Transform OFDM System

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***Abstract* — The aim of this paper is to present a novel technique to reduce the PAPR in OFDM signals. The main advantage of the proposed technique, named Double WHT-OFDM reduces the SER in frequency selective channels without incrementing the error floor in non-linear channels, which happens in the conventional WHT-OFDM. Therefore, the technique proposed in this paper is a simple and efficient solution to allow the use of WHT combined with OFDM in frequency-selective non-linear channels.**

I. Introduction

Wireless communication systems are evolving to be the main solution for future Internet networks. Today, 4° Generation of mobile communication systems, such as LTE (Long Term Evolution) [1], promise high data rates that are compatible with the a large number of application, i. e., video and photo sharing, online gaming, Web surfing and others. However, since 4° Generation systems use reserved frequency bands to operate, their overall capacity is limited. Also, operators must have a license to provide Internet access. It means that deploying these systems in low populated areas is not economically feasible.

IEEE 802.22 [2] rises as the first proposal to provide ADSL (Asymmetric Digital Subscriber Line) [3] compatible data rates in rural areas using wireless communication. In order to achieve this goal, IEEE 802.22 uses cognitive radio technology [4] to use UHF band frequencies as secondary incumbent. Since DTV broadcasters, wireless microphones and others radio communication systems are the primary users of the UHF channels, the user's terminals and the base station of the IEEE 802.22 system shall use vacant channels without interfering with the primary incumbents. In band and out band spectral sensing techniques [5] shall be implemented to guarantee non-interference operation.

IEEE 802.22 employs OFDM (Orthogonal Frequency Division Multiplexing) [6] as aerial interface. OFDM is suitable for broadband wireless communication systems because its robustness against frequency selective channels. An important drawback of this technique is the high PAPR (Peak to Average Power Ratio) [7] presented by the OFDM signal. The high peaks usually drive the power amplifier into saturation, clipping the transmitted waveform and introducing in-band and out-band interference. The in-band interference reduces the system performance while the out-band interference affects other systems that operates at surrounding frequencies. This is a critical situation for the IEEE 802.22 since the out-band emissions are likely to cause a prohibitive interference in primary users.

There are several techniques to reduce the PAPR in OFDM signals [8]-[13]. One proposal uses the Wash-Hadamard Transform (WHT) [14] to reduce the probability of high values of PAPR. This solution increases the performance of OFDM system in frequency selective channels and reduces the probability of the PAPR to assume high values [15]. However, given that a clipping has occurred, the impact in the WHT-OFDM is more severe than conventional OFDM signal [13].

The main goal of this paper is to present a new scheme to reduce the PAPR of OFDM signals based on Walsh-Hadamard Transform. Different Walsh-Hadamard matrices will be used in frequency and time domains, forcing the reduction of PAPR. This approach maintains the performance in frequency selective channels and increases the performance in non-linear channels, allowing the WHT-OFDM to be efficiently used in cognitive radio systems.

II. WHT-OFDM Principles

The WHT-OFDM consist on applying the Walsh-Hadamard Transform in the data symbols prior applying the IDFT (Inverse Discrete Fourier Transform) [16]. Wash-Hadamard Transform of a N -length vector, $\vec{c}_n$, is given by

$$\vec{S}_\Omega = \frac{1}{\sqrt{N}} \cdot \vec{c}_n \cdot \Omega_N \quad , \tag{1}$$

where

$$\Omega_N = \begin{bmatrix} \Omega_{N/2} & \Omega_{N/2} \\ \Omega_{N/2} & -\Omega_{N/2} \end{bmatrix} , \tag{2}$$

is the Walsh-Hadamard matrix, N is a base-2 integer and $\Omega_1 = 1$.

Assuming $\vec{c}_n$ is a data vector with N random symbols from an *M*-QAM (*M*-ary Quadrature Amplitude Modulation) constellation [17] it is possible to conclude that each element from vector $\vec{S}_\Omega$ is a linear combination of all symbols from $\vec{c}_n$. Thus, each subcarrier will be modulated by a component that carries information of all data symbols. It means that if a subcarrier is suppressed by a frequency selective channel all symbols will suffer a slight interference, instead a single symbol suffering a severe attenuation. This procedure increases the performance of the system in frequency selective channels. The vector $\vec{s}_\Omega$ is applied to the IDFT (Inverse Discrete Fourier Trasnform) block to generate the time domain samples of the WHT-OFDM symbols. Thus, the time domain samples are given by

$$\vec{s}_\Omega = \frac{1}{N} \sum_{k=0}^{N-1} \vec{S}_{\Omega_k} \cdot e^{j 2\pi \frac{k}{N} n} \quad . \tag{3}$$

Cyclic Prefix [1] samples are added to time domain signal and the signal is transmitted in a frequency selective non-linear channel to the receiver.

On the receiver side, the cyclic prefix is removed and the time domain samples received from the communication channel, $\vec{r}_\Omega$, are applied to the DFT (Discrete Fourier Transform), leading to

$$\vec{R}_\Omega = \sum_{k=0}^{N-1} \vec{r}_{\Omega_k} \cdot e^{-j2\pi \frac{k}{N} n} \quad . \tag{4}$$

The received vector in the time domain is applied to the IWHT (Inverse Walsh Hadamard Transform), which also consists on multiplying the vector by the same Wash-Hadamard matrix used in the transmission. In others words,

$$\vec{c}_n' = \frac{1}{\sqrt{N}} \cdot \vec{R}_\Omega \cdot \Omega_N \quad . \tag{5}$$

Finally, the received symbols are detected and the estimated bits are delivered to destination. Figure 1 presents the block diagram of the WHT-OFDM transmitter, while Figure 2 presents the block diagram of the WHT-OFDM receiver.

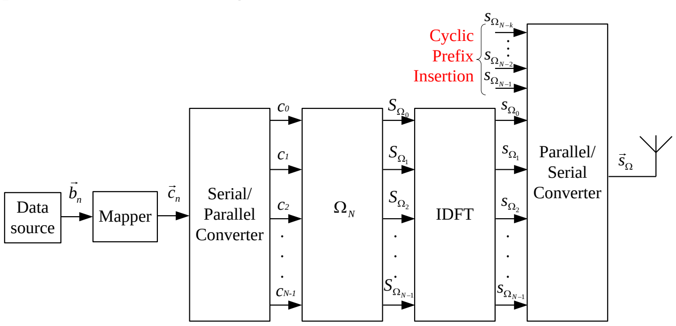

Fig.1. Block diagram of the WHT-OFDM transmitter.

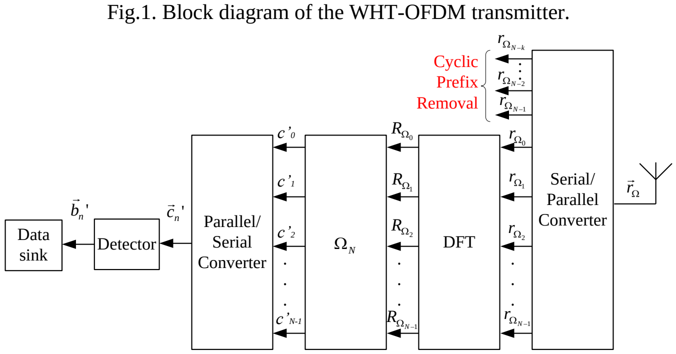

Fig.2. Block diagram of the WHT-OFDM receiver.

The use of WHT combined with OFDM increases the performance of the system in frequency selective channels and reduces the PAPR, as can be seen in Figure 3 and Figure 4, respectively. The considerable performance gain in frequency selective channels makes WHT-OFDM an interesting solution for high data rate wireless systems. At first glance, WHT-OFDM also seems to be a good solution for non-linear channels because of the PAPR reduction. However, the ICI caused by clipping the high peaks of the signal is more severe on WHT-OFDM than on conventional OFDM. Figure 5 shows the probability of the root mean square error between in the received data symbols be larger than a threshold, considering conventional OFDM and WHT-OFDM in a non-linear channel. Noise has been neglected in this result.

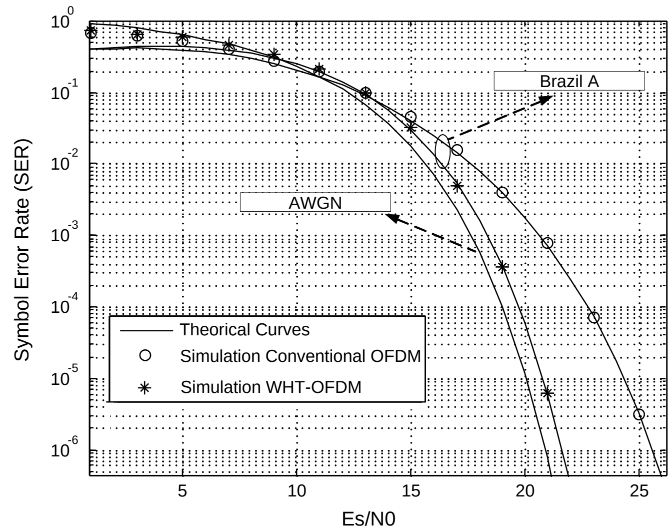

Fig.3. Symbol error rate of OFDM and WHT-OFDM on frequency selective channel.

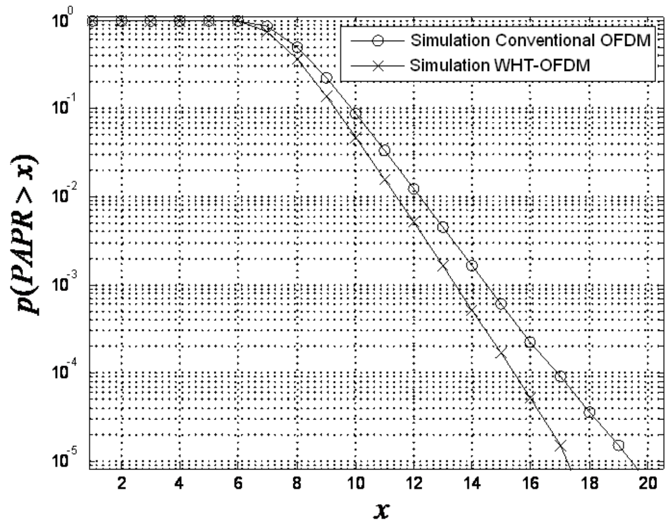

Fig.4. Complementary Cumulative Distribution Function of the PAPR of conventional OFDM symbols and WHT-OFDM symbols.

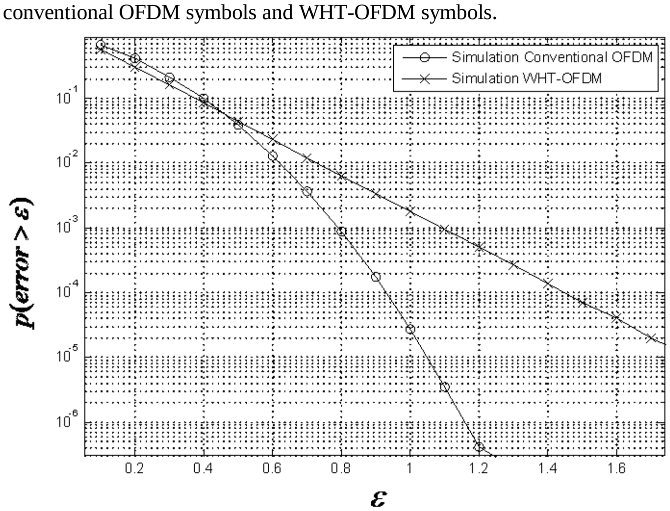

Fig.5. Cumulative Density Function of the modulation error.

Although the WHT-OFDM presents a lower PAPR, resulting in a lower clipping probability, the symbol error rate is very high, given that a clipping has occurred. This results on the high symbol error floor that can be seen in Figure 6.

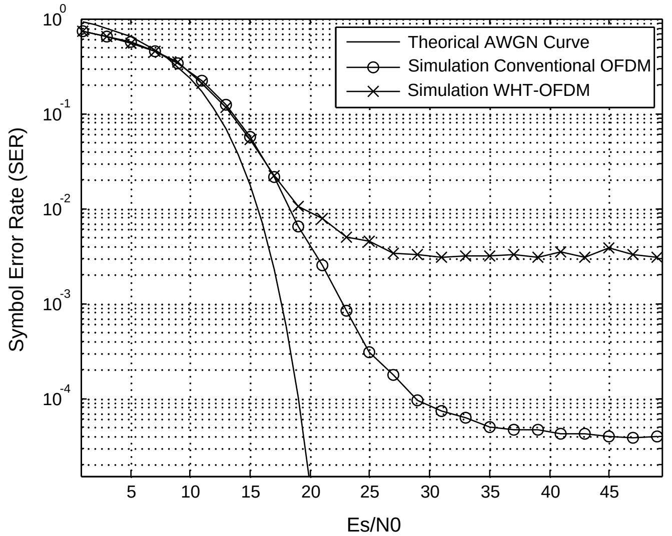

Fig.6. Symbol error rate of conventional OFDM and WHT-OFDM on non-linear AWGN channel.

III. PROPOSED SYSTEM

The results presented in the past section show that WHT-OFDM is an interesting solution for linear frequency selective channels, but it suffers from a more severe interference on non-linear channels than conventional OFDM. Since the non-linearity of the channel is mainly due to the power amplifier, it is not possible to assure that a device will not clip the signal because this probability will depend on choices made by the manufacturer, such as amplifier class [18], pre-distortion system [19] and others.

In order to obtain the performance gain in frequency selective channels provided by the WHT and to avoid the high error floor in non-linear channels, it is necessary to reduce the influence of clipping in the WHT-OFDM signal. One proposal to achieve this goal is to use a Double Walsh-Hadamard Transform, based on different Walsh-Hadamard Matrices that can be obtained by permuting the columns of the matrix presented in (2). Let Ω_N^{kl} be a Walsh-Hadamard matrix where columns k and l have been permutated. In this case, the WHT can be redefined as

$$\vec{S}_\Omega = \frac{1}{\sqrt{N}} \cdot \vec{c}_n \cdot \Omega_N^{kl} \ , \tag{6}$$

while the IWHT is given by [1]

$$\vec{c}_n = \frac{1}{\sqrt{N}} \cdot \vec{R}_\Omega \cdot \left(\Omega_N^{kl}\right)^T \ . \tag{7}$$

where $(\cdot)^T$ denotes the transpose operation.

In the proposed scheme, the data symbols are parallelized in a vector with N elements. The real component of this vector multiplies the Walsh-Hadamard matrix presented in (2), while the imaginary part multiplies the matrix given by (6). After both WHTs, the real and imaginary parts are combined to generate the data vector $\vec{S}_\Omega$, which is applied to IDFT. The cyclic prefix is added to the symbol in the time domain before its transmission. Figure 7 presents the block diagram of the transmitter, while Figure 8 presents the block diagram of the receiver.

On the receiver side, the signal is sampled and the cyclic prefix is removed. The useful samples are applied to the DFT, resulting in the vector $\vec{R}_\Omega$, whose real part is delivered to the original IWHT, while its imaginary part is applied to the IWHT with the modified Wash Hadamard matrix. The resulting in-phase and quadrature components are combined and the received symbols are delivered to the decision process, resulting in an estimative of the transmitted bits.

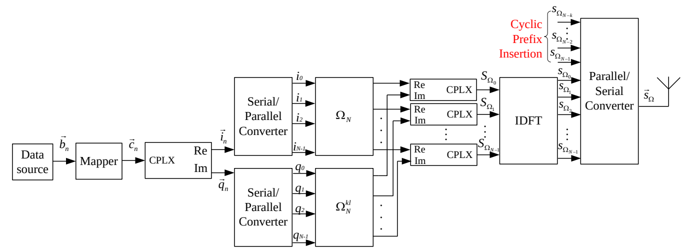

Fig.7. Block diagram of the Double WHT-OFDM transmitter.

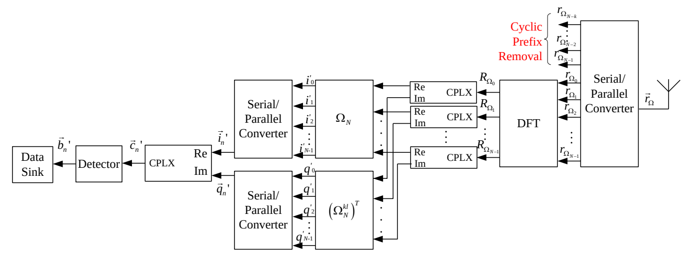

Fig.8. Block diagram of the Double WHT-OFDM receiver.

IV. PERFORMANCE OVER NON-LINEAR AWGN CHANNEL

Figure 9 presents the performance of the Double WHT-OFDM in a non-linear AWGN channel, while Table I presents the simulation parameters used to obtain this result.

TABLE I
PARAMETERS USED IN SIMULATIONS.

PARAMETER	VALUE
Mapper	16-QAM
# subcarriers	2048
Clipping threshold	2 times OFDM standard Deviation

From Figure 9 it is possible to conclude that the performance of Double WHT-OFDM approaches the performance of conventional OFDM in non-linear AWGN channels, while the WHT-OFDM presents an error floor significantly higher than conventional OFDM or Double WHT-OFDM. Since the principle of distributing the data symbols in all subcarriers is still present in the Double WHT-OFDM, this technique suffers from additional ICI distortion when compared with OFDM over non-linear AWGN channels.

The error in the received symbol due the clipping has also been analyzed for the Double WHT-OFDM. Figure 10 shows that the modulation error due the clipping of the proposed technique approaches the modulation error of the conventional OFDM, which allows one to conclude that the new technique leads to a better SER performance over non-linear AWGN channel, as it has been observed in Figure 9.

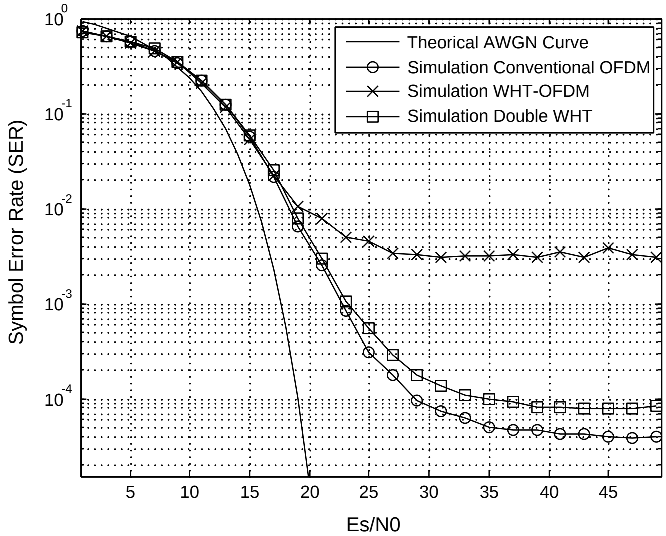

Fig.9. Symbol error rate of conventional OFDM, WHT-OFDM and Double WHT over non-linear AWGN channel.

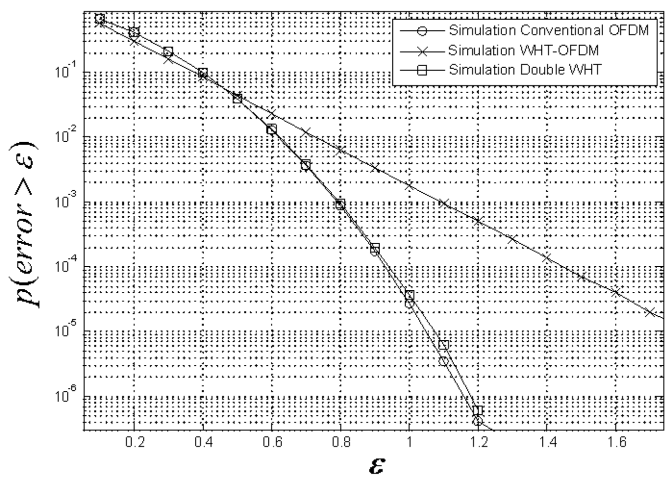

Fig.10. Cumulative density functions of the modulation error.

V. PERFORMANCE OVER FREQUENCY-SELECTIVE CHANNELS

The main advantage of the WHT-OFDM is the better SER performance over frequency-selective channels. The drawback of the WHT-OFDM is the high error floor caused by the clipping. The main aim of this session is to explore the SER performance of Double WHT-OFDM in frequency-selective non-linear channels. Three different channels with delay profiles presented in Table II will be considered [20]. These delay profiles have been chosen because they are typically used to model broadband communication over UHF channels [21].

Figure 11 presents the SER performance of Double WHT-OFDM over channel Brazil A and compares its performance with WHT-OFDM and conventional OFDM. The same conclusion can be drawn from Figure 12 and 13, which respectively present the SER performance over channel Brazil C and Brazil E. Since these channels are more severe, with smaller coherence bandwidth, the better performance of Double WHT-OFDM for low SNR, when compared with conventional OFDM, becomes more evident.

TABLE II
CHANNEL DELAY PROFILES.

Profile	Description	Path 1	Path 2	Path 3	Path 4	Path 5	Path 6
Brazil A	Delay (μs)	0	0.15	2.22	3.05	5.86	5.93
	Atten. (dB)	0	13.8	16.2	14.9	13.6	16.4
Brazil C	Delay (μs)	0	0.089	0.419	1.506	2.322	2.799
	Atten. (dB)	2.8	0	3.8	0.1	2.5	1.3
Brazil E	Delay (μs)	0	1	2	-	-	-
	Atten. (dB)	0	0	0	-	-	-

Two major conclusions can be drawn from Figure 11. The first one is that Double WHT-OFDM outperformances conventional OFDM for low values of SNR, where the clipping effect is not significant compared with the interferences introduced by the channel and noise. The reason for this result is the expected gain that the WHT introduces in frequency selective channels. For high SNR, conventional OFDM outperforms Double WHT-OFDM because of the relevant non-linear interferences. In this case, the slightly higher modulation error due clipping present in Double WHT-OFDM leads to a higher error floor when compared with conventional OFDM. Nevertheless, the performance of Double WHT-OFDM is significantly better than WHT-OFDM.

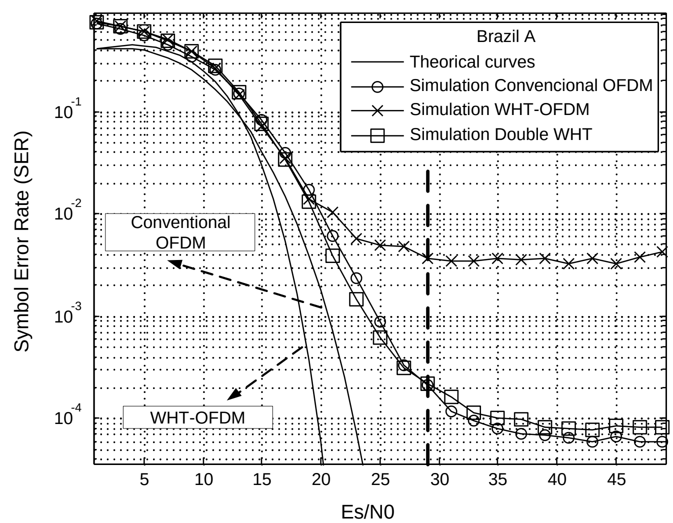

Fig.11. Symbol error rate of conventional OFDM, WHT-OFDM and Double WHT over non-linear Brazil A channel.

Figures 14 and 15 presents the modulation error due clipping for conventional OFDM, WHT-OFDM and WHT-OFDM over channel Brazil A for SNR=45 and SNR=25, respectively. From Figure 14 it is clear that the modulation error for conventional OFDM is smaller than the modulation error for Double-WHT for high values of SNR. However, observing Figure 15, it is clear that the modulation error of Double WHT-OFDM is smaller than the modulation error of conventional OFDM for low values of SNR.

VI. CONCLUSIONS

OFDM system is being largely used to provide a reliable high data rate air interface to new mobile communication systems. One of its major drawbacks is the high PAPR, which reduces the performance of the power amplifiers and may cause severe degradation in the system performance in case of saturation of the amplifier. Also, clipping the OFDM signal results in out of band transmission that may cause interference in others users. This interference is highly harmful in Cognitive Radio systems, since the interference in primary users is not tolerable.

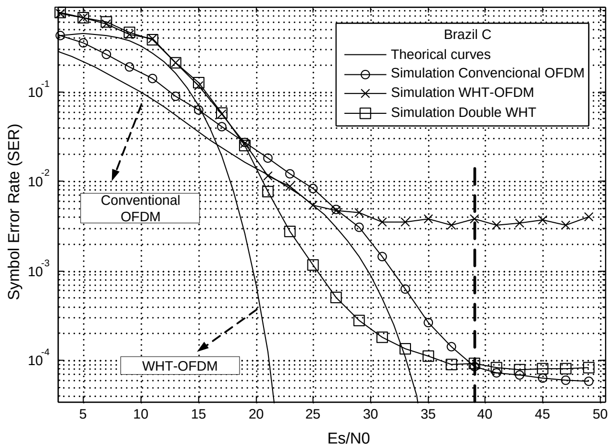

Fig.12. Symbol error rate of conventional OFDM, WHT-OFDM and Double WHT over non-linear Brazil C channel.

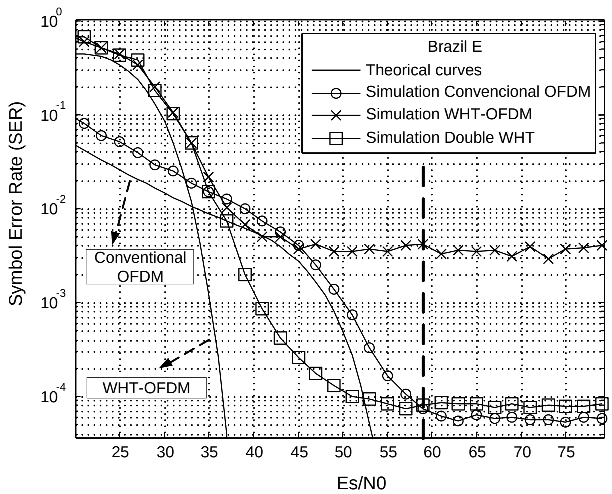

Fig.13. Symbol error rate of conventional OFDM, WHT-OFDM and Double WHT over non-linear Brazil E channel.

The use of WHT combined with OFDM presents two major advantages: i) reduction of PAPR and; ii) better performance in frequency selective channels. Despite these advantages, the use of WHT-OFDM in non-linear channels is not recommended, because WHT-OFDM symbols are more sensitive to Intracarrier Interference introduced by the nonlinearities. Thus, the use of WHT-OFDM shall be restricted to linear channels.

This paper has presented a new scheme that combines two different Walsh-Hadamard Transforms to produce a Double WHT-OFDM system. Simulations results have shown that the use of two different matrices to perform independent Walsh Hadamard Transforms of the real and imaginary components of the serial data symbols culminates in a more robust WHT-OFDM system against nonlinearities.

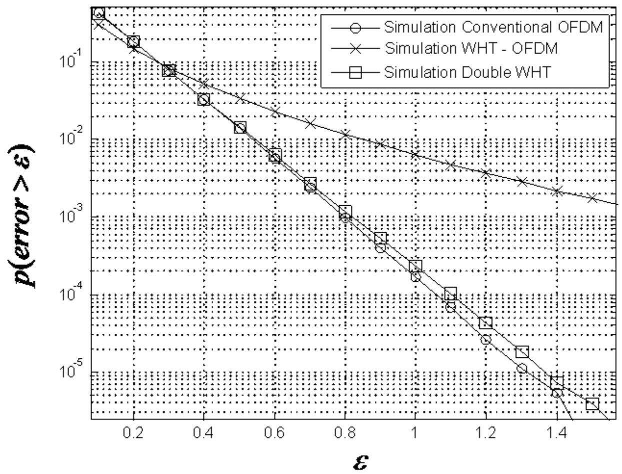

Fig.14. Cumulative density functions of the modulation error. Brazil A and SNR=45 dB.

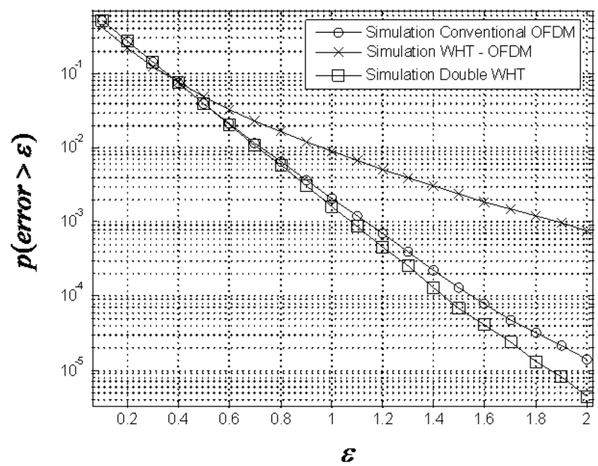

Fig.15. Cumulative density functions of the modulation error. Brazil A and SNR=25 dB.

The simulation results have shown that Double WHT-OFDM has a lower clipping probability than OFDM, but the effects of clipping on the new technique is more severe than the effect of clipping on the conventional technique. However, this effect is significantly smaller than the error observed in WHT-OFDM. In fact, the error floor presented by Double WHT-OFDM over non-linear channels is slightly higher than the error floor presented by conventional OFDM.

The SER performance curves also show that Double WHT-OFDM outperforms OFDM in non-linear frequency-selective channels for low values of SNR, where the interferences introduced by the clipping are not more significant than the interferences introduced by the channel impulse response and noise. The main conclusion is that Double WHT increases the performance of the system in frequency selective channels without resulting in severe performance degradation in non-linear channels.

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